Modelling of Ring Resonators with Magneto-optic Materials using the Finite Element Method

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Abstract: In this work we focus on the modal analysis of ring resonators realized with magneto-optic materials [1] by solving numerically the Maxwell's equations. Considering the lossless case (including no bending loss), we have implemented the finite element method in a cylindrical coordinate systems using the node-based formulation with second order shape functions. The penalty function [2] has been introduced to move out the spurious solutions and the final quadratic eigenvalue problem has been solved using the Krylov method.

Introduction

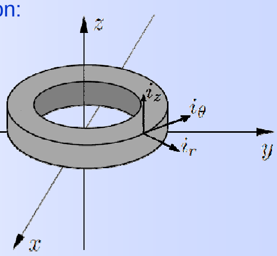
In the last years, several photonics research groups have focus their attention on ring resonators and their very promising applications in the field of integrated optics. Due to their superior selectivity, compactness, and possibility of dense integration, micro-ring and micro-disk cavities have been used to realize add-drop filters, switches, format converters, modulators and demodulators. In order to optimize the device structure and to improve their performance a rigorous analysis and modeling approach is very important, often requiring numerical techniques.

Rayleigh-Ritz Formulation

To find the electromagnetic field distribution and resonant frequencies of the ring cavity, we have to solve the Maxwell equation:

$$\nabla\times\underline{\mathbf{E}} = -j\omega\underline{\underline{\boldsymbol{\mu}}}\underline{\mathbf{H}}, \quad \nabla\times\underline{\mathbf{H}} = j\omega\underline{\underline{\boldsymbol{\varepsilon}}}\underline{\mathbf{E}}+\underline{\mathbf{J}},$$

$$\nabla\cdot\underline{\mathbf{E}} = \frac{\rho}{\varepsilon_0}, \qquad \nabla\cdot\underline{\mathbf{H}} = 0.$$



E, **H** are the electric and magnetic field;
J, ρ are the electric current density and the charge density;
ε, **μ** are the permittivity and permeability of the medium.

Hp. 1

$$\underline{\mathbf{E}}(r,\theta,z;t) = \underline{\mathbf{E}}(r,z)\,e^{j\omega t - j\gamma\theta} \quad \text{and} \quad \underline{\mathbf{H}}(r,\theta,z;t) = \underline{\mathbf{H}}(r,z)\,e^{j\omega t - j\gamma\theta}$$

Hp. 2

$$\underline{\underline{\boldsymbol{\mu}}} = \begin{pmatrix} \mu_{rr} & \mu_{r\theta} & \mu_{rz} \\ -\mu_{r\theta} & \mu_{\theta\theta} & -\mu_{\theta z} \\ \mu_{rz} & \mu_{\theta z} & \mu_{zz} \end{pmatrix} \quad \text{and} \quad \underline{\underline{\boldsymbol{\varepsilon}}} = \begin{pmatrix} \varepsilon_{rr} & \varepsilon_{r\theta} & \varepsilon_{rz} \\ -\varepsilon_{r\theta} & \varepsilon_{\theta\theta} & -\varepsilon_{\theta z} \\ \varepsilon_{rz} & \varepsilon_{\theta z} & \varepsilon_{zz} \end{pmatrix}$$

From ***Hp.2*** the problem is not self-adjoint, let us introduce the adjoint problem to compute the variational formulation. Considering the real-inner product:

$$\langle f, g\rangle = \int_V f\cdot g\,dV \quad \Rightarrow \quad \underline{\underline{\boldsymbol{\varepsilon}}}^a = \underline{\underline{\boldsymbol{\varepsilon}}}^t, \quad \underline{\underline{\boldsymbol{\mu}}}^a = \underline{\underline{\boldsymbol{\mu}}}^t, \quad \gamma^a = -\gamma.$$

Direct Field vs Adjoint Field

Hp. 3 Without sources (**J=0**, ρ=0), we have:

$$\begin{pmatrix} E_r^a \\ E_\theta^a \\ E_z^a \end{pmatrix} = \begin{pmatrix} -E_r \\ E_\theta \\ -E_z \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} H_r^a \\ H_\theta^a \\ H_z^a \end{pmatrix} = \begin{pmatrix} H_r \\ -H_\theta \\ H_z \end{pmatrix}$$

Rayleigh-Ritz Functional

$$\tilde{F}(\underline{\mathbf{H}}) = \iiint_V \left[\nabla\times\underline{\mathbf{H}}^a\cdot\left(\underline{\underline{\boldsymbol{\varepsilon}}}^{-1}\nabla\times\underline{\mathbf{H}}\right) - \omega^2\underline{\mathbf{H}}^a\cdot\underline{\underline{\boldsymbol{\mu}}}\underline{\mathbf{H}}\right]dV + \overbrace{\frac{\alpha}{\varepsilon_0}\iiint_V\left[\nabla\cdot\underline{\mathbf{H}}^a\nabla\cdot\underline{\mathbf{H}}\right]dV}^{\textit{Penalty Function}}$$

The terms exp(±jγθ) cancel out in the functional. Finally, the integral depends only on r, and z, and not on θ.

Boundary Condition: we assume **H**,**E**≈**0** far from the ring.

FEM – Real Formulation

Let consider the case when $\varepsilon_{r\theta}$, $\varepsilon_{\theta z}$, $\mu_{r\theta}$, $\mu_{\theta z}$ are **purely imaginary**, and ε_{rr}, $\varepsilon_{\theta\theta}$, ε_{rz}, ε_{zz}, μ_{rr}, $\mu_{\theta\theta}$, μ_{rz}, μ_{zz} are **purely real**.

Those tensors can used to describe:

- Magneto-optical materials [3] (e.g. Ce:YIG)
- Anisotropic materials (e.g. $LiNbO_3$)
- High refractive index contrast materials (e.g. Si/SiO_2)

Let assume the magnetic field is

$$\underline{\mathbf{H}} = \left[H_r(r,z)\underline{i}_r + jH_\theta(r,z)\underline{i}_\theta + H_z(r,z)\underline{i}_z\right]e^{j\omega t - jm\theta}$$

where m=Re{γ}. The complex unit *j* cancels out with the ones in the tensors.

We approximate **H** with the basic quadratic function $\varphi_i(r,z)$, where i is the node index

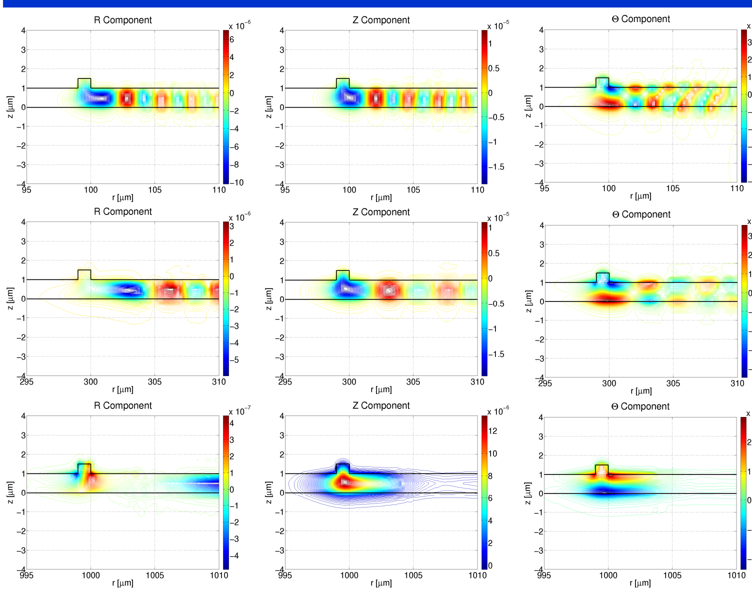
$$\underline{\mathbf{H}} \cong \sum_{i=1}^{N}\sqrt{r}\left[h_{ri}\begin{pmatrix}\varphi_i(r,z)\\0\\0\end{pmatrix} + jh_{\theta i}\begin{pmatrix}0\\\varphi_i(r,z)\\0\end{pmatrix} + h_{zi}\begin{pmatrix}0\\0\\\varphi_i(r,z)\end{pmatrix}\right]e^{j\omega t - jm\theta}$$

replacing it in the Rayleigh-Ritz functional, we obtain the quadratic form (QF)

$$\mathbf{h}^t\left[m^2\underline{\underline{\mathbf{M}}} + m\underline{\underline{\mathbf{C}}} + \underline{\underline{\mathbf{K}}} - \omega^2\underline{\underline{\mathbf{S}}}\right]\mathbf{h} = 0, \qquad \mathbf{h} = \left(h_{r1},\cdots,h_{rN},h_{\theta 1},\cdots,h_{\theta N},h_{z1},\cdots,h_{zN}\right)^t.$$

The minimum of the QF with respects to **h** is the numerical solution.

Numerical Results



➢ γ-problem (inverse problem)
*fix ω, we look for (m,**H**) → Quadratic Eigenvalue Problem*

$$\left[m^2\underline{\underline{\mathbf{M}}} + m\underline{\underline{\mathbf{C}}} + \left(\underline{\underline{\mathbf{K}}} - \omega^2\underline{\underline{\mathbf{S}}}\right)\right]\mathbf{h} = 0$$

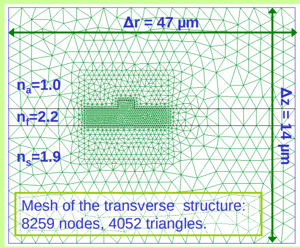
➢ ω-problem (direct problem)
*fix m, we look for (ω,**H**) → Generalized Eigenvalue Problem*

$$\left[m^2\underline{\underline{\mathbf{M}}} + m\underline{\underline{\mathbf{C}}} + \underline{\underline{\mathbf{K}}}\right]\mathbf{h} = \omega^2\underline{\underline{\mathbf{S}}}\,\mathbf{h}$$

Propagation constant m, γ-problem

Quasi-TE mode, n_s = 1.9, n_f = 2.2, n_a = 1, λ = 1.3 μm

Radius	This Work	Prkna [4]	Ref. [5]	Ref. [6]
100 μm	1033,555	1032,789	1032,8417	1032,787
300 μm	3107,886	3109,071	3109,1709	3109,513
500 μm	5185,805	5184,769	5184,9136	5184,959
1000 μm	10372,872	10373,070	10373,33	10373,303
1500 μm	15559,016	15560,839	15561,219	15561,346
2000 μm	20745,948	20748,443	20748,944	20748,954



Conclusions and Future Developments

✓ Less memory and computation time are required with respect to the Finite Difference Method and the Finite Difference Time Domain Method;
✓ Graded index rings can be studied more efficiently (mesh adaptivity);
✗ Bending loss has been neglected;
Use the complex formulation with Perfectly Matched Layer (PML);
The code can be easily parallelized.

References

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