A Keller-Segel model in chemotaxis with blow-up solutions

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We study the Neumann initial-boundary value problem for the fully parabolic Keller-Segel type system [1] with time dependent coefficients

$$
\begin{align*}
  &u_t = \Delta u + k_1(t) \text{div}(u \nabla v), \quad x \in \Omega, t \in (0, t^*), \\
  &v_t = k_2(t) \Delta v - k_3(t)v + k_4(t)u, \quad x \in \Omega, t \in (0, t^*), \\
  &\frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, \quad x \in \partial \Omega, \quad t \in (0, t^*), \\
  &u(x,t) = u_0(x), \quad v(x,t) = v_0(x), \quad x \in \Omega,
\end{align*}
$$

where $\Omega$ is a bounded domain in $\mathbb{R}^N$ with smooth boundary, $\frac{\partial}{\partial n}$ is the normal derivative on the boundary and $t^*$ is the blow up time. This system forms the core of numerous models used in mathematical biology to describe the spatio-temporal evolution of cell populations governed by both diffusive migration and chemotactic movement towards increasing gradients of a chemical that they produce themselves (chemotaxis). We derive conditions on the data and geometry of $\Omega$, sufficient to obtain an explicit lower bound for the blow-up time.

References

