A symbol approach in IgA matrix analysis (and in the design of efficient multigrid methods)

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We study the spectral properties of stiffness matrices that arise when isogeometric analysis is employed for the numerical solution of classical second order elliptic problems. Motivated by the applicative interest in the fast solution (by preconditioned Krylov or multigrid methods) of the related linear systems, we look for a spectral characterization of the involved matrices. In particular, we investigate non-singularity, conditioning (extremal behavior), spectral distribution in the Weyl sense, as well as clustering of the eigenvalues to a certain (compact) subset of the complex field. All the analysis is related to the notion of symbol in the Toeplitz setting and is carried out both for the cases of 1D and 2D problems.

The spectral properties represent the starting point for designing fast two-grid methods for which we provide a numerical confirmation of the optimality, meaning that the spectral radii of the related iteration matrices are bounded by a constant c_p for all n, $c_p < 1$: a formal proof of optimality for $p = 2$ and $p = 3$ is given. An extension of the results to the two-level case is provided, together with a wide set of numerical tests including the V-cycle and the W-cycle applied to approximated 1D and 2D problems.