Cornelis VAN DER MEE, Spring 2008, Math 3330, Sample Exam 4

Name:............................Grade:..........Rank:...........

To receive full credit, show all of your work. Neither calculators nor computers are allowed.

1. Compute the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 3 & -2 \\ 15 & -10 \end{pmatrix}.$$

   Use this information to diagonalize the matrix A if possible. Otherwise indicate why diagonalization is not possible.

2. Find a 2×2 matrix A such that $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ are eigenvectors of A, with eigenvalues -2 and 3, respectively.

3. Find the solution of the discrete dynamical system

$$x(n+1) = Ax(n), \qquad n = 0, 1, 2, 3, \ldots,$$

   where

$$A = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}, \qquad x(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

4. Find all eigenvalues (real and complex) of the matrix

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -4 & 1 \end{pmatrix}.$$

   Why or why not is it possible to diagonalize the matrix A?

5. Compute the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 3 & -3 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

   Use this information to diagonalize the matrix A if possible. Otherwise indicate why diagonalization is not possible.

6. Compute the eigenvalues (real and complex) and corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Explain why your result is in full agreement with the values of $\mathrm{Tr}(A)$ and $\det(A)$.

7. Why is $45 = 1+2+3+4+5+6+7+8+9$ an eigenvalue corresponding to any 9×9 sudoku matrix? What is the corresponding eigenvector?