Error bounds of Gauss-type quadratures with Bernstein-Szegő weights

A. V. Pejčev and M. M. Spalević
Department of Mathematics
University of Beograd
Faculty of Mechanical Engineering
Kraljice Marije 16, 11120 Belgrade 35, Serbia
apejcev@mas.bg.ac.rs

The kernels $K_n(z) = K_n(z, w)$ in the remainder terms $R_n(f)$ of the Gauss-type quadrature formulas

$$\int_{-1}^{1} f(t)w(t)\,dt = G_n[f] + R_n(f), \quad G_n[f] = \sum_{\nu=1}^{n} \lambda_\nu f(\tau_\nu) \quad (n \in \mathbf{N})$$

for analytic functions inside elliptical contours $\mathcal{E}_\rho$ with foci at ∓ 1 and the sum of semi-axes $\rho > 1$, where w is a nonnegative and integrable weight function of Bernstein-Szegő type, are studied. The derivation of effective bounds for $|R_n(f)|$ is possible if good estimates for $\max_{z\in\mathcal{E}_\rho} |K_n(z)|$ are available, especially if we know the location of the extremal point $\eta \in \mathcal{E}_\rho$ at which $|K_n|$ attains its maximum. In such a case, instead of looking for upper bounds for $\max_{z\in\mathcal{E}_\rho} |K_n(z)|$ one can simply try to calculate $|K_n(\eta, w)|$. In the case under consideration the location on the elliptic contours where the modulus of the kernel attains its maximum value is investigated. This leads to effective bounds for $|R_n(f)|$.