Automorphic Lie algebras with dihedral symmetry

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The idea of *Automorphic Lie Algebras* [1] arose from the concept of reduction groups studied in the early 80s in the field of integrable systems [2]. They are obtained by imposing a discrete group symmetry on a current algebra of Krichever Novikov type. That is, for $\mathfrak{g}$ a simple Lie algebra, $\mathcal{M}(\overline{\mathbb{C}})$ the field of meromorphic functions on the Riemann sphere, and G a finite subgroup of $\mathrm{Aut}(\mathfrak{g}\otimes\mathcal{M}(\overline{\mathbb{C}}))$, the Automorphic Lie Algebra is the space of invariants $(\mathfrak{g}\otimes\mathcal{M}(\overline{\mathbb{C}}))^G_\Gamma$ where $\Gamma\subset\overline{\mathbb{C}}$ is a single G-orbit where poles are allowed. Past work shows remarkable resemblance between Automorphic Lie Algebras with different reduction groups G [3], [4]. For example, if $\mathfrak{g}=\mathfrak{sl}(2,\mathbb{C})$ and Γ is an exceptional orbit, $|\Gamma|<|G|$, changing the group does not affect the Lie algebra structure, although the elements of the algebra are different. In the present research we fix G to be the dihedral group $\mathbb{D}_N$, and vary the orbit of poles and the Lie algebra, as well as the G-action on the Lie algebra. We find a uniform description of these algebras, valid both in the case of generic and exceptional orbits.

References

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