Equivalence between modified symplectic Gram-Schmidt and Householder SR algorithms

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The symplectic Gram-Schmidt (SGS) orthogonalization process is a crutial procedure for some important structure-preserving methods in linear algebra. The algorithm perfoms a factorization $A = SR$, where the ordered columns of the matrix S form a symplectic basis of the range of A, and R is J-upper triangular. There exist two versions of SGS, the classical (CSGS) and the modified (MSGS). Both are equivalent in exact arithmetic, but have different numerical behaviors. In this paper, a numerical equivalence is showed between the MSGS algorithm and Householder SR algorithm applied to an embedded matrix obtained from A by adding two blocks of zeros in the middle and the bottom of the matrix A. The latter algorithm is based on transformations which are symplectic and rank-one modification of the identity (symplectic Householder transformations).

References

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