Fractional boundary value problems: the stationary case

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Stochastic models based on Lévy processes lead to pseudo-differential equations driven by fractional operators of the following type:

$$(-\Delta)_p^s u(x) = 2 \lim_{\varepsilon \to 0^+} \int_{\mathbf{R}^N \setminus B_\varepsilon(x)} \frac{|u(x)-u(y)|^{p-2}(u(x)-u(y))}{|x-y|^{N+ps}} \, dx \, dy,$$

where $N \geq 1$, $p > 1$ and $0 < s < 1$. Though not explicitly involving any derivatives, the operator $(-\Delta)_p^s$ (which reduces to the fractional Laplacian for $p = 2$) exhibits many similarities to such classical second-order elliptic operators as the p-Laplacian. In this talk we will briefly review some recent results on *stationary* boundary value problems driven by $(-\Delta)_p^s$, including:

- regularity of weak solutions;
- maximum principles and sub-supersolutions;
- spectral properties;
- existence/multiplicity results based on variational methods.

For details we refer to the papers [1, 2, 3].

References

[1] A. I., M. Squassina, *Weyl-type laws for fractional p-eigenvalue problems*, Asymptot. Anal., 88 (2014), pp. 233–245.

[2] A. I., S. Liu, K. Perera, M. Squassina, *Existence results for fractional p-Laplacian problems via Morse theory*, Adv. Calc. Var., to appear.

[3] A. I., S. Mosconi, M. Squassina, *Global Hölder regularity for the fractional p-Laplacian*, preprint.