Numerical solutions and their superconvergence for fractional differential equations

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We propose and analyze a numerical method for solving initial and boundary value problems for fractional differential equations with Caputo type fractional derivatives. Usually, we cannot expect the solutions of such equations to be smooth on the whole interval of integration, which is a challenge to the convergence analysis of numerical methods. Therefore, using an integral equation reformulation of the original problem, we first study the regularity of the exact solution. Based on the obtained smoothness properties and spline collocation techniques, the numerical solution of the problem is discussed. Optimal convergence estimates are derived and aspects related to the superconvergence of the proposed algorithms are presented. A numerical illustration is also given.