# The inverse scattering transform for the defocusing nonlinear Schrödinger equation with nonzero boundary conditions

F. Demontis, B. Prinari, C. van der Mee, and **F. Vitale**
Department of Mathematics and Physics, University of Salento
Lecce, Italy
federica.vitale@le.infn.it

We have developed a rigorous theory of the inverse scattering transform for the defocusing nonlinear Schrödinger equation with nonvanishing boundary values $q_\pm \equiv q_0 e^{i\theta_\pm}$ as $x \to \pm\infty$. The direct problem is shown to be well-posed for potentials $q$ such that $q - q_\pm \in L^{1,2}(\mathbb{R}^\pm)$, for which analyticity properties of eigenfunctions and scattering data are established. The inverse scattering problem is formulated and solved both via Marchenko integral equations, and as a Riemann-Hilbert problem in terms of a suitable uniform variable. The asymptotic behavior of the scattering data is determined and shown to ensure the linear system solving the inverse problem is well-defined. Finally, the triplet method is developed as a tool to obtain explicit multisoliton solutions by solving the Marchenko integral equation via separation of variables.

## References

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