Markov's theorem and perturbed recurrence relations

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Let $j \geq 0$ be a fixed integer , $\{a_k\}_k$, $\{b_k\}_k$ be real sequences with $\forall k, b_k > 0$ and $N_{-1}(j,z) \equiv 0,\ N_0(j,z) \equiv 1$,

$$\forall k \geq 0,\ N_{k+1}(j,z) = (z - a_{k+j})N_k(j,z) - b_{k+j}N_{k-1}(j,z) \tag{1}$$

then we have the following classical result:

Theorem (Markov) If the sequence $\{a_k\}_k$ and $\{b_k\}_k$ are bounded then there exits an unique positive measure ν and a compact set $supp(\nu)$ in $\mathbf{R}$ such that

$$\forall z \in \mathbf{C} - supp(\nu) \quad \lim_{k\to\infty} \frac{N_{k-1}(1,z)}{N_k(z)} = c \int_{supp(\nu)} \frac{d\nu(x)}{z-x}$$

and this convergence is uniform on every compact subset of the complex plane that does not intersect $supp(\nu)$.
It is well known that the above result plays a fundamental role in the study of orthogonal polynomials, in the asymptotic behaviour of certain continous fractions and in many other types of numerical applications. Here, we give some new results about this Theorem when recurrence relation (1) is perturbed as well as its order. This work is the sequel of the paper [1].

References

[1] E. Leopold, *Asymptotic behaviour and general recurrence relations*, Appl. Numer. Math., 60 (2010), pp. 1352–1363.