$\mapsto$ f è continua

$f : [a,b] \to \mathbb{R}$ biunivoca, derivabile, $f[a,b] = [c,d]$

$g : [c,d] \to \mathbb{R}$ continua

$$\int_a^b g(f(x))\, f'(x)\,dx = \int_{f(a)}^{f(b)} g(t)\,dt$$

$t = f(x)$

$dt = f'(x)\,dx$

$$\int_1^4 \frac{1}{1+\sqrt{x}} \frac{dx}{2\sqrt{x}} \overset{t=\sqrt{x}}{\underset{dt = \frac{1}{2\sqrt{x}}dx}{=\!=}} \int_1^2 \frac{1}{1+t}\,dt = \Big[\ln(1+t)\Big]_1^2 = \ln 3 - \ln 2$$

$$= \ln(3/2)$$

$\int R(\cos x, \operatorname{sen} x)\, dx$

$t = \operatorname{tg}\frac{1}{2}x$ in un sottointervallo di $x \in (-\pi, \pi)$, $t \in (-\infty, \infty)$

$x = 2 \operatorname{arctg} t \qquad dx = \frac{2}{1+t^2}\, dt$

$$\frac{1}{1+t^2} = \frac{1}{1+\operatorname{tg}^2\frac{1}{2}x} = \frac{\cos^2\frac{1}{2}x}{\cos^2\frac{1}{2}x + \operatorname{sen}^2\frac{1}{2}x} = \cos^2\frac{1}{2}x$$

$$\operatorname{sen} x = 2 \operatorname{sen}\frac{1}{2}x \cos\frac{1}{2}x$$

$$\cos x = 2\cos^2\frac{1}{2}x - 1 = \frac{2}{1+t^2} - 1 = \frac{1-t^2}{1+t^2}$$

$$= 2\operatorname{tg}\frac{1}{2}x \cos^2\frac{1}{2}x = \frac{2t}{1+t^2}$$

$$\int \frac{dx}{\operatorname{sen} x} = \int \frac{\cancel{1+t^2}}{\cancel{2}t} \frac{\cancel{2}dt}{\cancel{1+t^2}} = \int \frac{dt}{t} = \ln|t| + \text{cost.}$$

$$= \ln\left|\operatorname{tg}\tfrac{1}{2}x\right| + \text{cost.}$$

$$\int R\left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}\right)\frac{2}{1+t^2}\,dt = \int R(\cos x, \operatorname{sen} x)\,dx$$

$$\int_{\pi/3}^{\pi/2} \frac{dx}{\operatorname{sen} x} = \int_{\frac{1}{3}\sqrt{3}}^{1} \frac{\cancel{1+t^2}}{\cancel{2}t}\frac{\cancel{2}\,dt}{\cancel{1+t^2}} = \Big[\ln t\Big]_{t=\frac{1}{3}\sqrt{3}}^{1} = \underset{\substack{\uparrow \\ 0}}{\ln 1} - \ln\left(\tfrac{1}{3}\sqrt{3}\right) = \tfrac{1}{2}\ln 3$$

$x = \pi/3 \longrightarrow t = \operatorname{tg}\frac{\pi}{6} = \frac{1}{3}\sqrt{3}$

$x = \pi/2 \longrightarrow t = \operatorname{tg}\frac{\pi}{4} = 1$

$$\Big[\ln\left|\operatorname{tg}\tfrac{1}{2}x\right|\Big]_{\pi/3}^{\pi/2}$$

$$\int_{0}^{\pi/3} \frac{dx}{\cos x} = \int_{0}^{\frac{1}{3}\sqrt{3}} \frac{\cancel{1+t^2}}{1-t^2} \, \frac{2\,dt}{\cancel{1+t^2}} = \Big[\ln|t+1| - \ln|t-1|\Big]_0^{\frac{1}{3}\sqrt{3}}$$

$$= \left[\ln \frac{1+t}{1-t}\right]_0^{\frac{1}{3}\sqrt{3}} = \ln \frac{1+\frac{1}{3}\sqrt{3}}{1-\frac{1}{3}\sqrt{3}}$$

$t = \operatorname{tg} \frac{1}{2} x$

$dx = \frac{2\,dt}{1+t^2}$

$\cos x = \frac{1-t^2}{1+t^2}$

$$\frac{2}{1-t^2} = \frac{A}{t-1} + \frac{B}{t+1} = \frac{A(t+1) + B(t-1)}{t^2-1}$$

$$= \frac{-1}{t-1} + \frac{1}{t+1}$$

$A + B = 0$

$A - B = -2$

$A = -B = -1$

$x = 0 \to t = \operatorname{tg} 0 = 0$

$x = \frac{\pi}{3} \to t = \operatorname{tg} \frac{\pi}{6} = \frac{1}{3}\sqrt{3}$

$$= \ln\left(\frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}\right) = \ln \frac{4+2\sqrt{3}}{2} = \ln\left(2+\sqrt{3}\right)$$

$$\int_{\emptyset}^{\pi/3} tg^2 x \, dx = \int_{\emptyset}^{\frac{1}{3}\sqrt{3}} \frac{2t}{1-t^2} \cdot \frac{2\,dt}{1+t^2}$$

$$t = tg \tfrac{1}{2} x$$

$$d\cancel{x} = \frac{2}{1+t^2}\, dt$$

$$tg\, x = \frac{sen \cancel{t}}{cos\, x} = \frac{2t}{1-t^2}$$

$$sen\, x = \frac{2t}{1+t^2}$$

$$cos\, x = \frac{1-t^2}{1+t^2}$$

$$\int R(\cos^2 x, \operatorname{sen}^2 x)\, dx$$

$t = \operatorname{tg} x$

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \longleftrightarrow t \in (-\infty, \infty)$

$x = \operatorname{arctg} t \to dx = \frac{dt}{1+t^2}$

$$\int_{0}^{\pi/3} \operatorname{tg}^2 x\, dx \overset{t=\operatorname{tg} x}{=\!=\!=} \int_{0}^{\sqrt{3}} t^2 \frac{dt}{1+t^2}$$

$$= \int_0^{\sqrt{3}} \left(1 - \frac{1}{1+t^2}\right) dt$$

$$= \Big[t - \operatorname{arctg} t\Big]_0^{\sqrt{3}} = \sqrt{3} - \operatorname{arctg}(\sqrt{3})$$

$$= \sqrt{3} - \frac{\pi}{3}$$

$$\int_0^{\pi/4} \cos^2 x \, dx$$

$$t = tg\, x \qquad dx = \frac{dt}{1+t^2}$$

$$\cos^2 x = \frac{\cos^2 x}{\cos^2 x + sen^2 x} = \frac{1}{1 + tg^2 x} = \frac{1}{1+t^2}$$

$$\|$$

$$\int_0^1 \frac{1}{1+t^2} \frac{dt}{1+t^2} = \int_0^1 \frac{dt}{(1+t^2)^2}$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\int_0^{\pi/4} \frac{1+\cos 2x}{2} dx = \left[\frac{1}{2}x + \frac{1}{4} sen\, 2x\right]_0^{\pi/4} = \frac{\pi}{8} + \frac{1}{4}$$

$$\int_{\pi/4}^{3\pi/4} \cos^2 x \, dx \overset{?}{=} \cancel{\int_{1}^{-1}} \frac{dt}{(1+t^2)^2} = -\int_{-1}^{1} \frac{dt}{(1+t^2)^2} < 0$$

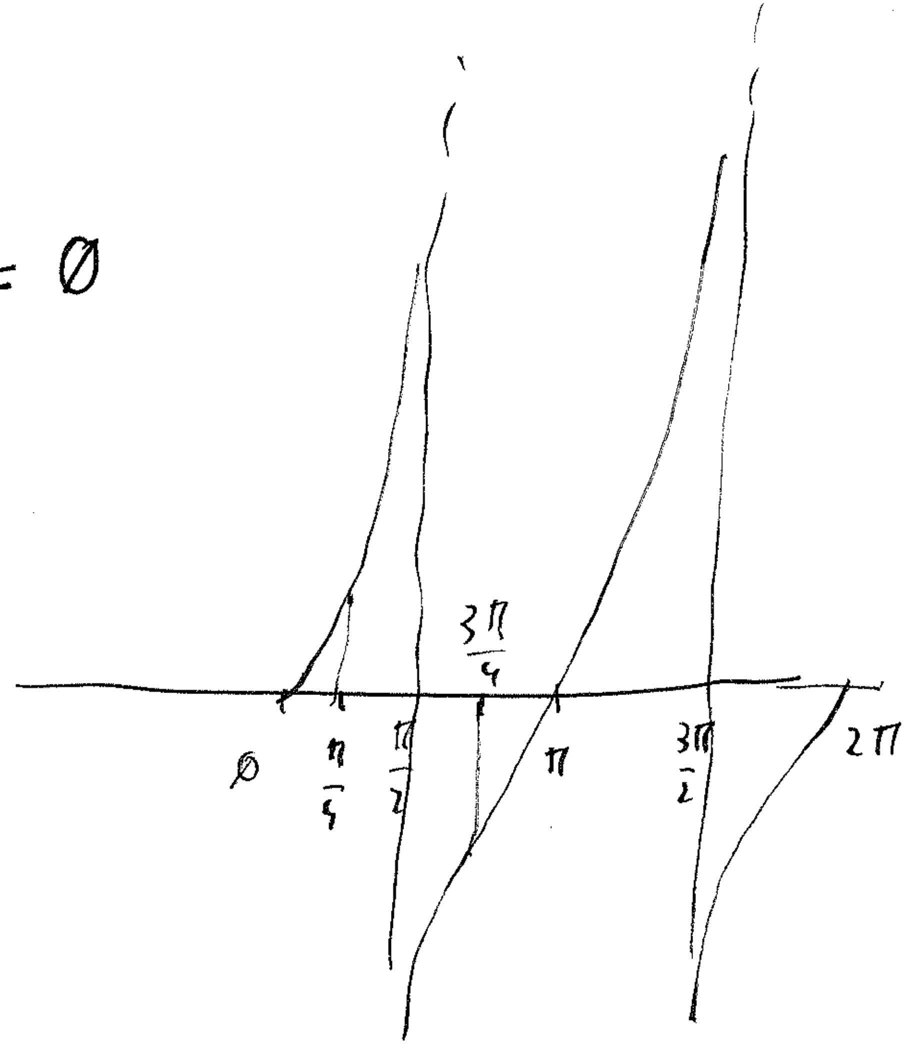
$$\int_1^{\infty} + \int_{-\infty}^{-1}$$

$$\int_0^{2\pi} \cos^2 x \, dx \overset{?}{=} \cancel{\int_0^0} \frac{dt}{(1+t^2)^2} = 0$$

$$\int_0^{\infty} + \int_{-\infty}^{\infty} + \int_{-\infty}^{0}$$

$t = \operatorname{tg} x$

$dx = \dfrac{dt}{1+t^2} \qquad \cos^2 x = \dfrac{1}{1+t^2}$



$$\int_{1/e}^{e} \frac{dx}{x(1+\ln^2 x)} = \int_{-1}^{1} \frac{dt}{1+t^2} = \left[\operatorname{arctg} t\right]_{-1}^{1} = \operatorname{arctg} 1 - \operatorname{arctg}(-1)$$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

$t = \ln x \qquad dt = \dfrac{dx}{x}$

$$\int_{\emptyset}^{4} x\sqrt{16-x^2}\,dx = \int_{16}^{\emptyset} -\frac{1}{2}\sqrt{t}\quad dt = \int_{\emptyset}^{16} \frac{1}{2}\sqrt{t}\,dt = \left[\frac{1}{3}t^{3/2}\right]_{0}^{16} = \frac{1}{3}(64-\emptyset) = \frac{64}{3}$$

$t = 16 - x^2 \qquad dt = -2x\,dx$

$$\int_{\emptyset}^{\frac{2}{3}\pi} \frac{dx}{2+\cos x} = \int_{\emptyset}^{\sqrt{3}} \frac{1}{2+\frac{1-t^2}{1+t^2}} \frac{2\,dt}{1+t^2} = \int_{\emptyset}^{\sqrt{3}} \frac{2\,dt}{2(1+t^2)+1-t^2}$$

$t = \operatorname{tg}\frac{1}{2}x \quad dx = \frac{2\,dt}{1+t^2}$

$$= \int_{0}^{\sqrt{3}} \frac{2\,dt}{3+t^2}$$

$t^2 = 3u^2$

$t = u\sqrt{3}$

$dt = du\sqrt{3}$

$\cos x = \frac{1-t^2}{1+t^2}$

$$= \int_{\emptyset}^{3} \frac{2\sqrt{3}\,du}{3(1+u^2)} = \left[\frac{2}{3}\sqrt{3}\ \operatorname{arctg} u\right]_0^3 = \frac{2}{3}\sqrt{3}\ \operatorname{arctg} 3$$

$$\int_0^1 \frac{x}{1+x^4}\,dx = \int_0^1 \frac{\frac{1}{2}\,dt}{1+t^2} = \left[\frac{1}{2}\operatorname{arctg} t\right]_0^1 = \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}$$

$t = x^2 \qquad dt = 2x\,dx$

$$\int \frac{x}{1+x^4}\,dx = \int \frac{\frac{1}{2}\,dt}{1+t^2} = \frac{1}{2}\operatorname{arctg} t + \text{cost.} = \frac{1}{2}\operatorname{arctg}(x^2) + \text{cost.}$$

$$\int_{0}^{\pi/6} \operatorname{sen}^5 x \cos^3 x \, dx = \int_{0}^{tg(\pi/12)} \left(\frac{2t}{1+t^2}\right)^5 \left(\frac{1-t^2}{1+t^2}\right)^3 \frac{2\,dt}{1+t^2}$$

$t = tg\,\frac{1}{2}x \qquad dx = \dfrac{2\,dt}{1+t^2}$

$$= \int_{0}^{2-\sqrt{3}} \frac{64\, t^5 (1-t^2)^3}{(1+t^2)^9}\, dt$$

$\operatorname{sen} x = \dfrac{2t}{1+t^2} \qquad \cos x = \dfrac{1-t^2}{1+t^2}$

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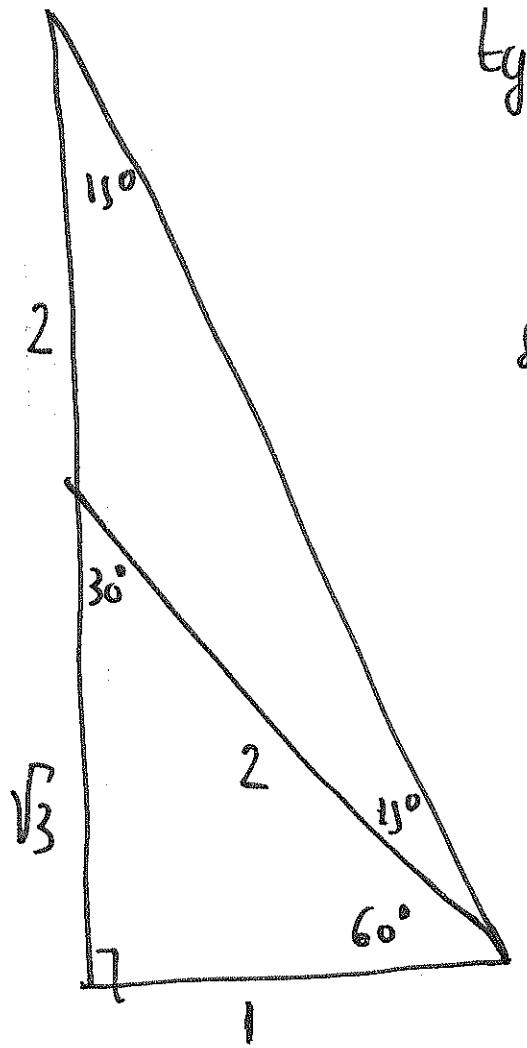
$t = \operatorname{sen} x \qquad dt = \cos x \, dx$

$$\underset{\operatorname{sen}^5 x}{t^5}\ \underset{\cos^2 x}{(1-t^2)}\ \underset{\cos x\, dx}{dt}$$

$$\Rightarrow \int_{0}^{1/2} t^5 (1-t^2)\, dt = \left[\frac{t^6}{6} - \frac{t^8}{8}\right]_0^{1/2} = \frac{1}{6.64} - \frac{1}{8.256} = \frac{48-3}{3*2048}$$

$\cos^3 x = \cos^2 x \cos x = (1-\operatorname{sen}^2 x)\cos x$




$$\operatorname{tg} 15° = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 2-\sqrt{3}$$

$$\operatorname{sen} 15° = \frac{\sqrt{6}-\sqrt{2}}{4} \qquad \cos 15° = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\int_0^{\pi/4} \operatorname{sen}^3 x \, dx = \int_0^{\pi/4} (1-\cos^2 x) \operatorname{sen} x \, dx$$

$$t = \cos x \qquad dt = -\operatorname{sen} x \, dx$$

$$\int_1^{\frac{1}{2}\sqrt{2}} (1-t^2) \cdot -dt = \int_{\frac{1}{2}\sqrt{2}}^{1} (1-t^2) \, dt = \left[ t - \frac{1}{3} t^3 \right]_{\frac{1}{2}\sqrt{2}}^{1}$$

$$= \frac{2}{3} - \left( \frac{1}{2}\sqrt{2} - \frac{1}{12}\sqrt{2} \right) = \frac{2}{3} - \frac{5}{12}\sqrt{2}$$

$$\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx = \int_0^{\pi} \frac{1}{2}\cos t\,dt = \left[\frac{1}{2}\operatorname{sen} t\right]_0^{\pi} = \emptyset$$

$t = x^2 \qquad dt = 2x\,dx$

$$\int_0^1 \frac{\operatorname{arcsen}\left(\frac{1}{2}x\right)}{\sqrt{4-x^2}}\,dx = \int_0^{1/2} \frac{2\operatorname{arcsen} t}{\sqrt{4(1-t^2)}}\,dt = \int_0^{1/2} \frac{\operatorname{arcsen} t}{\sqrt{1-t^2}}\,dt$$

$t = \frac{1}{2}x \qquad dt = \frac{1}{2}dx \qquad dx = 2\,dt \qquad\qquad u = \operatorname{arcsen} t \qquad du = \frac{dt}{\sqrt{1-t^2}}$

$$\rightarrow \int_{\emptyset}^{\pi/6} u\,du = \left[\frac{1}{2}u^2\right]_0^{\pi/6} = \frac{\pi^2}{72}$$

$$\int \frac{\operatorname{arcsen}\left(\frac{1}{2}x\right)}{\sqrt{4-x^2}}\,dx = \frac{1}{2}u^2 + \text{cost.} = \frac{1}{2}\operatorname{arcsen}^2\left(\frac{1}{2}x\right) + \text{cost.}$$