Cornelis VAN DER MEE, Spring 2008, Math 3330, Exam 3a

Name:................................**Grade**:..........**Rank**:...........

To receive full credit, show all of your work. Neither calculators nor computers are allowed.

1. Consider the two vectors

$$\vec{\boldsymbol{u}} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}, \qquad \vec{\boldsymbol{v}} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}.$$

   a. Compute the cosine of the angle between $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$.

   b. Compute the distance between $\vec{\boldsymbol{u}}$ and $\vec{\boldsymbol{v}}$.

   c. Does there exist an orthogonal 3×3 matrix A such that $A\vec{\boldsymbol{u}} = \vec{\boldsymbol{v}}$? If it exists, construct one. If it does not exist, explain why not.

2. Find an orthonormal basis for

$$V = \text{span}\left[\begin{pmatrix} -1 \\ 3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}\right]$$

   and use this information to write down the orthogonal projection of $\mathbb{R}^4$ onto V.

3. Find a least-squares solution to the system

$$\underbrace{\begin{pmatrix} 3 & 4 \\ -4 & 0 \\ 0 & 2 \end{pmatrix}}_{=A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}_{=\vec{\boldsymbol{x}}} = \underbrace{\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}}_{=\vec{\boldsymbol{u}}}.$$

   Compute the distance from the vector $\vec{\boldsymbol{u}}$ to the image of A.

4. Compute the volume of the parallelepiped spanned by the columns of the 3×3 matrix

$$A = \begin{pmatrix} 1 & 2 & -3 \\ -2 & 1 & 4 \\ 2 & 3 & 1 \end{pmatrix}.$$

5. Find the determinants of the matrices

$$A = \begin{pmatrix} 1 & 3 & 9 & 27 \\ 1 & 2 & 4 & 8 \\ 1 & -2 & 4 & -8 \\ 1 & -3 & 9 & -27 \end{pmatrix}, \qquad B = \begin{pmatrix} 3 & 0 & 0 & 9 \\ 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}.$$

6. Let A be a 6×6 matrix with $\det(A) = -3$.

   a. Compute $\det(\sqrt{5}\, A)$.

   b. Compute $\det(A^{-1} A^T A)$.

   c. Compute $\det(SAS^{-1}A)$, where S is a 6×6 matrix satisfying $\det(S) = 17$.

   d. Compute the determinant of the matrix obtained from A by first interchanging the last two columns, then interchanging the last two rows, and then multiplying the second row by 3.