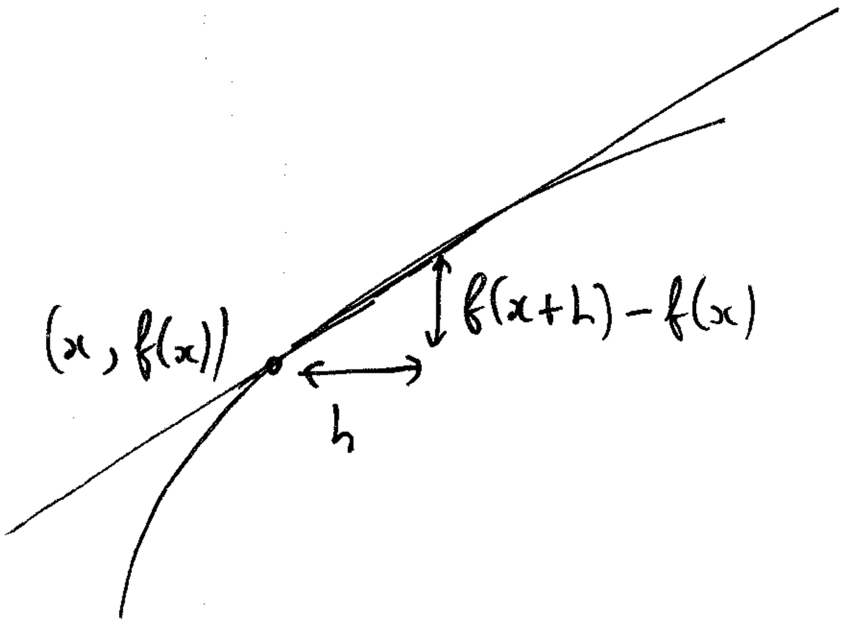
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Retta tangente in $(x_0, f(x_0))$



$$y - f(x_0) = f'(x_0)(x - x_0)$$

$f(x) \equiv$ costante $\quad f'(x) = 0$

$$f(x) = x^n \qquad f'(x) = n x^{n-1}$$

$$f(x) = e^x \qquad f'(x) = e^x$$

$$f(x) = \ln x = \log_e(x) \rightarrow f'(x) = \frac{1}{x}$$

$$(f \pm g)' = f' \pm g'$$

$$(cf)' = cf'$$

$$(fg)' = fg' + f'g$$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

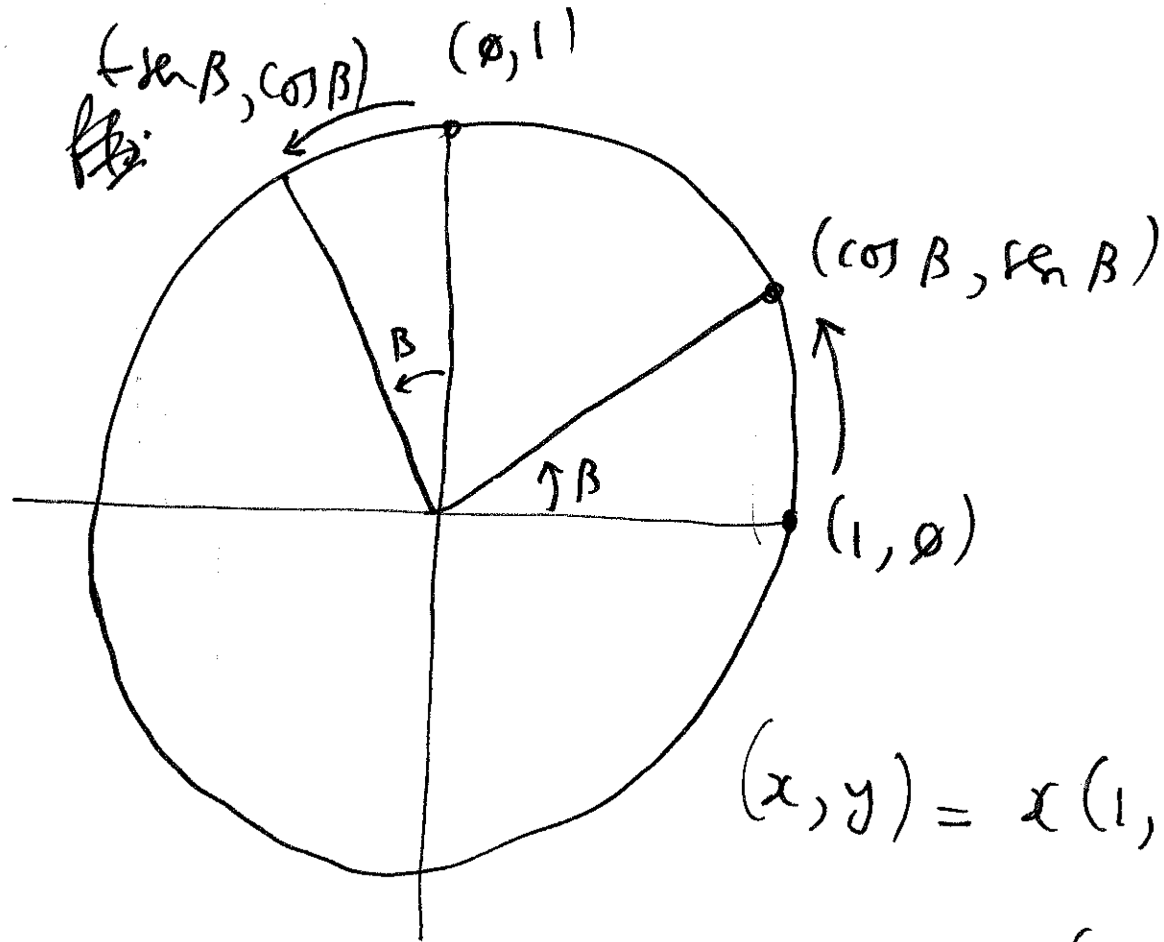
$$x \rightarrow 1$$

$$x^2 \rightarrow 2x$$

$$x^3 \rightarrow 3x^2$$

$$\frac{1}{x} = x^{-1} \rightarrow \frac{-1}{x^2} = -x^{-2}$$

$$\sqrt{x} = x^{1/2} \rightarrow \frac{1}{2\sqrt{x}} = \frac{1}{2} x^{-1/2}$$



$$(x, y) = x(1, \emptyset) + y(\emptyset, 1)$$

$$\Rightarrow x(\cos\beta, \operatorname{sen}\beta) + y(-\operatorname{sen}\beta, \cos\beta)$$

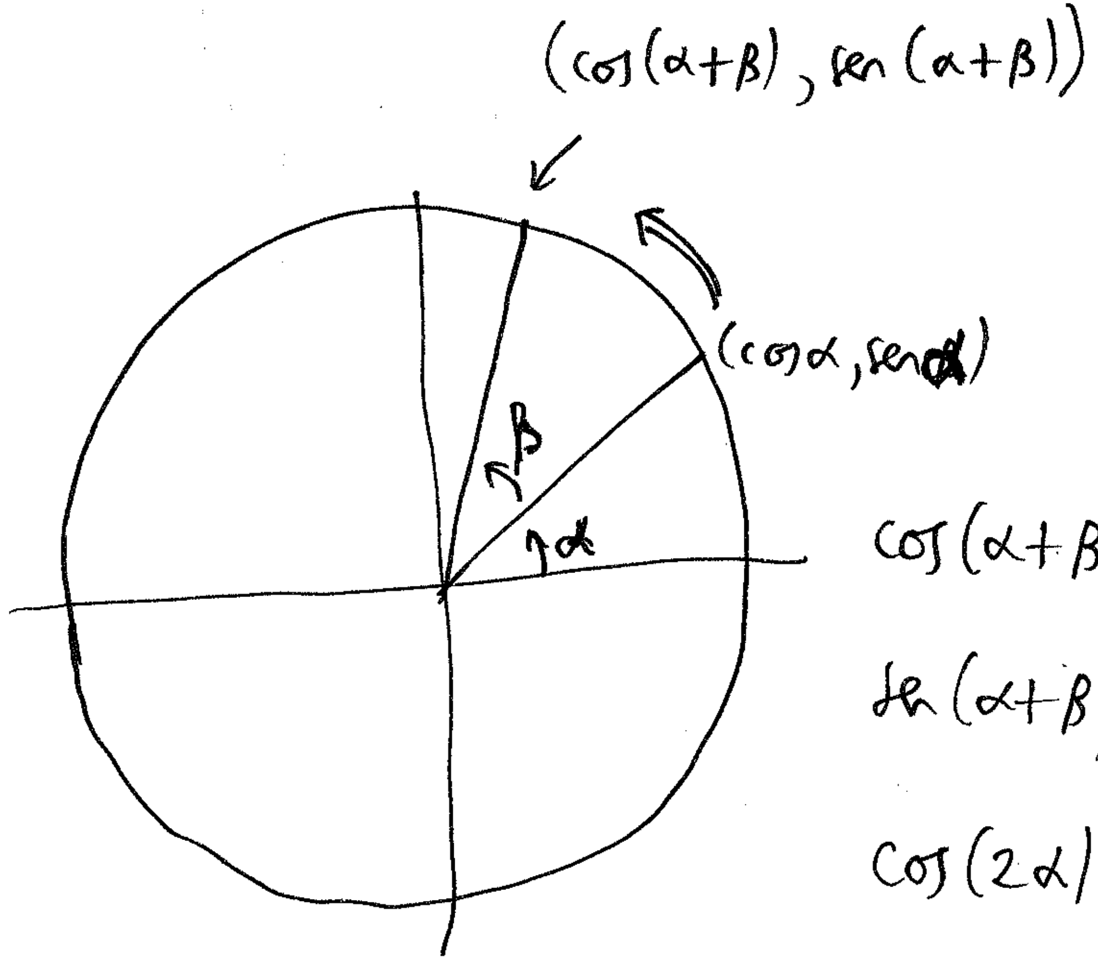
$$= (x\cos\beta - y\operatorname{sen}\beta,\ y\cos\beta + x\operatorname{sen}\beta)$$

$$\cos^2\alpha = 1 - \operatorname{sen}^2\alpha$$

$$\operatorname{sen}^2\alpha = 1 - \cos^2\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - [1 - \cos^2\alpha] = 2\cos^2\alpha - 1$$

$$\cos(2\alpha) = [1 - \operatorname{sen}^2\alpha] - \operatorname{sen}^2\alpha = 1 - 2\operatorname{sen}^2\alpha$$



$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cos\beta + \cos\alpha\operatorname{sen}\beta$$

$$\cos(2\alpha) = \cos^2\alpha - \operatorname{sen}^2\alpha$$

$$\operatorname{sen}(2\alpha) = 2\operatorname{sen}\alpha\cos\alpha$$

$$\frac{\operatorname{sen}(x+h)-\operatorname{sen}(x)}{h} = \frac{\operatorname{sen} x \cos h + \cos x \operatorname{sen} h - \operatorname{sen} x}{h}$$

$$= \operatorname{sen} x \, \frac{\cos h - 1}{h} + \cos x \, \frac{\operatorname{sen} h}{h} \longrightarrow \cos x$$

$$\frac{\cos(x+h)-\cos(x)}{h} = \frac{\cos x \cos h - \operatorname{sen} x \operatorname{sen} h - \cos x}{h}$$

$$= \cos x \, \frac{\cos h - 1}{h} - \operatorname{sen} x \, \frac{\operatorname{sen} h}{h} \longrightarrow - \operatorname{sen} x$$

$$\lim_{h \to 0} \frac{\operatorname{sen} h}{h} = 1$$

$$\frac{\cos h - 1}{h} \times \frac{\cos h + 1}{\cos h + 1} = \frac{-\operatorname{sen}^2 h}{h[\cos h + 1]}$$

$$= -\left(\frac{\operatorname{sen} h}{h}\right)^2 \frac{h}{\cos h + 1} \longrightarrow -(1)^2 \frac{\emptyset}{1+1} = \emptyset$$

$$f(x) = \operatorname{sen}(x) \longrightarrow f'(x) = \cos(x)$$

$$f(x) = \cos(x) \longrightarrow f'(x) = -\operatorname{sen}(x)$$

$$\left(\frac{f}{g}\right)' = \frac{g f' - f g'}{g^2}$$

$$\operatorname{tg} x = \frac{\operatorname{sen} x}{\cos x} \rightarrow \frac{d}{dx} \operatorname{tg} x = \frac{(\cos x)\frac{d}{dx}\operatorname{sen} x - \operatorname{sen} x \frac{d}{dx}\cos x}{\cos^2 x}$$

$$= \frac{(\cos x)\cos x - \operatorname{sen} x \cdot (-\operatorname{sen} x)}{\cos^2 x} = \frac{\cos^2 x + \operatorname{sen}^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\operatorname{cotg} x = \frac{\cos x}{\operatorname{sen} x} \rightarrow \frac{d}{dx} \operatorname{cotg} x = \frac{(\operatorname{sen} x)(-\operatorname{sen} x) - (\cos x)(\cos x)}{\operatorname{sen}^2 x} = \frac{-1}{\operatorname{sen}^2 x}$$

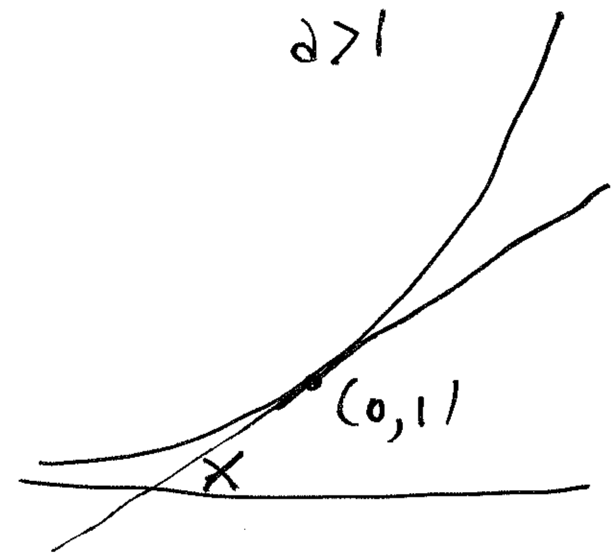
$$h(x) = (g \circ f)(x) = g(f(x))$$

$$h'(x) = g'(f(x)) . f'(x)$$

$$\frac{d}{dx} \operatorname{sen}(x^2) = \cos(x^2) . 2x = 2x \cos(x^2)$$

$$\frac{d}{dx} \operatorname{sen}^2 x = 2 \operatorname{sen} x \frac{d}{dx} \operatorname{sen} x = 2 \operatorname{sen} x \cos x = \operatorname{sen}(2x)$$

$a > 0 \qquad b = \ln a \longleftrightarrow e^b = a$



$$f(x) = a^x = \left(e^b\right)^x = e^{bx}$$

$$f'(x) = e^{bx} \frac{d}{dx}(bx) = a^x \cdot b = a^x \ln a$$

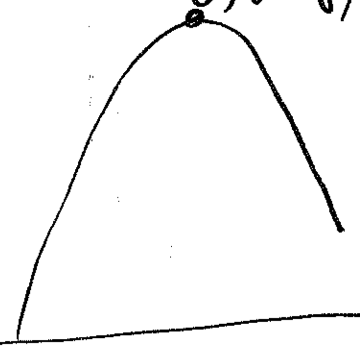
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h \to 0} \frac{a^x a^h - a^x}{h}$$

$$= a^x \underbrace{\lim_{h \to 0} \frac{a^h - 1}{h}}_{\ln a} = a^x \lim_{h \to 0} \frac{f(h) - f(0)}{h} = a^x f'(0)$$

MASSIMO:

$f(x) \leq f(x_0)$ se $|x - x_0|$ è abbastanza piccola

$(x_0, f(x_0))$



$$\frac{f(x_0+h) - f(x_0)}{h} = \frac{\leq 0}{h \text{ positiva}} \xrightarrow{h \to 0^+} \leq 0$$

$$\frac{f(x_0+h) - f(x_0)}{h} = \frac{\leq 0}{h \text{ negativa}} \xrightarrow{h \to 0^-} \geq 0$$

$\Big\} \; f'(x_0) = 0$

$\boxed{\text{Teorema di FERMAT}}$

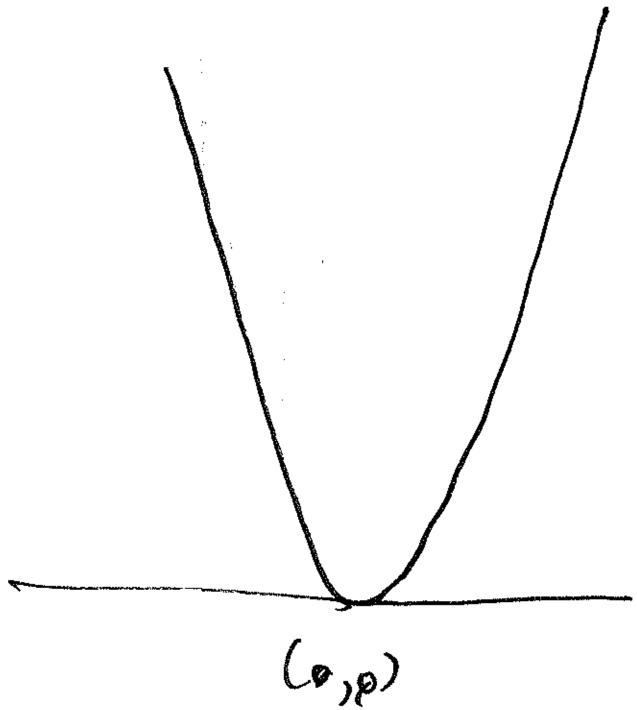
$(x_0, f(x_0))$

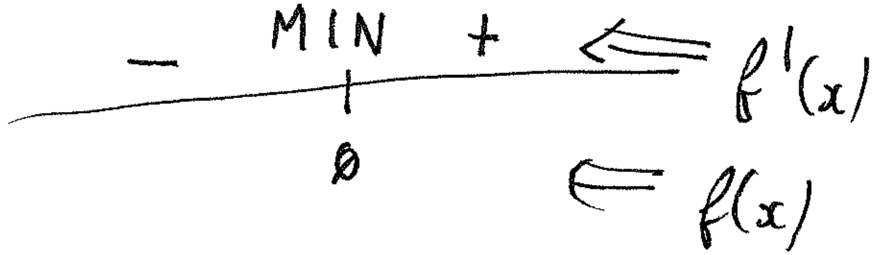
MINIMO:

$f(x) \geq f(x_0)$ se $|x - x_0|$ è piccola

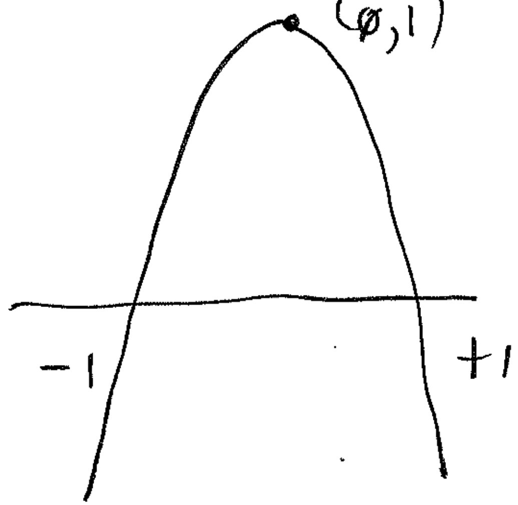
$f'(x_0) = 0$

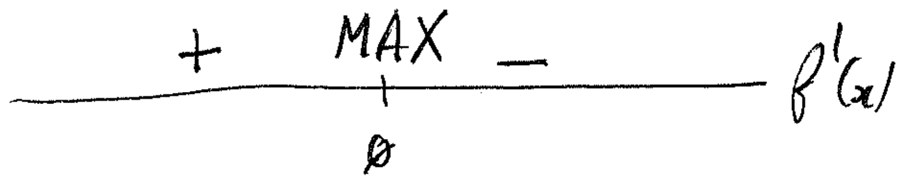
$f(x) = x^2$ $\quad$ $f'(x) = 2x$



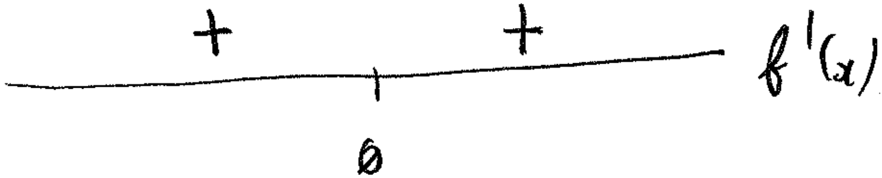


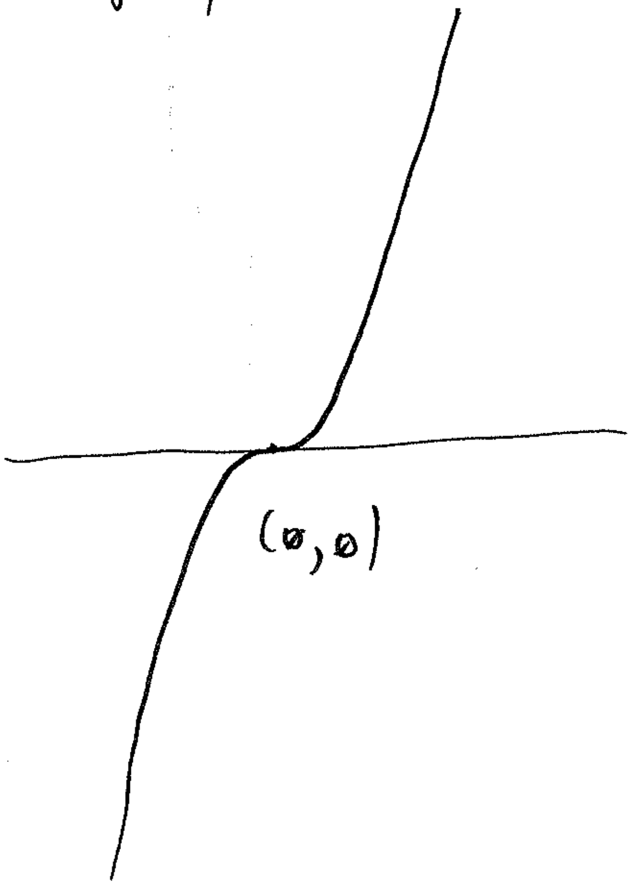
$f(x) = 1 - x^4 \longrightarrow f'(x) = -4x^3$





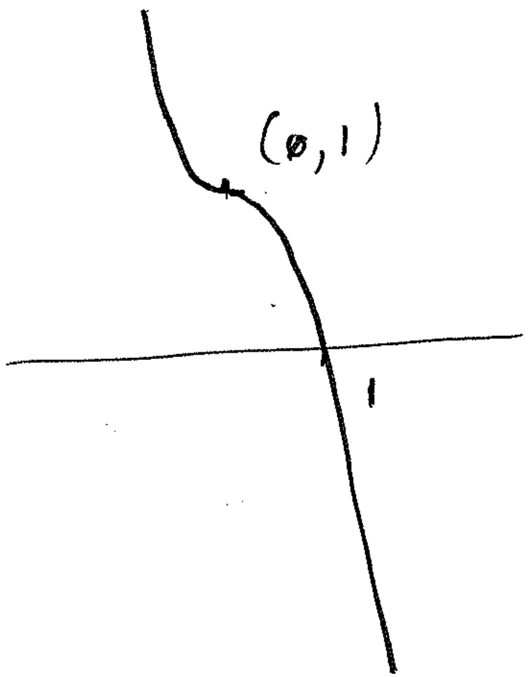
$f(x) = x^3 \rightarrow f'(x) = 3x^2$

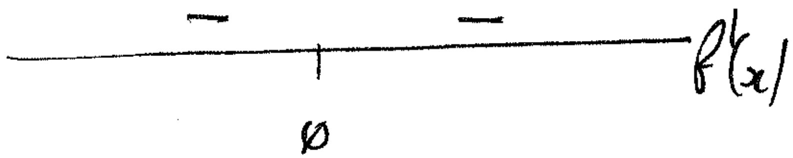




NÉ MASSIMO NÉ MINIMO

$f(x) = 1 - x^5 \rightarrow f'(x) = -5x^4$





TEOREMA DI ROLLE

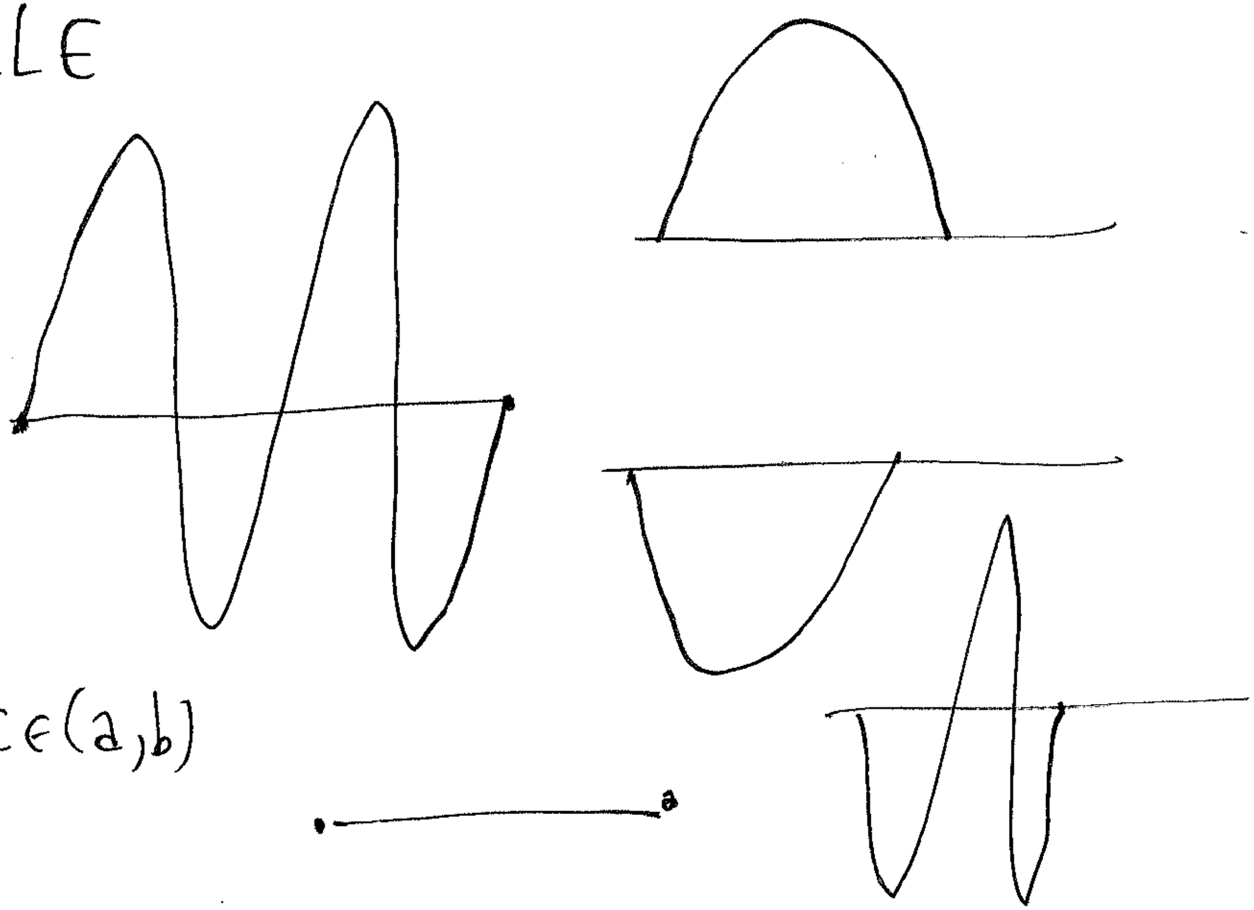
f continua in $[a, b]$

f derivabile in (a, b)

$f(a) = f(b)$

allora esiste almeno un $c \in (a, b)$

tale che $f'(c) = 0$



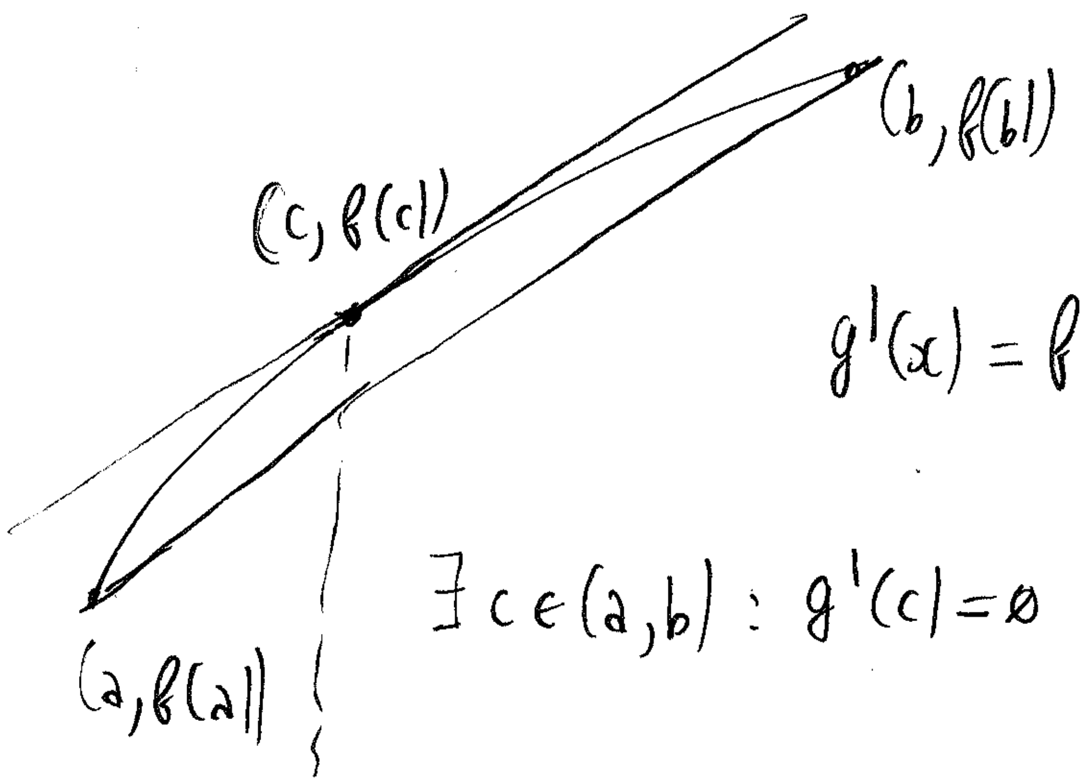
TEOREMA DI LAGRANGE

f continua in $[a,b]$

f derivabile in (a,b)

$$\cancel{g(x) = f(x) - f(a)}$$

$$g(x) = f(x) - \left\{ f(a) + \frac{f(b)-f(a)}{b-a}(x-a) \right\}$$



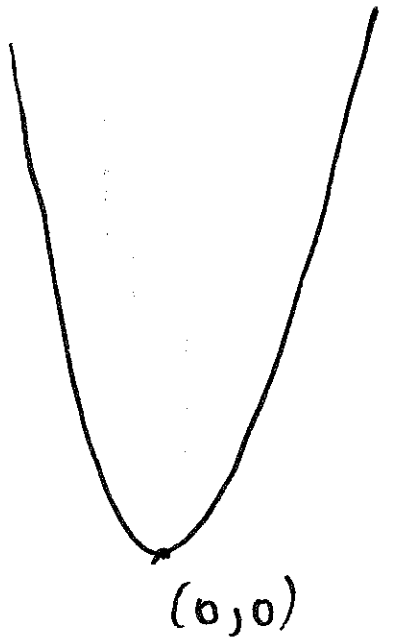
$$g(a) = g(b) = 0$$

$$g'(x) = f'(x) - \frac{f(b)-f(a)}{b-a}$$

$$\exists\, c \in (a,b) : g'(c) = 0 \iff f'(c) = \frac{f(b)-f(a)}{b-a}$$

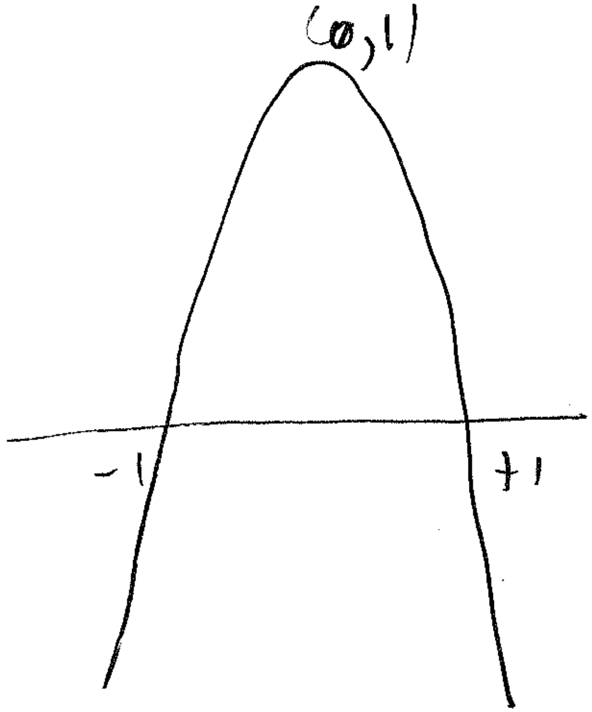
$f(x) = x^2 \quad f'(x) = 2x$

–	$\emptyset$	+	$\Leftarrow f'(x)$
decrescente	MIN	crescente	$\Leftarrow f(x)$



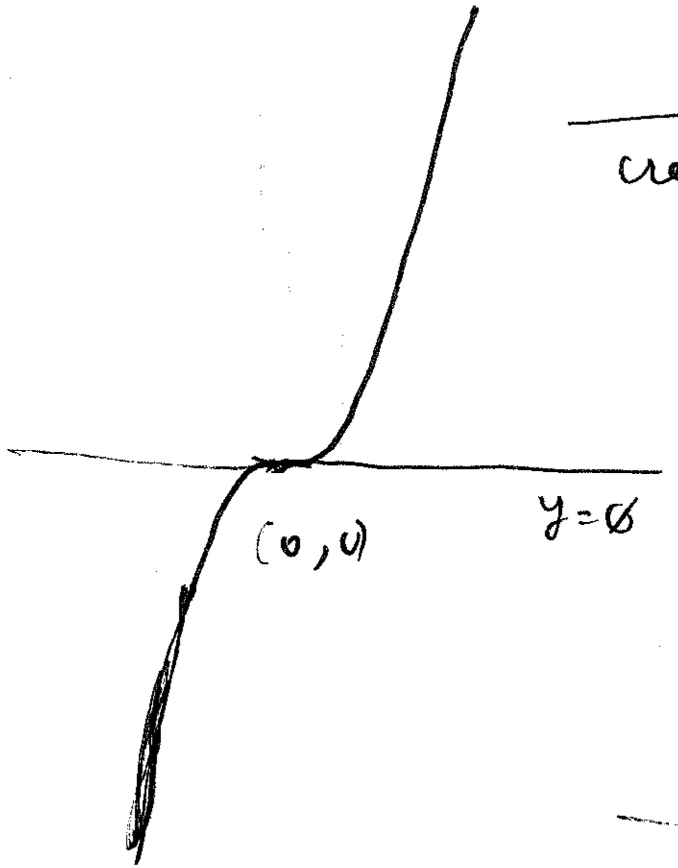
$f(x) = 1 - x^4 \quad f'(x) = -4x^3$

+	$\emptyset$	–	$\Leftarrow f'(x)$
crescente	MAX	decrescente	$\Leftarrow f(x)$



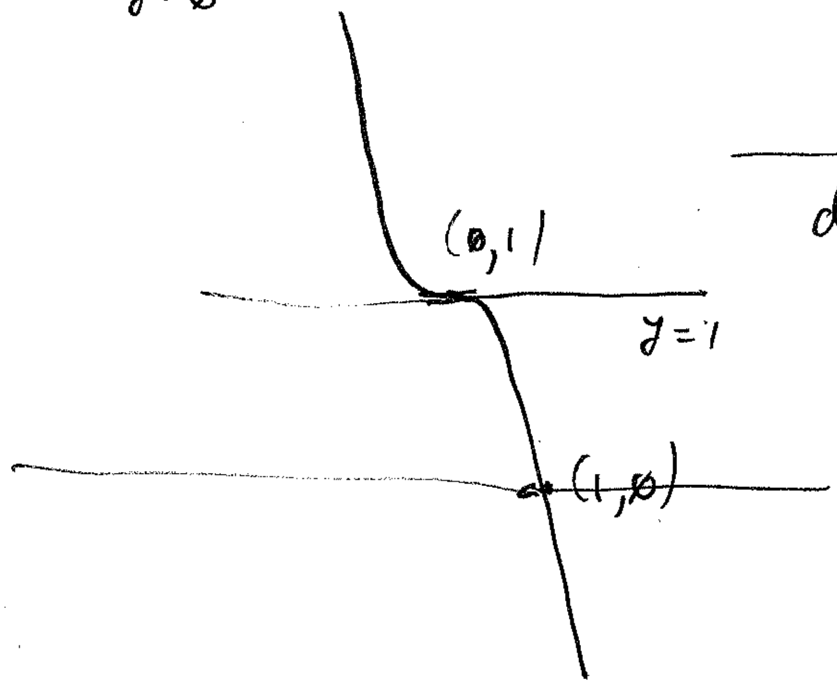
$f(x) = x^3 \qquad f'(x) = 3x^2$

+	$\emptyset$	+	$f'(x)$
crescente		crescente	



$f(x) = 1 - x^5 \qquad f'(x) = -5x^4$

−	$\emptyset$	−	$f'(x)$
decrescente		decrescente	$f(x)$



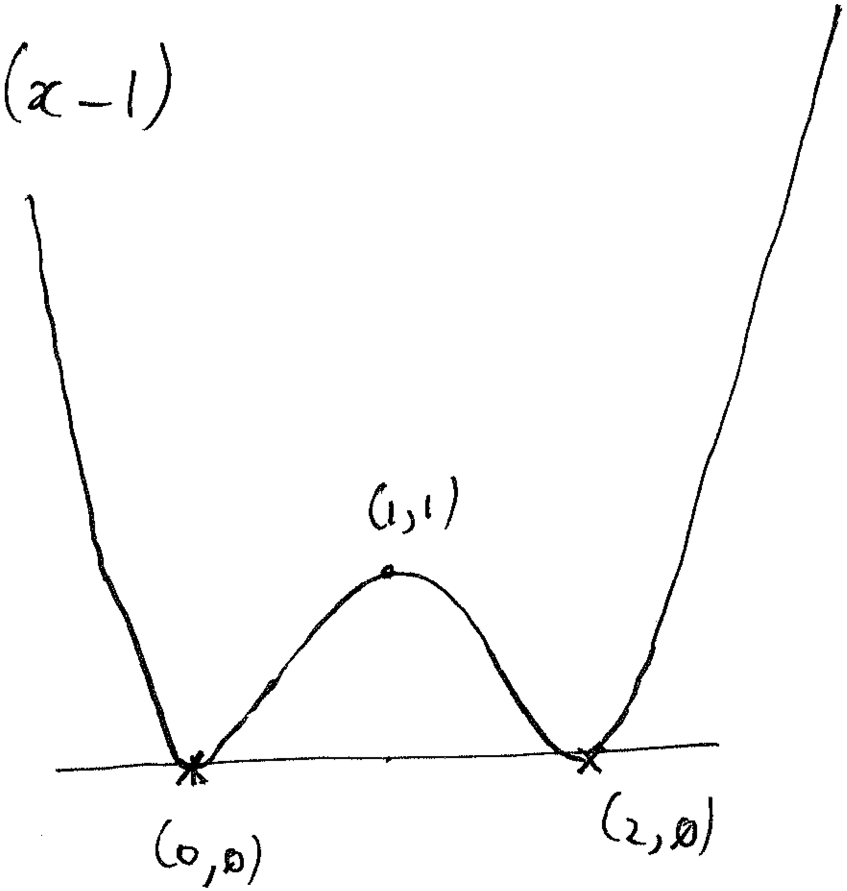
$$f(x) = x^2(x-2)^2 = x^2(x^2-4x+4) = x^4-4x^3+4x^2$$

$$= (x^2-2x)^2$$

$$f'(x) = 4x^3-12x^2+8x = 4x(x^2-3x+2) = 4x(x-1)(x-2)$$

$$= 2(x^2-2x)\cdot(2x-2) = 4x(x-2)(x-1)$$

	$\emptyset$		1		2		
$-$		$+$		$-$		$+$	$f'(x)$
decrescente	MIN	crescente	MAX	decrescente	MIN	crescente.	$f(x)$
	$\Downarrow$		$\Downarrow$		$\Downarrow$		
	$(\emptyset,\emptyset)$		$(1,1)$		$(2;\emptyset)$		



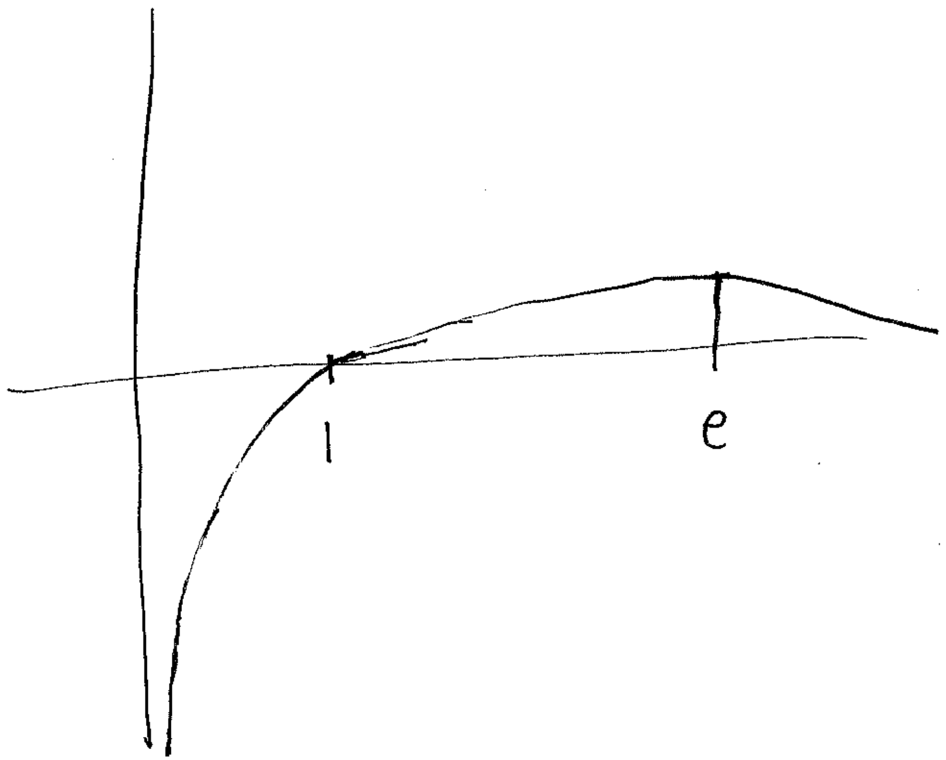
$$b = \ln x \leftrightarrow e^{b} = x$$

$$f(x) = \frac{\ln x}{x}, \quad x > 0$$

$$f(e) = \frac{\ln e}{e} = \frac{1}{e} \simeq 0{,}36$$

$$f'(x) = \frac{x \frac{d}{dx}\ln x - (\ln x)\frac{d}{dx}x}{\cancel{x}^2} = \frac{1 - \ln x}{x^2}$$

0	+	e	−	f'(x)
	crescente	MAX	decrescente	



$$\lim_{x \to +\infty} \frac{\ln x}{x} = 0$$

$$\lim_{x \to +\infty} \frac{1 - \ln x}{x} = 0$$