On the solution of certain algebraic Riccati equations arising fluid queues

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Consider a nonsymmetric algebraic Riccati equation (NARE) $C + XA + DX - XBX = 0$, where the unknown X has size $m \times n$, and where the coefficients A, B, C and D are real $n \times n$, $n \times m$, $m \times n$ and $m \times m$ matrices, respectively. Assume that the block coefficients are such that $M = \left[\begin{smallmatrix} A & -B \\ C & D \end{smallmatrix}\right]$ is a nonsingular M-matrix or a singular irreducible M-matrix. The solution X of interest is the minimal nonnegative.

In the modeling of an adaptive $MMAP[K]/PH[K]/1$ queue, the matrix D is a $K \times K$ block diagonal matrix with $h \times h$ blocks. We present a new algorithm for computing the minimal nonnegative solution of the NARE where we exploit the structure of the matrix D. The solution of the original NARE is computed by solving a set of correlated NAREs, with coefficients of small size, obtained by a suitable block partitioning of the coefficients A, B, C and D. More specifically, the sought solution X is partitioned in blocks X_i, $i = 1, \ldots, K$, and X_i is the minimal nonnegative solution of the i-th NARE

$$C_i + X\widetilde{A}_i + D_iX - XB_iX = 0, \quad \widetilde{A}_i = A - \sum_{j=1, j\neq i}^{K} B_jX_j, \tag{1}$$

for $i = 1, \ldots, K$, where the coefficient $\widetilde{A}_i$ of the above equation depends on the solutions X_j, with $j \neq i$, of the remaining $K - 1$ NAREs. To solve the above equations we propose an iterative scheme, where we replace the unknown coefficient $\widetilde{A}_i$ with an approximation. Convergence results are proved by using properties of nonnegative matrices and M-matrices. From the analysis of the computational cost and from the numerical experiments, the algorithm is more effective than the standard algorithms when the block size K is larger than h.