A free boundary numerical method for solving an overdetermined elliptic problem

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This work presents a mathematical problem related to the equilibrium analysis of a prestressed membrane with rigid and cable boundaries. The membrane is represented by a regular surface $z(x, y)$, its stresses (tensions) by a positive tensor $\boldsymbol{\sigma}(x, y)$, and its boundary by a set of regular curves; Γ^{r} and Γ^{c}, i.e. rigid and cable boundary respectively. The membrane-cable interface equilibrium requires taking into account a singular condition on Γ^{c}, and it makes the problem more difficult. Precisely, if H represents the Hessian matrix of z and $\boldsymbol{t}$ is the tangent unit vector to Γ^{c}, once $\boldsymbol{\sigma}$ is fixed we have to find z in a bounded domain D such that

$$\begin{cases} \operatorname{div}(\boldsymbol{\sigma} \cdot \nabla z) = 0 \text{ in } D, \\ z = g \text{ on } \Gamma^{\mathrm{r}}, \qquad z = h \text{ on } \Gamma^{\mathrm{c}} \text{ (Dirichlet boundary conditions)}, \\ \boldsymbol{t} \cdot (H \cdot \boldsymbol{t}) = 0 \text{ on } \Gamma^{\mathrm{c}} \text{ (unusual boundary condition)}, \end{cases} \tag{1}$$

g and h being two functions defined in Γ^{r} and Γ^{c} ($\partial D = \Gamma^{\mathrm{c}} \cup \Gamma^{\mathrm{c}}$). In the last system it is not possible to arbitrarily choose both functions g and h; in fact, an overdetermined elliptic problem would be obtained and its solution z would not necessarily solve also the unusual boundary condition on Γ^{c}. Therefore, we consider Γ^{c} as a *free portion* of ∂D and, by means of an iterative procedure, it is possible to fit the shapes of the cable (i.e. Γ^{c}) and of the membrane (i.e. z) so that system (1) is completely verified.

The aim of the talk is to define and discuss this mathematical problem and, successively, to present some numerical results regarding the iterative approach.

References

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