Cornelis VAN DER MEE, Spring 2008, Math 3330, Exam 3

Name:. . . . .**Grade**:. . . . .**Rank**:. . . . .

To receive full credit, show all of your work. Neither calculators nor computers are allowed.

ex.1	ex.2	ex.3	ex.4	ex.5	ex.6	S1	S2	S3

1. Consider the two vectors

$$\vec{\boldsymbol{u}} = \begin{pmatrix} -15 \\ 20 \end{pmatrix}, \qquad \vec{\boldsymbol{v}} = \begin{pmatrix} 7 \\ 24 \end{pmatrix}.$$

   a. Compute the lengths of $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$.

   b. Compute the cosine of the angle between $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$.

   c. Construct an orthogonal 2×2 matrix A such that $A\vec{\boldsymbol{u}} = \vec{\boldsymbol{v}}$.

   d. Is it possible to choose the matrix A in part c in such a way that $\det(A) = 1$? If it is possible, compute such an orthogonal matrix A and explain its geometrical meaning. If it is not possible, argue why not.

2. Find an orthonormal basis for

$$V = \text{span}\left[\begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 2 \end{pmatrix}\right]$$

   and use this information to write down the orthogonal projection of $\mathbb{R}^4$ onto V.

3. Find a least-squares solution to the system

$$\begin{pmatrix} 3 & 4 \\ -4 & 3 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}.$$

4. Find the determinant of the 3×3 matrix

$$A = \begin{pmatrix} 1 & 2 & -5 \\ -1 & 1 & 8 \\ 3 & 3 & 7 \end{pmatrix}.$$

Describe the parallelepiped whose volume is given by this determinant.

5. Find the determinants of the matrices

$$A = \begin{pmatrix} 0 & 0 & 0 & 5 \\ 4 & 3 & 9 & -7 \\ 0 & 3 & 2 & -2 \\ 0 & 0 & 2 & 7 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 9 & -8 & 0 & 0 & 5 \end{pmatrix}.$$

6. Let A be an 8×8 matrix with $\det(A) = -2$.

   a. Compute $\det(-\sqrt{2}\, A)$.

   b. Compute $\det(A^T A^3)$.

   c. Compute $\det(SA^2S^{-1})$, where S is an 8×8 matrix satisfying $\det(S) = 7$.

   d. Compute the determinant of the matrix obtained from A by first interchanging the first two columns, then interchanging the last two columns, and then dividing the second row by 2.