A modularity-based spectral graph analysis

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The modularity matrix of a graph is a special rank-one modification of the adjacency matrix, introduced by Newman and Grivan in the framework of graph clustering and community detection problems, see e.g., [1]. In fact, eigenvectors of modularity matrices can be exploited in community detection algorithms in the same way as eigenvectors of Laplacian matrices are currently utilized for solving graph partitioning or bandwidth reduction problems.

We perform an in-depth spectral analysis of modularity matrices. In particular, we prove certain properties of nodal domains induced by eigenvectors of modularity matrices, analogous to those known for graph Laplacian matrices, see e.g., [2]; and we outline the relationship between eigenvalues of modularity matrices and certain combinatorial descriptions of tightly connected subgraphs.

References

[1] M. E. J. Newman and M. Grivan, *Finding and evaluating community structure in networks*, Phys. Rev. E, 69 (2006), 026113.

[2] M. Fiedler, *A property of eigenvectors on non-negative symmetric matrices and its application to graph theory*, Czechoslovak Math. J., 25 (1975), pp. 619–633.