Cornelis VAN DER MEE, Spring 2008, Math 3330, Final Exam

Name:. . . . .**Grade**:. . . . .**Rank**:. . . . .
To receive full credit, show all of your work. Neither calculators nor computers are allowed.

ex.1	ex.2	ex.3	ex.4	ex.5	ex.6	ex.7	ex.8	ex.9	ex.10	ex.11	ex.12
				XX XX							
S1		S2		S3		S4		Final		Final Score	

1. Bring the following matrix to reduced row echelon form:

$$A = \begin{pmatrix} 0 & 0 & 0 & -1 & 2 & 3 \\ 1 & -3 & 2 & 0 & 1 & 7 \\ 0 & 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 & 0 & 6 \end{pmatrix},$$

and determine its rank and nullity.

2. Find **all** solutions of the linear system $A\vec{x} = \mathbf{0}$, where

$$A = \begin{pmatrix} 3 & 6 & 7 & -3 \\ 0 & 0 & 2 & -8 \end{pmatrix}.$$

3. Consider the following 4×4 matrix:

$$A = \begin{pmatrix} 0 & 1 & 2 & -8 \\ 1 & 0 & 0 & -5 \\ 0 & 0 & 2 & -8 \\ 0 & 3 & 0 & 0 \end{pmatrix}.$$

   a. Find a basis of the image of A and show that it really is a basis.
   b. Find a basis of the kernel of A and show that it really is a basis.

4. Argue why or why not the set of polynomials

$$35x^4 - 30x^2 + 3, \qquad 5x^3 - 3x, \qquad 3x^2 - 1, \qquad x, \qquad 1,$$

is a basis of the vector space of polynomials of degree ≤ 4.

6. Apply the Gram-Schmidt process to the given basis vectors of

$$V = \text{span}\left[\begin{pmatrix} 5 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 0 \\ 1 \end{pmatrix}\right]$$

to obtain an orthonormal basis of V.

7. Find a least-squares solution to the system

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix}.$$

8. Find the factors Q and R in the QR factorization of the matrix

$$M = \begin{pmatrix} 12 & 0 \\ 3 & 4 \\ 4 & -3 \end{pmatrix}$$

by using the Gram-Schmidt process.

9. Find the determinant of the 3×3 matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

Explain why or why not A^{-1} exists.

10. Compute the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 6 & -3 \\ -2 & 7 \end{pmatrix}.$$

Use this information to diagonalize the matrix A if possible. Otherwise indicate why diagonalization is not possible.

11. Find all eigenvalues (real and complex) of the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 4 & -2 & 0 \end{pmatrix}.$$

Why or why not is it possible to diagonalize the matrix A?

12. Consider the discrete dynamical system

$$x(n+1) = Ax(n), \qquad n = 0, 1, 2, 3, \dots,$$

where

$$A = \begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix}, \qquad x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

a. Write $x(1) = Ax(0)$ as a multiple of an eigenvector of A.

b. Compute the solution $x(n)$ for $n = 1, 2, 3, \dots$.