Fractional convexity maximum principle

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The celebrated *convexity maximum principle* was proved by Nick Korevaar [12, 13] in the 80's to answer a question posed by his advisor, prof. Robert Finn, concerning convexity of capillary surfaces in convex pipes. Korevaar's idea gave birth to a number of subsequent contributions, especially due to Kawohl [7, 8] and Kennington [9, 10, 11]. Instead of arguing by contradiction, Porru and the speaker in 1993 gave an alternative proof [2] based on the construction of a suitable elliptic inequality, in the spirit of Larry Payne's P-functions. This talk deals with a possible extension [1] of the convexity maximum principle to continuous solutions of equations involving the fractional Laplacian, which is a (non-local) pseudodifferential operator currently investigated by several authors: see, for instance, [3] and the series of papers by Iannizzotto *et alii* [4, 5, 6]. Further results on convexity for the fractional Laplacian have been recently obtained by Kulczycki [14].

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