A Keller-Segel model in chemotaxis with blow-up solutions

S. Vernier Piro
Dip. di Matematica e Informatica, Viale Merello 92, Cagliari, Italy
svernier@unica.it

We study the Neumann initial-boundary value problem for the fully parabolic Keller-Segel type system [1] with time dependent coefficients

$$u_t = \Delta u + k_1(t)\ div(u\nabla v), \quad x \in \Omega, t \in (0, t^*),$$

$$v_t = k_2(t)\Delta v - k_3(t)v + k_4(t)u, \quad x \in \Omega, t \in (0, t^*),$$

$$\frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, \quad x \in \partial\Omega, \quad t \in (0, t^*),$$

$$u(x, t) = u_0(x), \quad v(x, t) = v_0(x), \quad x \in \Omega,$$

where Ω is a bounded domain in R^N with smooth boundary, $\frac{\partial}{\partial n}$ is the normal derivative on the boundary and t^* is the blow up time. This system forms the core of numerous models used in mathematical biology to describe the spatio-temporal evolution of cell populations governed by both diffusive migration and chemotactic movement towards increasing gradients of a chemical that they produce themselves (chemotaxis). We derive conditions on the data and geometry of Ω, sufficient to obtain an explicit lower bound for the blow-up time.

References

[1] E. F. Keller and L. A. Segel, *Model for chemotaxis*, J. Theor. Biol., 530 (1971) pp. 225-234.

[2] L.E. Payne and J. C. Song, *Lower bound for blow-up in a model of chemotaxis*, J. Math. Anal. Appl., 385 **2** (2012) pp. 672-676..

[3] L.E. Payne, G.A. Philippin and S. Vernier-Piro, *Blow-up phenomena for a semilinear heat equation with nonlinear boundary condition I*, Z. Angew. Math. Phys., 61 (2010) pp. 971-978.