A survey of inverse problems relevant in applied geophysics

Giuseppe Rodriguez

Department of Mathematics and Computer Science
University of Cagliari, Italy

Opening Meeting for the Research Project GNCS 2016
"PING - Inverse Problems in Geophysics"
Florence, April 6, 2016

Rough classification of devices (and models)

- Low frequency: low resolution, large depth.
- High frequency: high resolution, small depth.

- Seismic prospecting, ground penetrating radar (GPR): observations are essentially a blurred version of reality.
- Frequency domain electromagnetics (FDEM): the amplitude and phase of an EM induced field are measured; the device works at a single frequency (or a finite number).
- Time domain electromagnetics (TDEM): the device measures the decaying of an impulse ($\sim \delta(x)$); infinite frequencies are involved.
- Electrical resistivity tomography, seismic tomography, magnetotellurics, ...

Functioning principle and applications

- Seismic and GPR: waves propagate in a ground and are sensed at a finite number of observation points.
- EM prospecting: a primary EM field induces eddy currents in the subsoil, which in turn produce a secondary EM field.

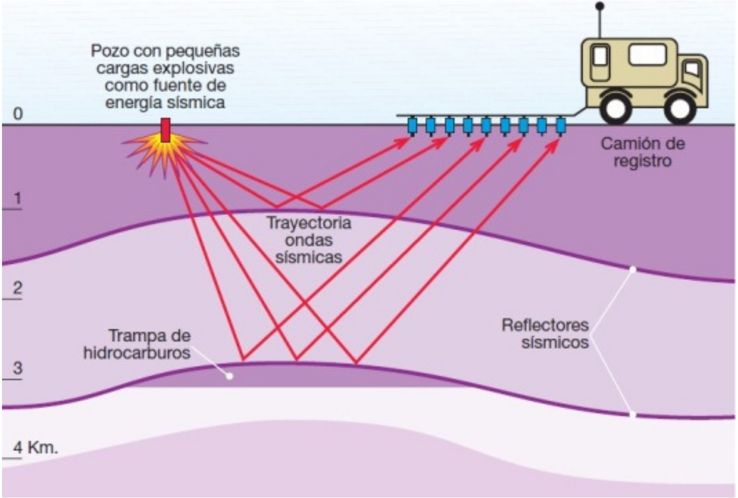
Functioning principle and applications

- Seismic and GPR: waves propagate in a ground and are sensed at a finite number of observation points.
- EM prospecting: a primary EM field induces eddy currents in the subsoil, which in turn produce a secondary EM field.

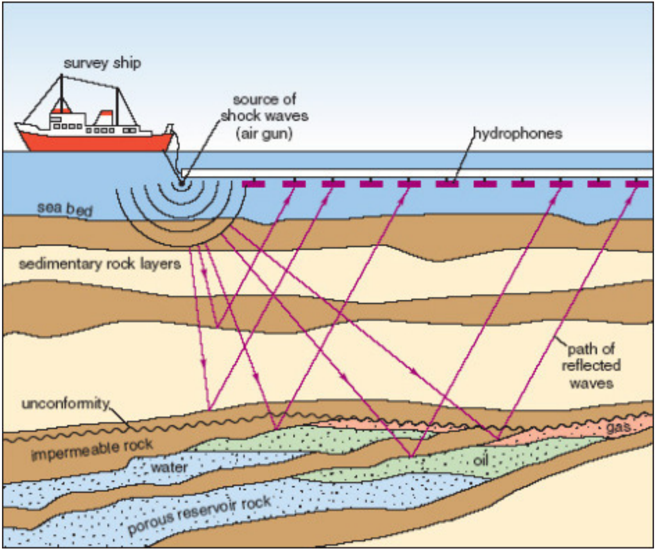
Applications are countless:

- hydrological and hydrogeological characterizations
- hazardous waste studies
- precision-agriculture applications
- archaeological surveys
- geotechnical investigations
- unexploded ordnance (UXO) detection
- ...

Land seismic/GPR prospecting



Marine seismic/GPR prospecting



GPR/EM land prospecting

Seismic wavefield modeling

$$
\left\{
\begin{array}{rcll}
\frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} + \beta \frac{\partial U}{\partial t} & = & \Delta U + F & \text{in } \ \Omega \times (0, T) \\
U(\mathbf{x}, t) & = & \Phi_0(\mathbf{x}, t) & \text{on } \Gamma_D \times (0, T) \\
\frac{\partial U}{\partial n}(\mathbf{x}, t) & = & \Phi_1(\mathbf{x}, t) & \text{on } \Gamma_N \times (0, T) \\
U(\mathbf{x}, 0) & = & U_0(\mathbf{x}) & \text{in } \Omega \\
\frac{\partial U}{\partial t}(\mathbf{x}, 0) & = & U_1(\mathbf{x}) & \text{in } \Omega
\end{array}
\right.
$$

$c(\mathbf{x})$ wave propagation velocity, $\beta(\mathbf{x})$ energy dissipation.

Seismic wavefield modeling

$$
\begin{cases}
\frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} + \beta \frac{\partial U}{\partial t} = \Delta U + F & \text{in } \Omega \times (0, T) \\
U(\mathbf{x}, t) = \Phi_0(\mathbf{x}, t) & \text{on } \Gamma_D \times (0, T) \\
\frac{\partial U}{\partial n}(\mathbf{x}, t) = \Phi_1(\mathbf{x}, t) & \text{on } \Gamma_N \times (0, T) \\
U(\mathbf{x}, 0) = U_0(\mathbf{x}) & \text{in } \Omega \\
\frac{\partial U}{\partial t}(\mathbf{x}, 0) = U_1(\mathbf{x}) & \text{in } \Omega
\end{cases}
$$

$c(\mathbf{x})$ wave propagation velocity, $\beta(\mathbf{x})$ energy dissipation.

In applications, it is common to assume that F as well as the boundary conditions have harmonic time-dependent behavior. As a consequence, the solution U is expected to exhibit a similar behavior as $t \to \infty$, that is, $U(\mathbf{x}, t) = u(\mathbf{x})\, e^{i\omega t}$. This leads to

$$
\begin{cases}
\Delta u + \kappa u = f & \text{in } \Omega \\
u = \varphi_0 & \text{on } \Gamma_D \\
\frac{\partial u}{\partial n} = \varphi_1 & \text{on } \Gamma_N
\end{cases}
$$

Some remarks

- Ω is often a semi-unbounded domain
- Sommerfeld (radiation) conditions are needed
- One must solve an identification problem
  - The incoming wave is (approximately) known
  - The outgoing wave is measured only at a small number of observation points
  - The physical parameters of the propagation medium must be determined
- This is often referred to as Full Waveform Inversion (FWI).
- A nonlinear, severely ill-conditioned, noisy, data fitting problem must be solved (least-squares or L^p norm)

$$\min_{\mathbf{p}} \|F(\mathbf{p}) - \mathbf{m}\|, \qquad \begin{cases} \mathbf{p}(\mathbf{x}) & \text{parameters of the model} \\ \mathbf{m} & \text{measurements} \end{cases}$$

Groung penetrating radar (GPR)

It is very similar, in principle, to seismic prospecting.

- The fundamental model consists of Maxwell's equations.
- Under suitable assumptions they can be reduced to Helmholtz's equation.
- All above remarks are valid.

The general approach is so difficult to cope with, that in both cases (seismic and radar) it is often simplified:

- from 3D to 2D, and even to 1D;
- keep into account the physical and geometrical peculiarities of a particular experimental setting.

Deconvolution

The easiest simplification assumes that the subsoil and the incoming wave interact via a convolution

$$s(t) = \int_0^\infty r(\lambda) w(t - \lambda)\, d\lambda,$$

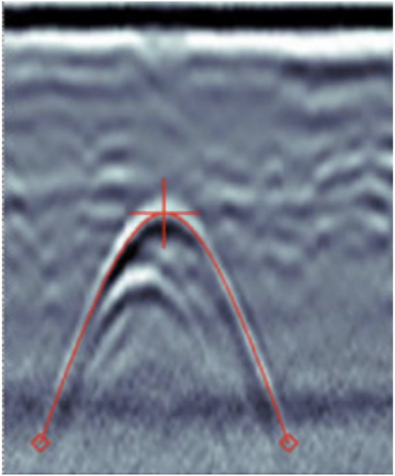
$s(t)$ is the measured trace, $w(t)$ is the probing signal (*wavelet*).

The impulse response $r(t)$ represents how the physical features of the ground modify the travelling wave.

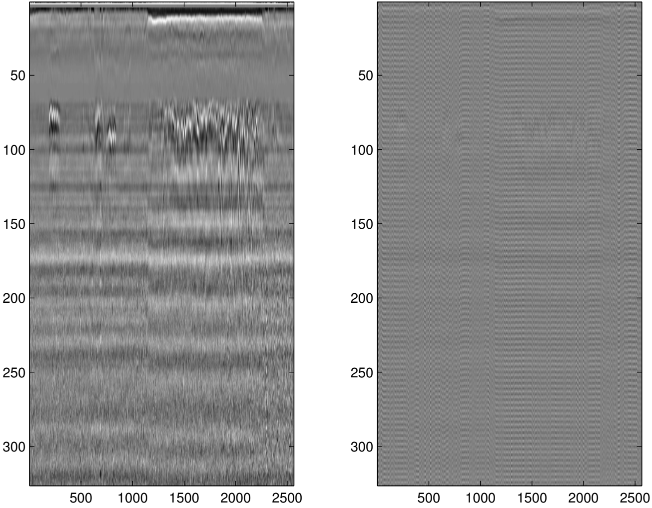
As the *wavelet* only approximates a *delta* function, the image produced by the device is blurred.

The *wavelet* is often not perfectly known, so blind deconvolution is an option.

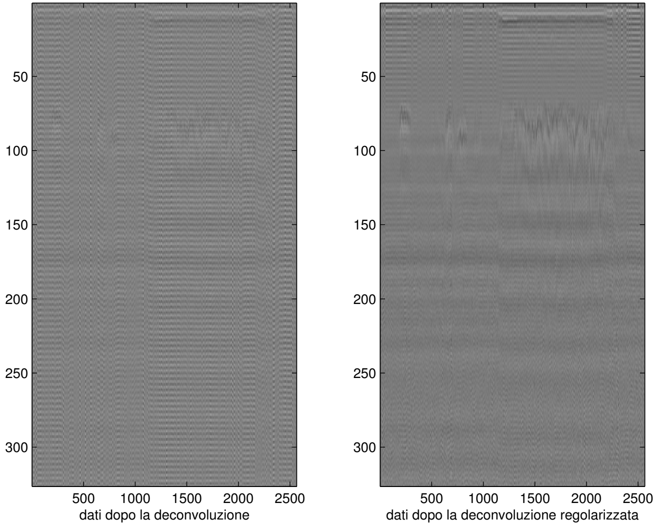
GPR deconvolution



GPR deconvolution



GPR deconvolution



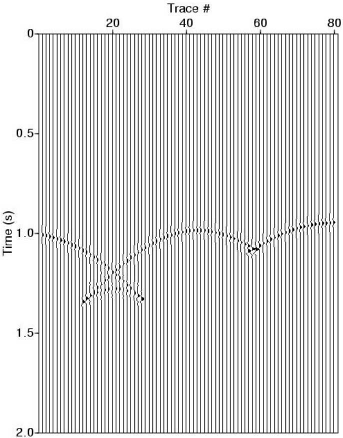
Migration

Each 1D deconvolution is independent, there is no feedback between adjacent inversions.
A very common procedure for coupling measurements is migration.

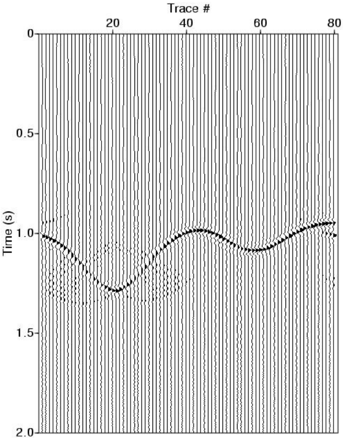
From Wikipedia: "Seismic migration is the process by which seismic events are geometrically re-located in either space or time to the location the event occurred in the subsurface rather than the location that it was recorded at the surface, thereby creating a more accurate image of the subsurface."

There are various migration procedures, some can be formulated as either 2D or 3D integral equations (Kirkhoff, Stolt, RTM, etc.).

Migration

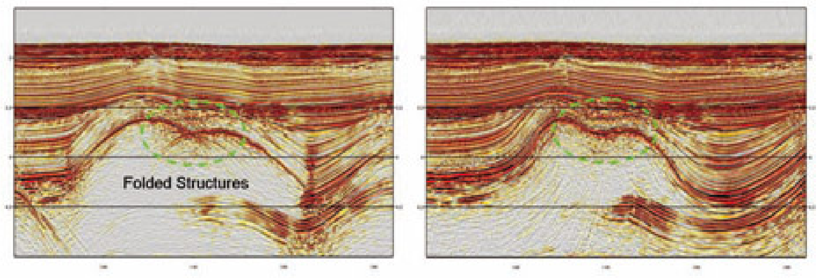


Simple Dataset

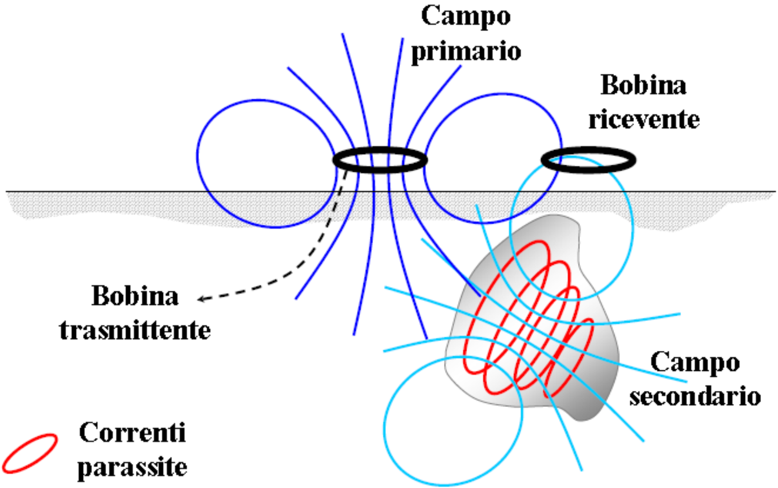


Phase shift

Migration



TDEM/FDEM prospecting



TDEM/FDEM prospecting

Instruments are generally constituted by a transmitting coil and one or more receiving coils.

In TDEM an electromagnetic pulse is generated, and the device senses the decay time of the induced EM field.

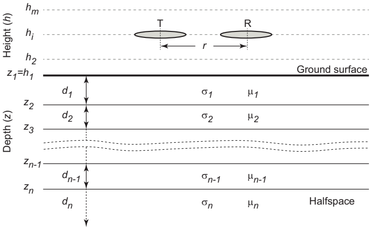
In FDEM the instrument generates a primary field at a single frequency, and measures the induced secondary field.

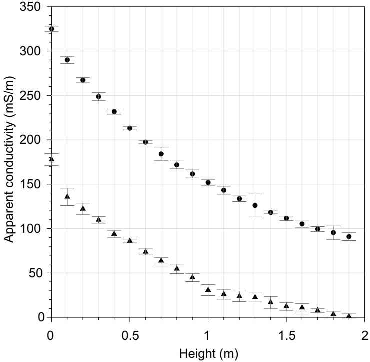
To obtain multiple measurements in FDEM one can vary:

- the frequency of the primary wave
- the orientation of the coils
- the distance between the coils
- the height above the ground

TDEM/FDEM prospecting

A data set contains information on the electromagnetic properties of the subsoil, assumed to possess a layered structure, but the graphical interpretation is less obvious.





A linear model for FDEM

$$m^V(h) = \int_0^\infty \phi^V(h+z)\sigma(z)\,dz$$
$$m^H(h) = \int_0^\infty \phi^H(h+z)\sigma(z)\,dz$$

where $\sigma(z)$ is the real conductivity,

$$\phi^V(z) = \frac{4z}{(4z^2+1)^{3/2}}, \qquad \phi^H(z) = 2 - \frac{4z}{(4z^2+1)^{1/2}},$$

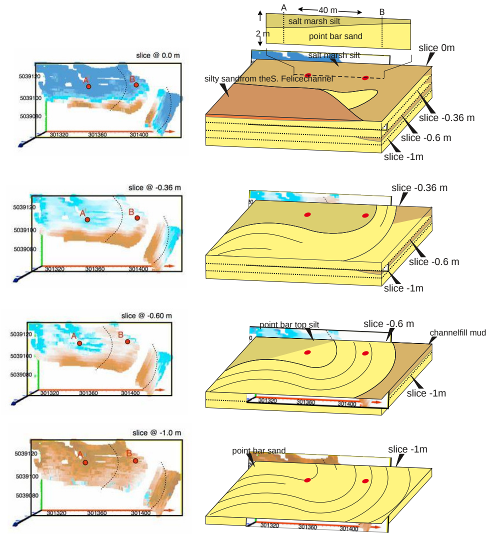
and z is the ratio between the depth and the inter-coil distance r.

The assumptions for this model to be applicable are very restrictive, while nonlinear models should be closer to reality.

Field data from the Venice Lagoon

$\sim$ 5000 measurements, 5 frequencies

Field data from the Venice Lagoon



$\sim$ 5000 measurements, 5 frequencies

Dealing with data inversion

- nonlinear minimization (Gauss-Newton, trust region, etc.)
- severe ill-conditioning implies regularization
- troubles in estimating the regularization parameter
  - unknown (and large) noise level
  - noise may not be equally distributed
- each 1D inversion is independent (but they can be coupled)
- in principle one can combine different data sets

$$\|F_0(\mathbf{x}) - \mathbf{m}_0\|^2 + \mu_1 \|F_1(\mathbf{x}) - \mathbf{m}_1\|^2 + \cdots$$

(how to choose μ_i?)

Inverse Problems in Science and Engineering

Giulio Vignoli

Geological Survey of Denmark and Greenland
Højbjerg, Denmark

Cagliari, May 23–27, 2016