Rational Krylov methods and Gauss quadrature

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The need to evaluate expressions of the form $f(A)v$ or $v^H f(A)v$, where A is a large sparse or structured matrix, v is a vector, f is a nonlinear function, and ^{H} denotes transposition and complex conjugation, arises in many applications. Rational Krylov methods can be attractive for computing approximations of such expressions. These methods project the approximation problem onto a rational Krylov subspace of fairly small dimension, and then solve the small approximation problem so obtained. We are interested in the situation when the rational functions that define the rational Krylov subspace have few distinct poles. We discuss the case when A is Hermitian and an orthogonal basis for the rational Krylov subspace can be generated with short recursion formulas. Rational Gauss quadrature rules for the approximation of $v^H f(A)v$ will be described. When A is non-Hermitian, the recursions can be described by a generalized Hessenberg matrix. Applications to pseudospectrum computations are presented.