Series acceleration through precise remainder asymptotics

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Many modern series acceleration methods are built around an input of a sequence $\{s_n\}$ of partial sums of the series, and a remainder estimate ρ_n; they then proceed by extrapolating from the s_n and ρ_n a limit or antilimit of the sequence, based on the corrections to the remainder estimate assumed by the particular acceleration method. Generally, the remainder estimates are quite simple, at best correct to leading order, because precise forms for the remainders are usually not known. In this talk we show that whenever an asymptotic expansion for the ratio of the *terms* of the series is known in inverse powers of n, we can derive an asymptotic expansion $\rho_n \sim \omega_n \sum_k c_k/n^k$. Here the leading term ω_n may contain a power or factorial in n and can be determined analytically, but more importantly the asymptotic coefficients c_k can be explicitly computed to any desired order from the asymptotic expansion of the term ratio. We outline the derivation of this method and the circumstances under which it either accelerates the convergence or improves the divergence of an analytic series, and we give several examples of its application: to generalized hypergeometric functions, zeta functions, and some slowly convergent Fourier series. We thereby extend the results of [1], which considered only generalized hypergeometric series ${}_{q+1}F_q$.

References

[1] J. L. Willis, *Acceleration of generalized hypergeometric functions through precise remainder asymptotics*, preprint arxiv.org/1102.3003.