A new framework for multi-parameter regularization

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This talk proposes some new general strategies for the analysis and implementation of multi-parameter regularization methods. We will focus on multi-parameter Tikhonov regularization, i.e., Tikhonov-regularization involving at least two regularization terms weighted by two regularization parameters. Methods like this are very effective when the solution has different features that should be simultaneously preserved in the regularization process, and they are potentially superior to any one-parameter regularization method. However, a key issue linked to multi-parameter Tikhonov regularization is the selection of each regularization parameter; see [1, 2]. The goal of this talk is to introduce a new strategy to select a proper set of regularization parameters that satisfies the discrepancy principle and maximizes a suitable quantity, whose size reflects the quality of the computed approximate solution. This strategy can be applied to direct multi-parameter Tikhonov regularization, but also to the so-called class of (multi-parameter) Krylov-Tikhonov methods [4], which are iterative methods based on the projection of a (multi-parameter) Tikhonov-regularized problem onto Krylov subspaces of increasing dimensions. In the case of multi-parameter Krylov-Tikhonov methods, we review the strategy originally introduced in [3], and we propose a different and more robust implementation. Further details about the new strategy can be found in [5].

References

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