Computation of relevant scattering data in the Zakharov-Shabat system

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In the numerical solution of non-linear PDEs of integrable type by means of the Inverse Scattering Transform technique, the first step consists of identifying the relevant scattering parameters of the associated Zakharov-Shabat system. In particular, it is important to identify both the bound state parameters with their multiplicities, and the so-called norming constants.

To this end, it is necessary to compute the coefficients $\{c_{js}\}_{j=1,\,s=0}^{M,\,n_j-1}$ and the parameters $\{f_j\}_{j=1}^{M}$ of a monomial-exponential sum of the type

$$h(x) = \sum_{j=1}^{M} \sum_{s=0}^{n_j-1} c_{js} x^s e^{f_j x},$$

where M and $\{n_j\}_{j=1}^{M}$ are positive integers and $\{c_{js}\}_{j=1,\,s=0}^{M,\,n_j-1}$ and $\{f_j\}_{j=1}^{M}$ are complex or real parameters with $c_{j,n_j-1} \neq 0$, given $2N$ sampled data $h(k)$ for $k = k_0, k_0+1, \ldots, k_0+2N$ with $k_0 \in \mathbb{N}^+ = \{0, 1, 2, \ldots\}$ and $N \geq L = n_1 + \cdots + n_M$.

In this talk we illustrate a linearization technique to numerically solve this non-linear approximation problem. It is based on the following steps:

1. Identification of the common rank of two square Hankel matrices H_0 and H_1 of order N generated by the $2N$ given data;

2. Computation of the parameters M, $\{n_j\}_{j=1}^{M}$ and $\{f_j\}_{j=1}^{M}$ by solving a generalized eigenvalue problem;

3. Computation of the coefficients $\{c_{js}\}_{j=1,\,s=0}^{M,\,n_j-1}$ by solving an overdetermined linear system.