Asymptotics of the smallest singular value of a class of Toeplitz-generated matrices and related finite rank perturbations

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Square matrices of the form $X_n = T_n + f_n(T_n^{-1})^*$, where T_n is an $n \times n$ invertible banded Toeplitz matrix and f_n some positive sequence are considered. The norms of their inverses are described asymptotically as their size n increases. As an example, for

$$X_n = \begin{bmatrix} 1+\frac{1}{n} & -1 & 0 & \cdots & \cdots & 0 \\ \frac{1}{n} & 1+\frac{1}{n} & -1 & 0 & & \vdots \\ \vdots & & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & -1 & 0 \\ \vdots & \ddots & \ddots & \ddots & & -1 \\ \frac{1}{n} & \cdots & \cdots & \cdots & \frac{1}{n} & 1+\frac{1}{n} \end{bmatrix},$$

it will be shown that

$$\lim_{n\to\infty} \frac{2\|X_n^{-1}\|}{\sqrt{n}} = 1.$$

Certain finite rank perturbations of these matrices are shown to have no effect on this behaviour. In the concrete example above, for the matrix K_n obtained from X_n by adding one to each entry in the first column, one also has

$$\lim_{n\to\infty} \frac{2\|K_n^{-1}\|}{\sqrt{n}} = 1.$$