Some new results in geometrical optics

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In this talk I want to present some recent results obtained in the framework of Geometrical Optics from the inverse point of view. We shall be concerned with the propagation of light in a continuous transparent inhomogeneous and isotropic medium, dispersive or not. We put and solve the two following inverse problems of geometrical optics:
1) **3-dimensional inverse problem**: Given a two-parametric family of curves $\mathcal{F}_2$: $f(x,y,z) = c_1, \quad g(x,y,z) = c_2$, inside a 3-dimensional medium $\mathcal{M}_3$, we want to find the refractive-index distributions $n(x,y,z)$ allowing for the creation of the given family of curves as a family of monochromatic light rays.
2) **2-dimensional inverse problem**: Given a monoparametric family of curves $\mathcal{F}_1$: inside a 2-dimensional medium $\mathcal{M}_2$, lying on a regular surface S, we want to find the refractive-index distributions $n = n(u,v)$ allowing for the creation of the given family of curves as a family of monochromatic light rays. Our main results are:
Proposition 1: Given a family $\mathcal{F}_2$ lying on a medium $\mathcal{M}_3$, all the refractive-index distributions $n(x,y,z)$ allowing for the creation of the given family of curves as a family of monochromatic light rays, are solutions of the system of two first order linear PDE: $\alpha n_x - n_y + \Omega_1 n = 0$, $\beta n_x - n_z + \Omega_2 n = 0$, in the unique unknown function $n(x,y,z)$ where $\alpha(x,y,z)$, $\beta(x,y,z)$, $\Omega_1(x,y,z)$, $\Omega_2(x,y,z)$ are functions depending only on the given family of light rays.
Proposition 2: Given a family $\mathcal{F}_1$, inside a medium $\mathcal{M}_2$ lying on a regular surface S, with a line element given by $ds^2 = Edu^2 + 2Fdudv + Gdv^2$, all the refractive-index distributions $n(u,v)$ allowing for the creation of the given family of curves as a family of monochromatic light rays, are solutions of the linear first order PDE: $(G - \gamma F)n_u - (F - \gamma E)n_v + \Omega n = 0$, in the unknown function $n(u,v)$, where $\gamma = \frac{f_v}{f_u}$ is a function of u, v depending only on the given family; E, F, G are the coefficients of the assigned metric on S, and Ω is a functions of u, v depending both of the family and on the metric.