R. Boll consistent around the cube systems and their linearizability.

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Consistency around the cube has proved to be one of the most useful concept in studying discrete, multilinear, integrable, nonlinear systems defined on a quad-graph, soon becoming a definition itself of integrability. An algorithmic procedure in fact provides a (true) Lax pair and Bäcklund transformations for any consistent system. A first classification of these equations was presented in [1], later extended in [2]. A complete classification (under the additional hypothesis of the tetrahedron property) has been finally obtained in [3]-[4].

When one assume the more general context of [3]-[4], where a priori different equations live on the faces of the consistency cube, the strictly equivalence between existence of a (non trivial) Lax pair, Bäcklund transformations and consistency has been somehow criticized in [5]. The notion of *weak Lax pair* was thus introduced.

In this seminar we show how this critics can be fully reabsorbed, once the consistent systems of Boll are properly extended on the lattice. After all the independent equations inside an equivalence class have been identified, an algebraic entropy test reveals their *linearizability* ($C-$integrability). We construct the Lax representation, whose weakness obviously reflects the linearizability property. We also provide explicit examples of the linearization procedure.

References

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