International Workshop on Applied Mathematics & Quantum Information

Cagliari, November 3-4, 2016

Contents

1 INTRODUCTION AND ACKNOWLEDGMENTS

We are delighted to be hosting the *International Workshop on Applied Mathematics* & *Quantum Information* here at the University of Cagliari (Italy). The aim of such an activity is to gather European researchers, specialist in applied mathematics and quantum informations, as well as related subjects, and let them share their results and discuss further developments and open problems.

Mainly, this meeting was possible thanks to the generous availability of all the speakers who participated in the activity. Moreover, we also appreciate the effort of all the other colleagues who, directly or indirectly, offered their useful contribution in the organization of the Workshop.

Cagliari, 3 November 2016

The Scientific Committee

Maria Luisa Dalla Chiara (*University of Firenze*)
Roberto Giuntini (*University of Cagliari*)
Cornelis van der Mee (*University of Cagliari*)
Giuseppe Rodriguez (*University of Cagliari*)

The Organizers

Gustavo Martin Bosyk (*Instituto de Física La Plata (IFLP), CONICET, Argentina*)
Luisa Fermo (*University of Cagliari*)
Hector Freytes (*University of Cagliari*)
Federico Holik (*Instituto de Física La Plata (IFLP), CONICET, Argentina*)
Giuseppe Sergioli (*University of Cagliari*)
Giuseppe Viglialoro (*University of Cagliari*)

2 Invited speakers

1. Gustavo Martin Bosyk - Instituto de Física La Plata (IFLP), CONICET, Argentina
2. Maria Luisa Dalla Chiara - University of Florence
3. Patricia Diaz de Alba - University of Cagliari
4. Luisa Fermo - University of Cagliari
5. Hector Freytes - University of Cagliari
6. Federico Holik - Instituto de Física La Plata (IFLP), CONICET, Argentina
7. Giuseppe Rodriguez - University of Cagliari
8. Enrica Santucci - University of Cagliari
9. Giuseppe Sergioli - University of Cagliari
10. Ranjith Venkatrama - University of Cagliari
11. Cornelis van der Mee - University of Cagliari
12. Giuseppe Viglialoro - University of Cagliari
13. Steeve Zozor - GIPSA Lab, Département Image et Signal, University of Grenoble

3 ABSTRACTS OF THE CONTRIBUTIONS

Majorization and entanglement transformations

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Abstract

First, I will briefly introduce the concept of majorization between probability vectors and its main properties [1]. In particular, I will present the *majorization lattice* obtained by Cicalese and Vaccaro [2]. Then, I will address the problem of entanglement transformations by using *local operations and classical communications* (LOCC). More precisely, the problem consists in two parties, Alice and Bob, that share an entangled pure-state $|\psi\rangle$ (initial state) and their goal is to transform it in another entangled pure-state $|\phi\rangle$ (target state), by using only LOCC. A celebrated result of Nielsen gives the necessary and sufficient condition that makes possible this entanglement transformation process [3]. Indeed, this process can be achieved if and only if the majorization relation $\psi \prec \phi$ holds, where ψ and ϕ are probability vectors obtained by taking the squares of the Schmidt coefficients of the initial and target states, respectively. In general, this condition is not fulfilled. However, one can look for an *approximate* entanglement transformation. Vidal *et. al* have proposed a deterministic transformation using LOCC in order to obtain a state most approximate to target in terms of maximal fidelity between them [4]. In this talk, I will present an alternative proposal by exploiting the fact that majorization is indeed a lattice for the set of probability vectors [5].

References

[1] A.W. Marshall, I. Olkin, and B.C. Arnold, *Inequalities: Theory of Majorization and Its Applications*, Springer Verlag, 2011.

[2] F. Cicalese and U. Vaccaro, *Supermodularity and Subadditivity Properties of the Entropy on the Majorization Lattice*, IEEE Trans. Inf. Theory **48** (2002), pp.933-38.

[3] M.A. Nielsen, *Conditions for a Class of Entanglement Transformations*, Phys. Rev. Lett. **83** (1999), pp.436-39.

[4] G. Vidal, D. Jonathan, and M.A. Nielsen, *Approximate transformations and robust manipulation of bipartite pure-state entanglement*, Phys. Rev. A **62** (2000), pp.01230401-01230410

[5] G.M. Bosyk, G. Sergioli, H. Freytes, F. Holik and G. Bellomo, *Approximate transformations of bipartite pure-state entanglement from the majorization lattice*, arXiv:1608.04818v1 (2016), pp.1-6

A Many-valued Approach to Quantum Computational Logics

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Abstract

Quantum computational logics are special examples of quantum logic where formulas are supposed to denote pieces of quantum information (qubit-systems or mixtures of qubit-systems), while logical connectives are interpreted as reversible quantum logical gates. Hence, any formula of the quantum computational language represents a synthetic logical description of a quantum circuit. We investigate a many-valued approach to quantum information, where the basic notion of qubit has been replaced by the more general notion of qudit. The qudit-semantics allows us to represent as reversible gates some basic logical operations of Lukasiewicz many-valued logics. In the final part of the article we discuss some problems that concern possible implementa- tions of gates by means of optical devices.

References

[1] A. Dvurečenskij, S. Pulmannová, *New Trends in Quantum Structures*, Kluwer, Dordrecht, 2000.

[2] M.L. Dalla Chiara, R. Giuntini, R. Greechie, *Reasoning in Quantum Theory*, Kluwer, Dordrecht, 2004.

[3] M.L. Dalla Chiara, R. Giuntini, R. Leporini, "Logics from quantum computation", *Int. J. Quantum Inf.* **3**, 293–337, 2005.

[4] M.L. Dalla Chiara, R. Giuntini, R. Leporini, G. Sergioli, "Holistic logical arguments in quantum computation", *Math. Slovaca* **66** (2), 1–22, 2016.

[5] M.L. Dalla Chiara, R. Giuntini, R. Leporini, G. Sergioli, "A first-order epistemic quantum computational semantics with relativistic-like epistemic effects", *Fuzzy Sets and Systems* **298**, 69–90, 2016.

[6] N. Matsuda, R. Shimizu, Y. Mitsumori, H. Kosaka, K. Edamatsu, "Observation of optical-fibre Kerr nonlinearity at the single-photon level", *Nature Photonics* **3**, 95–98, 2009.

Reconstructing the electrical conductivity and the magnetic permeability of the soil by a FDEM data inversion

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Abstract

This work aims to detect or infer, by non destructive investigation of soil properties, inhomogeneities in the ground or the presence of particular conductive substances. A non-linear model is used to describe the interaction of an electromagnetic field with the soil.

Starting from electromagnetic data collected by a ground conductivity meter (GPR), we reconstruct, assuming that the electrical conductivity is known in every layer, the magnetic permeability of the soil with respect to depth, with a regularized Gauss-Newton method. We propose an inversion method, based on the low-rank approximation of the Jacobian of the nonlinear model. This algorithm has been tested by numerical experiments on synthetic data sets.

This is a joint investigation with Giuseppe Rodriguez.

References

[1] G. P. Deidda, C. Fenu, and G. Rodriguez, *Regularized solution of a nonlinear problem in electromagnetic sounding*, Inverse Problems **30**:125014 (2014), 27 pages.

[2] P. Díaz de Alba, and G. Rodriguez, *Regularized solution of a nonlinear problem in applied Geophysics*, Book Trends in Differential Equations and Applications, SEMA SIMAI Springer Series 8 (2016), pp.357–369. ISBN 978-3-319-32012-0.

[3] P. Díaz de Alba, and G. Rodriguez, *Identifying the magnetic permeability in multi-frequency FDEM data inversion*, *Work in progress, 2016.*

Recovering monomial-exponential sums via matrix-pencil methods

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Abstract

In this talk we describe a numerical procedure to solve the following non-linear approximation problem: recover the positive integers M and $\{m_j\}_{j=1}^M$, the distinct complex or real parameters $\{f_j\}_{j=1}^M$ and the complex or real coefficients $\{c_{js}\}_{j=1,s=0}^{M,m_j-1}$ of the following monomial-exponential sum

$$h(x) = \sum_{j=1}^{M} \sum_{s=0}^{m_j-1} c_{js} x^s e^{f_j x}, \tag{1}$$

knowing h in $2N$ equidistant data points with $N \geq \widehat{M} = m_1 + \ldots + m_M$.

The problem arises in different fields such as signal processes, electromagnetism and optical fibers [1].

The method we use is based on theoretical results proved in [2, 3] where different algorithms are also proposed and where the technique is also generilized to the case of bi-variate sums.

References

[1] L. Fermo, C. van der Mee and S. Seatzu, *Scattering data computation for the ZakharovShabat system*, Calcolo **53** (2016), pp. 487-520.

[2] L. Fermo, C. van der Mee and S. Seatzu, *Parameter estimation of monomial-exponential sums*, Electronic Transactions on Numerical Analysis **41** (2014), pp. 249-261.

[3] L. Fermo, C. van der Mee and S. Seatzu, *Parameter estimation of monomial-exponential sums in one and two variables*, Applied Mathematics and Computation **258** (2015), pp. 576-586.

Implementing Łukasiewicz operations in quantum computational with mixed states

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Abstract

In order to establish a connection between infinite-valued Łukasiewicz logic and quantum computation with mixed states, a representation of the Lukasiewicz truncated sum, in terms of quantum operations is given. In [1], authors provided a first attempt to achieve this result. In order to improve the efficiency of this representation, without coming into an undesirable increasing of the dimension of the Hilbert, we here propose to use a quantum cloner which acts on 1 original qubits and generates 2 clones [2].

References

[1] H. Freytes, G. Sergioli, A. Arico, *Representing continuous t-norms in quantum computation with mixed states*, J. Phys. A, 43(46):465306, 12 (2010).

[2] N. Gisin, S. Massar, *Optimal quantum cloning machines* , Phys.Rev.Lett. 79 (1997) 2153-2156.

Quantum Probability and The Problem of Pattern Recognition

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Abstract

We discuss a possible generalization of the problem of pattern recognition to arbitrary probabilistic models. We discuss how to deal with the problem of recognizing an individual pattern among a family of different species or classes of objects which obey probabilistic laws which do not comply with Kolmogorov's axioms. Our framework allows for the introduction of non-trivial correlations (as entanglement or discord) between the different species involved, opening the door to a new way of harnessing these physical resources for solving pattern recognition problems.

Large scale computation of the trace of a matrix function

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Abstract

During the last decade many new indices have been introduced, in order to characterize either the nodes of a complex network or the network itself in its entirety, which can be expressed as functions of the adjacency matrix A associated to the network. While such matrix functions can be defined in terms of the spectral decomposition of A, most interesting applications lead to so large adjacency matrices that it is unfeasible to compute their spectral factorization. In these situations, projecting the original matrix in Krylov spaces of suitable dimension may provide an effective approach for analyzing complex networks [1, 3]. Block algorithms lead to very efficient computations when the size of the problem is very large [2, 5].

In this talk, after a brief introduction, we will discuss a block method recently introduced in [4] which exploits the global block Lanczos decomposition for the computation of the trace of a matrix function. This quantity is connected to the Estrada index for a network. The resulting algorithm presents substantial computational advantages with respect to other approaches, especially when a parallel processing environment is available.

References

[1] C. Fenu, D. Martin, L. Reichel, and G. Rodriguez. *Network analysis via partial spectral factorization and Gauss quadrature.* SIAM J. Sci. Comput., **35** (2013), pp. A2046–A2068.

[2] C. Fenu, D. Martin, L. Reichel, and G. Rodriguez. *Block Gauss and anti-Gauss quadrature with application to networks.* SIAM J. Matrix Anal. Appl., **34** (2013), pp. 1655–1684.

[3] J. Baglama, C. Fenu, L. Reichel, and G. Rodriguez. *Analysis of directed networks via partial singular value decomposition and Gauss quadrature.* Linear Algebra Appl., **456** (2014), pp. 93–121.

[4] M. Bellalij, L. Reichel, G. Rodriguez, and H. Sadok. *Bounding matrix functionals via partial global block Lanczos decomposition.* Appl. Numer. Math., **94** (2015), pp. 127–139.

[5] L. Reichel, G. Rodriguez, and T. Tang. *New block quadrature rules for the approximation of matrix functions.* Linear Algebra Appl., **502** (2016), pp. 299–326.

On the Thermodynamic Equivalence between Hopfield Networks and Hybrid Boltzmann Machines

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Abstract

The main result of this work is to numerically and analytically prove the thermodynamic equivalence between two different kinds of neural networks, widely used for solving common machine learning problems. In particular, we compared the well known Hopfield network [1, 5], which exhibits a fully connected structure of neurons, with a restricted version [2, 4] of a Boltzmann Machine [3] (rBM), consisting of one layer of visible units and two (or more) disconnected layers of hidden units. We stress that in this work a completely novel structure of rBM was introduced, called *hybrid*, in which the visible units are digital and the hidden units are analog (while previous studies have investigated the cases in which both types of layers are either analog or digital). The statistical mechanical analogy between the models implies that it is possible to simulate the dynamics of the visible units in a Boltzmann Machine by means of a Hopfield network with a consequent and meaningful computational advantage. In addition, the phase diagram of the Hopfield model has a counterpart in this type of Boltzmann Machine and this result permits to adjust the ratio between the sizes of hidden and visible layers in order to obtain the optimal generative model of the observed data. Furthermore, we rigorously proved that the efficiency of our models decreases by introducing two different sources of noise, in particular when the connection between the hidden layers and a system subjected to an external field are considered.

References

[1] D. J. Amit, *Modeling brain function: the world of attractor neural network*, Cambridge University Press (1992)

[2] Y. Bengio, Learning deep architectures for AI, *Foundations and Trends in Machine Learning*, **2**(1) (2009), pp.1-127.

[3] G. E. Hinton, Learning multiple layers of representation, *Trends in Cognitive Science*, **11**(10) (2007), pp. 428-434.

[4] G. E. Hinton, R. R. Salakhutdinov, Reducing the dimensionality of data with neural networks, *Science*, **313**(5786) (2006), pp. 504-507

[5] J. J. Hopfield, Neural networks and physical systems with emergent collective computational abilities, *Proceedings of the National Academy of Sciences of the United States of America*, **79**(8) (1982), pp. 2554-2558

A Quantum-inspired version of the Nearest Mean Classifier

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Abstract

We introduce a framework suitable for describing standard classification problems using the mathematical language of quantum states. In particular, we provide a one-toone correspondence between real objects and pure density operators. This correspondence enables us: i) to represent the Nearest Mean Classifier (NMC) in terms of quantum objects, ii) to introduce a quantum-inspired version of the NMC called Quantum Classifier (QC). By comparing the QC with the NMC on different datasets, we show how the first classifier is able to provide additional information that can be beneficial on a classical computer with respect to the second classifier.

References

[1] H. Freytes, G. Sergioli, A. Arico, *Representing continuous t-norms in quantum computation with mixed states*, J. Phys. A, 43(46):465306, 12 (2010).

[2] M. Schuld, I. Sinayskiy, F. Petruccione, *An introduction to quantum machine learning*, Contemp. Phys., 56(2), arXiv:1409.3097 (2014).

[3] C. A. Trugenberger, *Quantum pattern recognition*, Quantum Inf. Process, 1(6):471-493 (2002).

[4] G. Sergioli, E. Santucci, L. Didaci, J. Miszczak, R. Giuntini *Pattern Recognition in the Quantum Bloch Sphere*, arXiv:1603.00173.

Exact solutions of integrable nonlinear evolution equations

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Abstract

A review of the inverse scattering transform (IST) to solve the so-called integrable nonlinear evolution equations is given. Among these equations we count the nonlinear Schrödinger, sine-Gordon, Korteweg-de Vries, Hirots, and Toda equations. In particular, we explain a matrix triplet method to obtain the soliton solutions. This is a joint work with Francesco Demontis.

Fuzzy Representation of Quantum Fredkin Gate

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Abstract

In this talk, a fuzzy representation of a generalized quantum version of the Fredkin gate will be discussed in the framework of Quantum Computation with Mixed gates.

Existence and boundedness properties of solutions to a chemotaxis-system with logistic growth

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Abstract

In this talk we study the chemotaxis-system

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla v) + g(u) & x \in \Omega, t > 0, \\ v_t = \Delta v - v + u & x \in \Omega, t > 0, \end{cases}$$

defined in a convex smooth and bounded domain Ω of $\mathbb{R}^3$, $\chi > 0$ and endowed with homogeneous Neumann boundary conditions. If the source g behaves similarly to the logistic function and verifies $g(s) \leq a - bs^{\alpha}$, for $s \geq 0$, with $a \geq 0$, $b > 0$ and $\alpha > 1$, we discuss the questions concerning the existence of very weak solutions and their uniformly-in-time boundedness properties (see [1] and [2]). In addition, for the two-dimensional setting some numerical results are also presented.

References

[1] G. Viglialoro, *Very weak global solutions to a parabolic-parabolic chemotaxis-system with logistic source*, J. Math. Anal. Appl. **439**(1) (2016), pp. 197–212

[2] G. Viglialoro, *Boundedness properties of very weak solutions to a fully parabolic chemotaxis-system with logistic source*, Nonlinear Anal.-Real. **34** (2017), pp. 520–535

Generalized quantum entropies: a definition and some properties

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Abstract

In this presentation, a family of quantum entropies inspired by the classical (h,phi)-entropies proposed by Salicrú et al. (more precisely, inspired by the Csiszár's divergencies) will be introduced. The proposed family includes several well-known entropies such that the von Neumann entropy or quantum versions of the Rényi's and Havrda-Charvát-Daróczy-Tsallis's entropies, among many others. The main properties of the proposed quantum (h,phi)-entropies lie on the fundamental concept of majorization. The behavior of these entropies when a quantum state is subject to some quantum operations (unitaria, measurement) will be characterized, and its behavior when dealing with composite systems as well. Some potential applications in detection of entanglement will be exposed. Finally, we will present possible definitions of associate measures such that conditional generalized quantum entropies.

4 Program of the meeting

All the invited speakers will present their contributions at *Polo Umanistitico "sa Duchessa" (Aula Specchi)*, University of Cagliari, Italy (Via Is Mirrionis 1 - 09123 Cagliari), and according to the following program.

Workshop Schedule

3 November 2016

- 15:15-15:30 - **OPENING**

Chair: Roberto Giuntini

- 15:30-16:00 - **Steeve Zozor**
- 16:00-16:30 - **Gustavo Martin Bosyk**
- 16:30-17:00 - *Coffee break*

Chair: Giuseppe Viglialoro

- 17:00-17:30 - **Patricia Diaz de Alba**
- 17:30-18:00 - **Cornelis van der Mee**
- 18:00-18:30 - **Luisa Fermo**
- 21:00 - *Social Dinner*

4 November 2016

Chair: Federico Holik

- 10:00-10:30 - **Maria Luisa Dalla Chiara**
- 10:30-11:00 - **Hector Freytes**
- 11:00-11:30 - *Coffee break*

Chair: Maria Luisa Dalla Chiara

- 11:30-12:00 - **Giuseppe Rodriguez**
- 12:00-12:30 - **Enrica Santucci**

Chair: Hector Freytes

- 15:30-16:00 - **Federico Holik**
- 16:00-16:30 - **Giuseppe Sergioli**
- 16:30-17:00 - *Coffee break*

Chair: Luisa Fermo

- 17:00-17:30 - **Giuseppe Viglialoro**
- 17:30-18:00 - **Ranjiith Venkatrama**