# From classical approaches to $C^*$-algebra techniques in the numerical analysis of singular integral equations

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The application of Cauchy singular and hypersingular integral equations, for example in airfoil theory and elasticity theory, and the theory of their numerical solution have a long history. The so-called classical collocation method for equations of the type

$$a(x)u(x)+\frac{b(x)}{\pi}\int_{-1}^{1}\frac{u(y)\,dy}{y-x}+\int_{-1}^{1}h(x,y)u(y)\,dy=f(x)\,,\quad -1<x<1\,,\tag{1}$$

is based on formulas like

$$\frac{1}{\pi}\int_{-1}^{1}\frac{T_n(y)\,dy}{(y-x)\sqrt{1-y^2}}=U_{n-1}(x)\,,\quad -1<x<1\,,\ n=0,1,2,\ldots\,,\tag{2}$$

where $T_n(x)$ and $U_n(x)$ are the normalized Chebyshev polynomials of degree $n$ and of first and second kind, respectively. Originally, this method was restricted to equations (1) with constant coefficients $a(x)\equiv a$ and $b(x)\equiv b$.

Basically, there exist two different ways to generalize the classical collocation method for equations with variable coefficients. The first one is by construction of generalized Jacobi polynomials satisfying a relation like (2) which is closely connected with the coefficients $a(x)$ and $b(x)$. The second one is still based on classical Chebyshev polynomials and their zeros independ from the coefficients in the equation (1). The numerical methods based on the first approach need much time for preprocessing, namely for the computation of the nodes and weights of generalized Jacobi polynomials. But the essential condition for their applicability is only the unique solvability of equation (1). The preprocessing for the methods based on the second approach is very cheap, but in general the invertibility of more than one operator is necessary and sufficient for their applicability. The investigation of the stability of such methods is based on the application of $C^*$-algebra techniques.

The talk gives an overview of these developments during the last 25 years and concentrates on recent results for equations of the form (1), where kernel functions $h(x,y)=k\left(\frac{1+x}{1+y}\right)\frac{1}{1+y}$ of Mellin type occur which are important in applications, for example in two-dimensional elasticity theory.