The inverse scattering transform for the focusing nonlinear Schrödinger equation with a one-sided non-zero boundary condition

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We present the inverse scattering transform (IST) as a tool to solve the initial-value problem for the focusing nonlinear Schrödinger (NLS) equation with one-sided non-zero boundary value $q_r(t) \equiv A_r e^{-2iA_r^2 t + i\theta_r}$, $A_r \geq 0$, $0 \leq \theta_r < 2\pi$, as $x \to +\infty$. The direct scattering problem is shown to be well-defined for NLS solutions $q(x,t)$ such that $[q(x,t) - q_r(t)\vartheta(x)] \in L^{1,1}(\mathbb{R})$ [$\vartheta(x)$ denotes the Heaviside function] with respect to $x \in \mathbb{R}$ for all $t \geq 0$, for which analyticity properties of eigenfunctions and scattering data are established. The inverse scattering problem is formulated both via (left and right) Marchenko integral equations and as a Riemann-Hilbert problem on a single sheet of the scattering variables $\lambda_r = \sqrt{k^2 + A_r^2}$, where k is the usual complex scattering parameter in the IST. Unlike the case of fully asymmetric boundary conditions [2] and similarly to the same-amplitude case dealt with in [1], the direct and inverse problems are also formulated in terms of a suitable uniformization variable that maps the two-sheeted Riemann surface for k into a single copy of the complex plane. The time evolution of the scattering coefficients is then derived, showing that, unlike the case of solutions with the same amplitude as $x \to \pm\infty$, here both reflection and transmission coefficients have a nontrivial (although explicit) time dependence. These results will be instrumental for the investigation of the long-time asymptotic behavior of physically relevant NLS solutions with nontrivial boundary conditions, either via the nonlinear steepest descent method on the Riemann-Hilbert problem, or via matched asymptotic expansions on the Marchenko integral equations.

References

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