

Answers to Sample Exam 1

$$1) \quad A = \begin{pmatrix} 1 & 2 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 2 & -6 \end{pmatrix}, \quad \text{rank}(A) = 3.$$

$$2) \quad \begin{cases} x_1 = 9 - 3x_2 + 5x_5, \\ x_3 = -3 - 2x_5, \\ x_4 = 3 + 2x_5. \end{cases}$$

3) Solutions: $x_1 = -5x_3$ and $x_2 = 2x_3$.

$$4) \quad A^{-1} = \frac{1}{103} \begin{pmatrix} 7 & -8 \\ 5 & 9 \end{pmatrix}.$$

$$5) \quad A = \begin{pmatrix} \frac{2}{3} & 6 & -8 \\ \frac{3}{3} & 4 & -3 \end{pmatrix}.$$

$$6) \quad P = \begin{pmatrix} \frac{25}{169} & -\frac{60}{144} \\ \frac{169}{60} & \frac{169}{169} \end{pmatrix}.$$

$$7) \quad R = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}.$$

$$8) \quad A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & \frac{1}{4} & 0 \\ \frac{13}{4} & -\frac{7}{4} & 1 \end{pmatrix}.$$

$$9) \quad A^4 = \begin{pmatrix} 1 & 0 \\ 40a & 81 \end{pmatrix}.$$

$$10) \quad ABC = \begin{pmatrix} 6 & -18 & 33 \\ 4 & -6 & 15 \\ 0 & 24 & -8 \end{pmatrix}.$$