Conjugate Gradient method for ill-posed linear problems in L^p spaces

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In linear image deblurring problems, the "geometry" of the L^p Banach spaces, for $1 < p < 2$, can substantially reduce the over-smoothness effects with respect to any classical iterative restoration algorithm in L^2 Euclidean space [2].
In this work we deal with a generalization of the conjugate gradient method for the minimization of the p-norm cost functional $\Phi(x) = ||Ax-y||_p^p$, related to the solution of the ill-posed operator equation $Ax = y$, where $A : X \to Y$ is a linear operator between L^p Banach spaces. The high convergence speed of conventional conjugate gradient in L^2 Hilbert space gives rise to a fast iterative method in L^p Banach spaces too.
The algorithm is applied to enhance the spatial resolution of microwave radiometer data [1]. The problem which describes the relationship between the coarse but partially correlated measurements and the brightness temperature belongs to the class of Fredholm integral equation of the first kind. After the convergence proof, we compare the performances of the proposed algorithm with the classical conjugate gradient method in Hilbert space.

References

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