“Good” points for multivariate polynomial interpolation and approximation

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For the interval $[-1, 1]$, it is well-known that interpolating in Chebyshev points is much better than using equidistant points in the same interval (Runge phenomenon). This fact forms the basis of Chebfun, a Matlab-toolbox that uses Chebyshev points for the interpolation and Chebyshev polynomials for the representation of the interpolant. Quantitatively, the fact that the Chebyshev points are “good” points for interpolation with a polynomial of degree δ corresponds to the fact that the Lebesgue constant grows as $\log \delta$ while this Lebesgue constant grows much faster in case of equidistant interpolation points. The Lebesgue constant is the maximum of the Lebesgue-function on the geometry considered, in this case the interval $[-1, 1]$.

Also for the multivariate case and for different geometries, sets of “good” points were investigated (e.g., Padua points on the unit square) and other “good” point configurations were computed by optimization algorithms. In this talk we will describe an alternative optimization method to compute point configurations with a small Lebesgue constant for different geometries. This method consists of several smaller optimization procedures, taking each more and more computational effort but leading to smaller and smaller Lebesgue constants. It will turn out that the choice of a good basis for a specific geometry is essential to be able to solve the polynomial interpolation problem over that geometry. We will use an orthonormal basis with respect to a discrete inner product where the points of the inner product are lying in the geometry that we are considering at that moment. No explicit representation for these basis polynomials will be computed but we will evaluate them using a recurrence relation, generalizing the three-term recurrence relation on the real line and the Szegö recurrence relation on the complex unit circle.