REGOLE PER IL PARZIALE DEL 17.11.2007

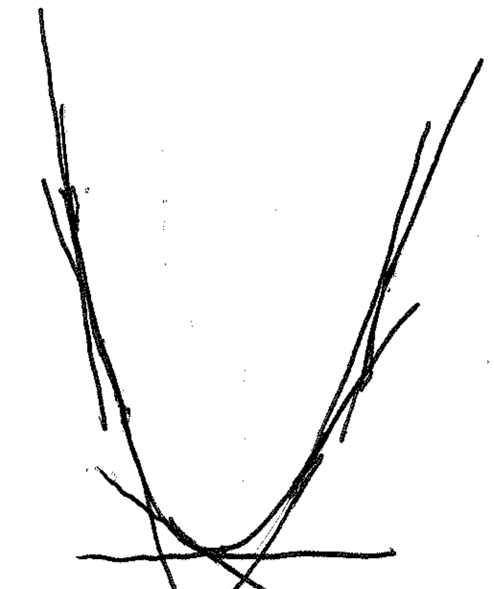
1. Gli studenti che seguono le lezioni ad Iglesias vengono a Cagliari.
2. È vietato uscire dall'aula nella prima ora del parziale.
3. È possibile il controllo dei libretti/documenti.

I risultati verranno esposti sul mio sito:

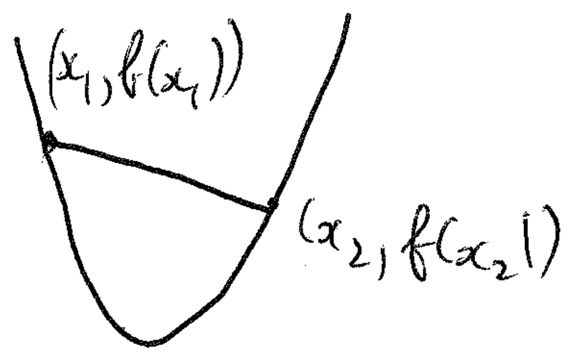
$\text{http://}\underbrace{\text{krein}}_{\text{bugs}}\text{.unica.it/\textasciitilde cornelis/DIDATTICA/ANALISI1INF/analisi1inf.html}$,

raggiungibile anche dallo spazio docenti del sito del CL Informatica.
Chi non ha mai comunicato la sua matricola, non troverà i suoi risultati, poichè la legge sulla privacy mi costringe a blindare i nomi.

$$f(x) = x^2 \longrightarrow f'(x) = 2x\,,\quad \underline{\underline{f''(x) = 2}}$$



$(0,0)$

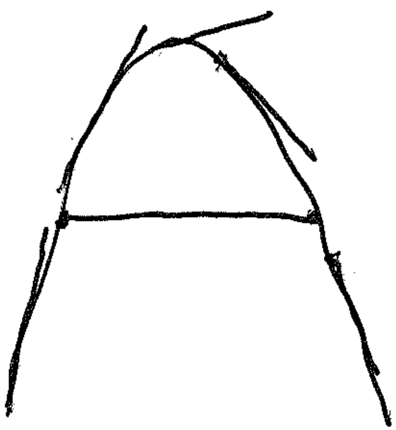


$$t x_1 + (1-t) x_2$$

f è convessa $\Leftrightarrow$ per ogni $t \in (0,1)$,

$$f(t x_1 + (1-t) x_2) \leq t f(x_1) + (1-t) f(x_2)$$

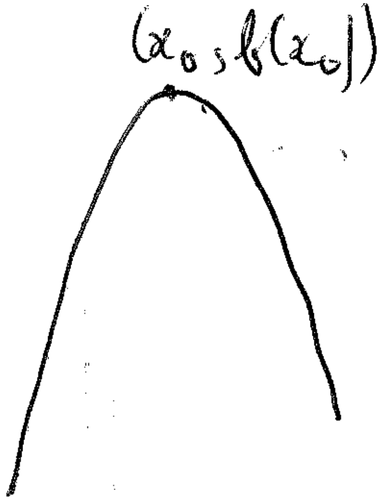
$\Leftrightarrow f'$ crescente $\Leftrightarrow f'' \geq 0$



f è concava $\Leftrightarrow$ per ogni $t \in (0,1)$,

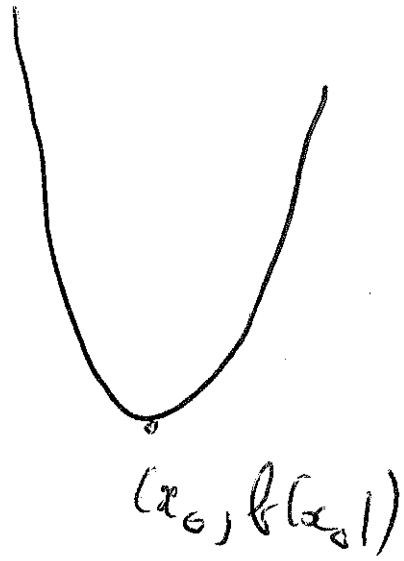
$$f(t x_1 + (1-t) x_2) \geq t f(x_1) + (1-t) f(x_2)$$

$\Leftrightarrow f'$ decrescente $\Leftrightarrow f'' \leq 0$



+	$=0$	−	$f'(x)$
crescente	x_0	decrescente	$f(x)$
	MAX		

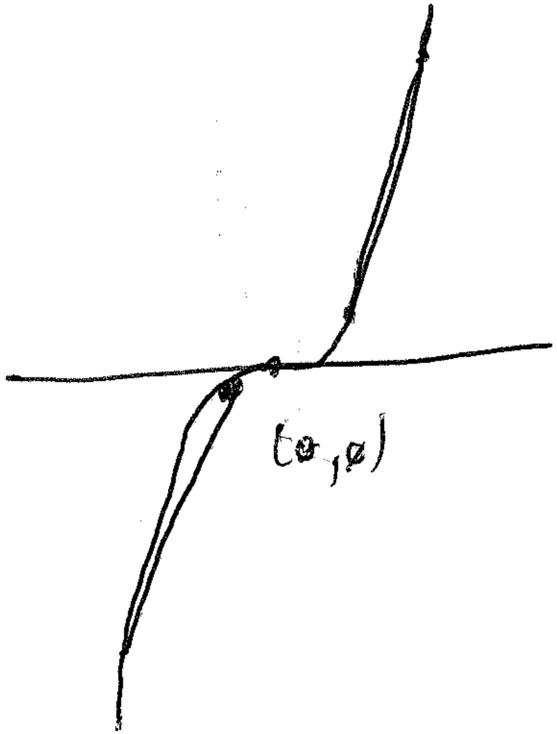
In tal caso $f''(x_0) < 0$



−	$=0$	+	$f'(x)$
decrescente	x_0	crescente	$f(x)$

In tal caso $f''(x_0) > 0$

$$f(x) = x^3 \rightarrow f'(x) = 3x^2 \rightarrow f''(x) = 6x$$



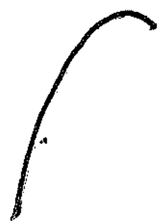
$$f'(0) = f''(0) = 0$$

+	$\overset{=0}{\mid}$	+	$f'(x)$
crescente	0	crescente	$f(x)$

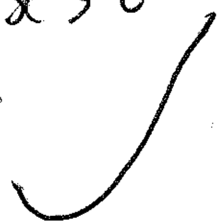
−	$\overset{=0}{\mid}$	+	$f''(x)$
concava	0	convessa	$f(x)$

x_0 è FLESSO $\overset{def}{\Longleftrightarrow}$ la funzione cambia da convessa a concava o viceversa

x < 0



x > 0



$\Longleftrightarrow f''(x_0) = 0$ e f'' cambia segno in x_0

$$f(x) = (x-1)(x-2)(x-3) = x^3 - 6x^2 + 11x - 6$$

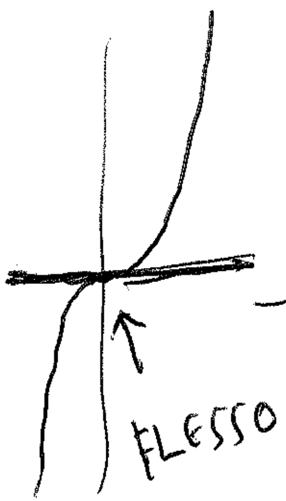
$$f'(x) = 3x^2 - 12x + 11 = 3(x^2 - 4x + 4) - 1 = 3(x-2)^2 - 1$$

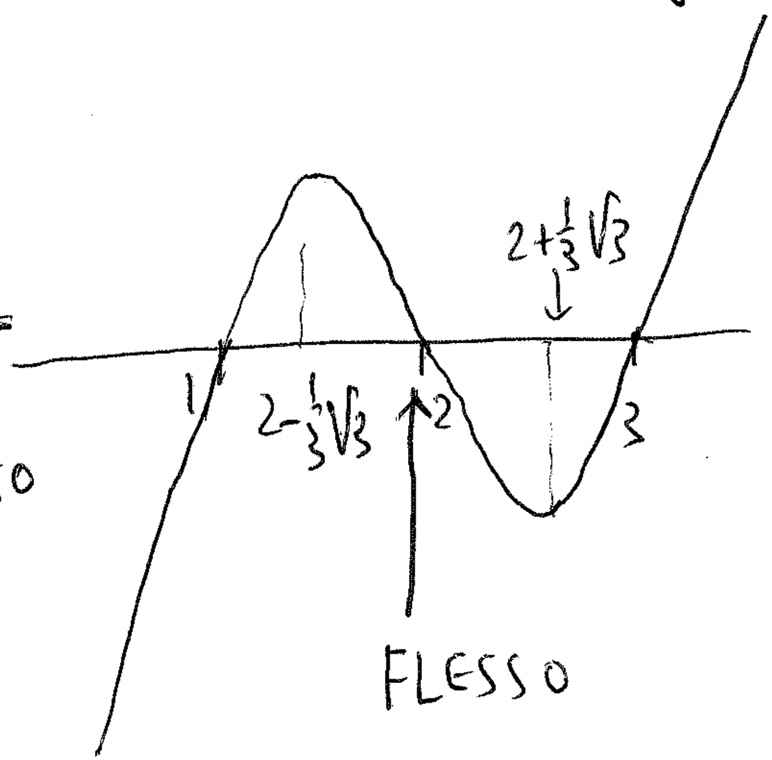
$$= 3\left(x - 2 - \tfrac{1}{3}\sqrt{3}\right)\left(x - 2 + \tfrac{1}{3}\sqrt{3}\right)$$

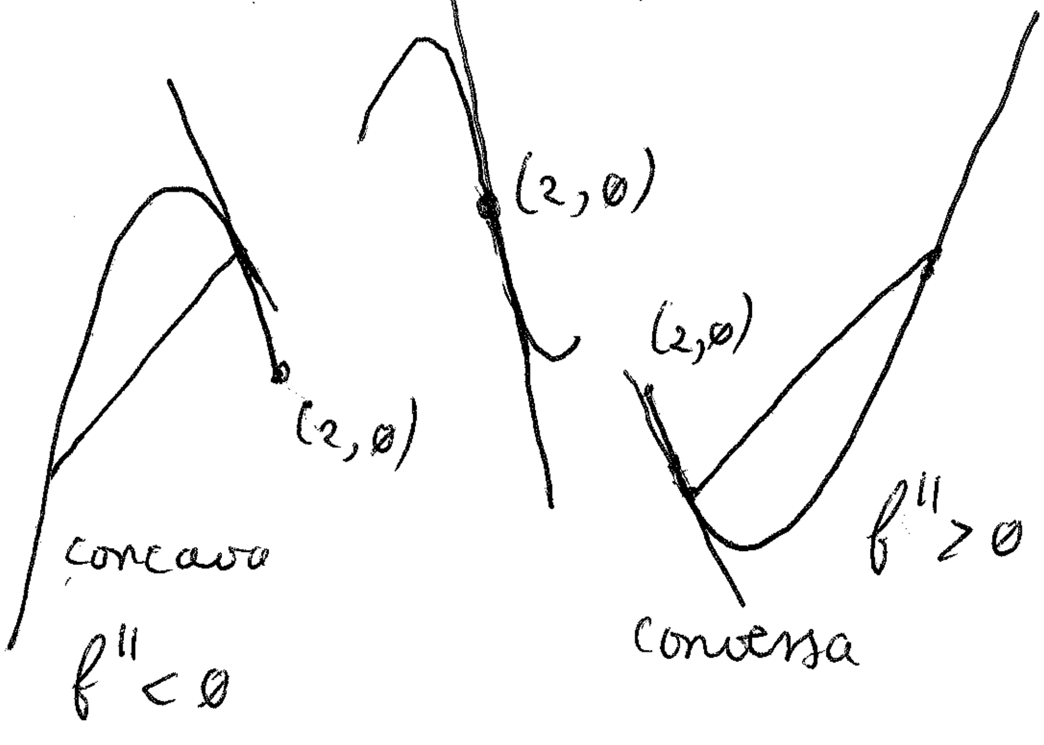
$f'(x)$: $+$ $\quad 2-\frac{1}{3}\sqrt{3} \quad$ $-$ $\quad 2+\frac{1}{3}\sqrt{3} \quad$ $+$

$$f''(x) = 6(x-2)$$

$$f''(x) = 6x - 12 = 6(x-2)$$







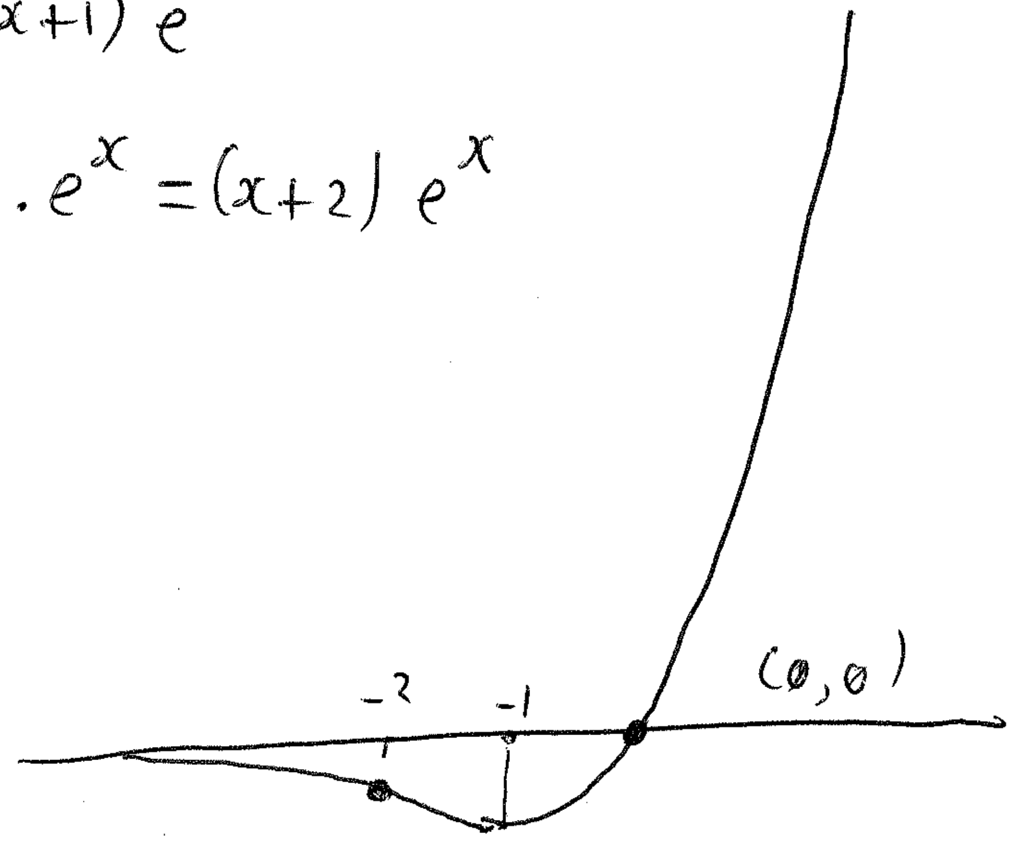
$$f(x) = x e^x \overset{f'(x)}{=} x e^x + 1 \cdot e^x = (x+1) e^x \qquad (fg)' = fg' + f'g$$

$$f''(x) = (x+1) e^x + 1 \cdot e^x = (x+2) e^x$$

$-$	-1 ($=0$)	$+$	$f'(x)$
	MIN		

$$f(-1) = -\frac{1}{e}$$

$-$	-2 (flesso)	$+$	$f''(x)$
concava		convessa	



$$\lim_{x \to -\infty} x e^x = 0$$

$y = 0 \Rightarrow$ asintoto orizzontale

$$f(x) = \frac{x-1}{x+3}, \quad x \neq -3$$

$$\lim_{x \to \pm\infty} \frac{x-1}{x+3} = 1$$

$y = 1$ asintoto orizzontale

$$f'(x) = \frac{(x+3)\cdot 1 - (x-1)\cdot 1}{(x+3)^2} = \frac{4}{(x+3)^2} > 0, \quad x \neq -3$$

$$= 4(x+3)^{-2}$$

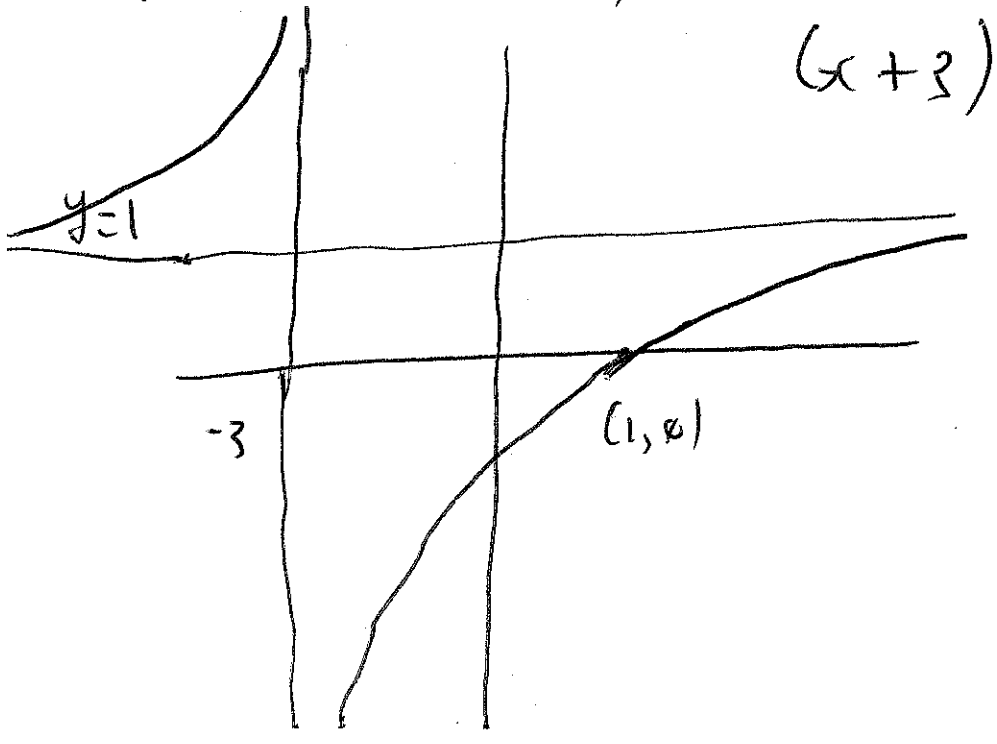
$$f''(x) = -8(x+3)^{-3} = \frac{-8}{(x+3)^3}, \quad x \neq -3$$

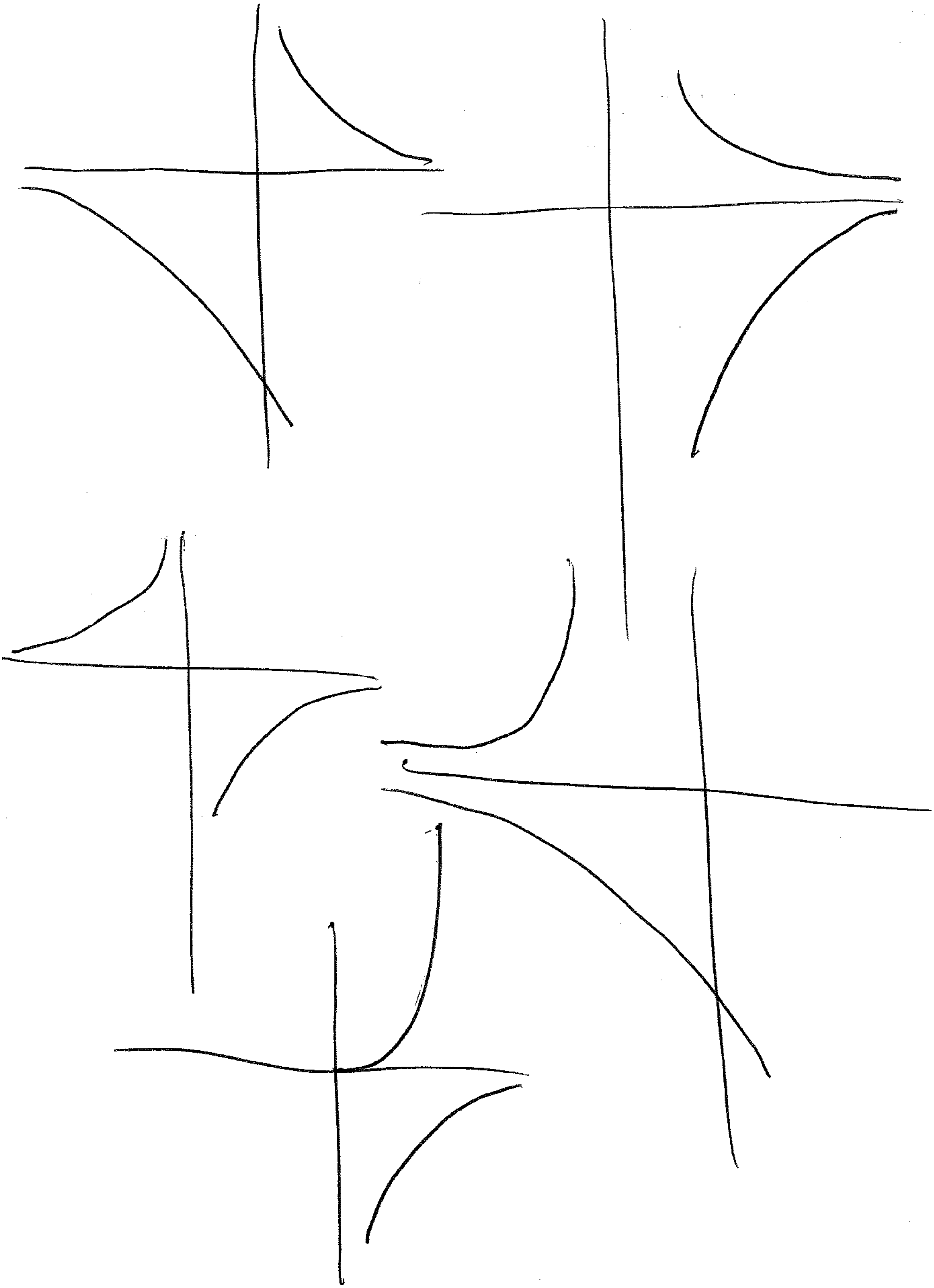
+	-3	−	$f''(x)$
convessa		concava	$f(x)$

$x = -3$ asintoto verticale

$$\lim_{x \to (-3)^-} f(x) = +\infty$$

$$\lim_{x \to (-3)^+} f(x) = -\infty$$

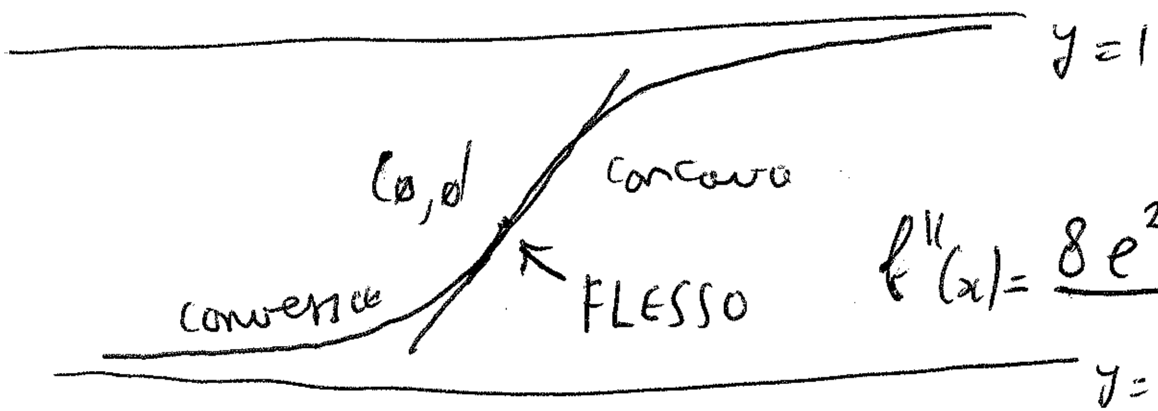




$$f(x) = \frac{e^{2x}-1}{e^{2x}+1} = \frac{1-e^{-2x}}{1+e^{-2x}} \qquad f(0)=1$$

$$\lim_{x\to+\infty} f(x) = \frac{1-0}{1+0} = 1 \qquad \lim_{x\to-\infty} f(x) = \frac{0-1}{0+1} = -1$$

$$f'(x) = \frac{(e^{2x}+1)\cdot 2e^{2x} - (e^{2x}-1)\cdot 2e^{2x}}{(e^{2x}+1)^2} = \frac{4e^{2x}}{(e^{2x}+1)^2} > 0$$



$$f''(x) = \frac{(e^{2x}+1)^{\cancel{2}}\cdot 8e^{2x} - 4e^{2x}\cdot 2\cancel{(e^{2x}+1)}\cdot \cancel{2e^{2x}}}{(e^{2x}+1)^{\cancel{4}3}}$$

$2e^{2x}$

$$f''(x) = \frac{8e^{2x}(1-e^{2x})}{(e^{2x}+1)^3}$$

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \lim_{x \to 0} \frac{\operatorname{sen} x}{2x} = \lim_{x \to 0} \frac{\cos x}{2} = \frac{1}{2}$$

De L'Hôpital

$$\lim_{x \to x_0} \boxed{\frac{f(x)}{g(x)}} = \frac{f(x)-f(x_0)}{x-x_0} \quad \frac{x-x_0}{g(x)-g(x_0)} \longrightarrow \frac{f'(x_0)}{g'(x_0)}$$

dove $f(x_0) = g(x_0) = 0$

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = \lim_{x \to 0} \frac{\cos x}{1} = 1$$

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = \lim_{x \to 0} \frac{\frac{1}{1+x}}{1} = 1$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

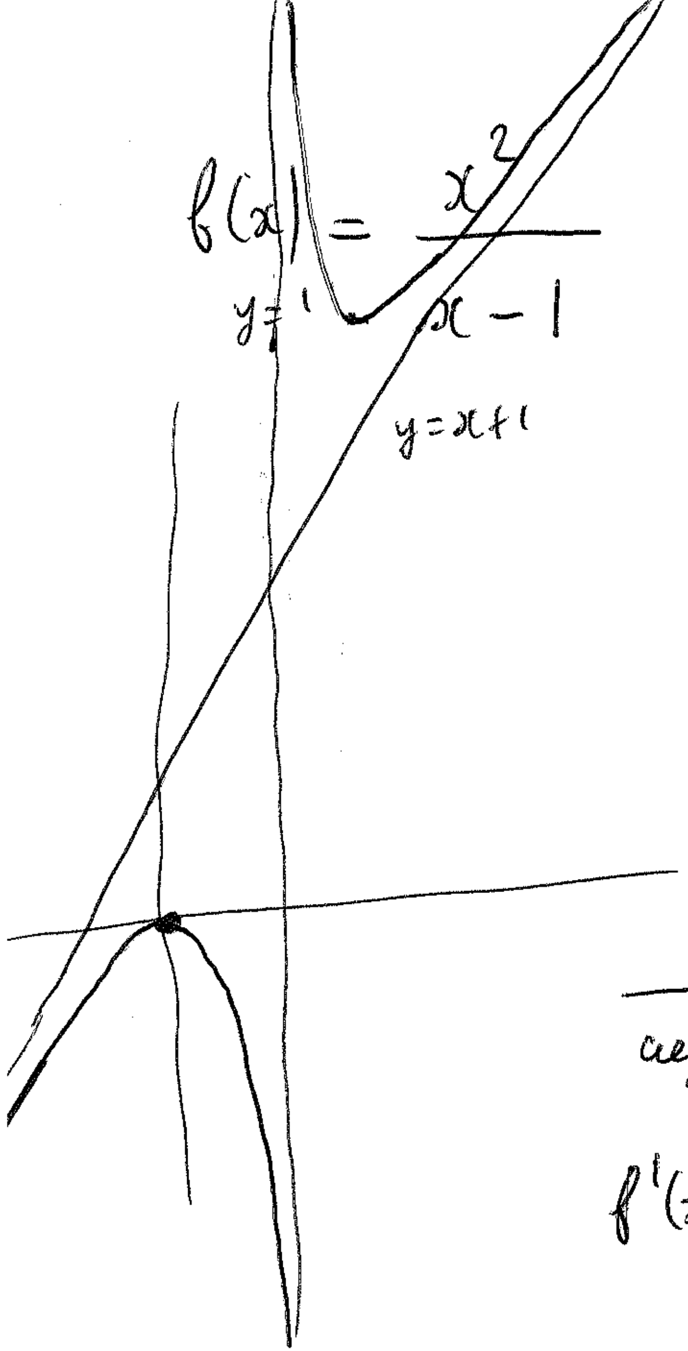
$$\lim_{x \to 0} \frac{e^x - 1 - x - \frac{1}{2}x^2}{x^3}$$

$$= \lim_{x \to 0} \frac{e^x - 1 - x}{3x^2} = \lim_{x \to 0} \frac{e^x - 1}{6x} = \lim_{x \to 0} \frac{e^x}{6} = \frac{1}{6}$$

$$\lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x \ln(1+3x)} = \lim_{x \to 0} \frac{2e^{2x} - 2}{\ln(1+3x) + x \frac{3}{1+3x}}$$

$$= \lim_{x \to 0} \frac{4e^{2x}}{\frac{1}{1+3x} \cdot 3 + 1 \cdot \frac{3}{1+3x} + x \cdot \frac{-9}{(1+3x)^2}} = \frac{4}{6} = \frac{2}{3}$$

$$f(x) = \frac{x^2}{x-1}$$



$x = 1$ asintoto vertical

$$f(x) = \frac{x^2 - 1}{x-1} + \frac{1}{x-1} = x+1 + \frac{1}{x-1}$$

$$\lim_{x \to \pm\infty} \{ f(x) - (x+1) \} = 0$$

$y = x+1$ asintoto obliquo

$$f'(x) = 1 - \frac{1}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

+	0 =	–	1 as.	–	2 =	+	$f'(x)$
crescente	MAX	decrescente			MIN	crescente	

$$f'(x) = 1 - \frac{1}{(x-1)^2} \rightarrow f''(x) = \frac{2}{(x-1)^3} \begin{cases} > 0, & x > 1 \\ < 0, & x < 1 \end{cases}$$

–	as	+	$f''(x)$
concava	1	convessa	

$f(x) = x \ln x \,, \quad x > 0$

$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} = 1 + \ln x$

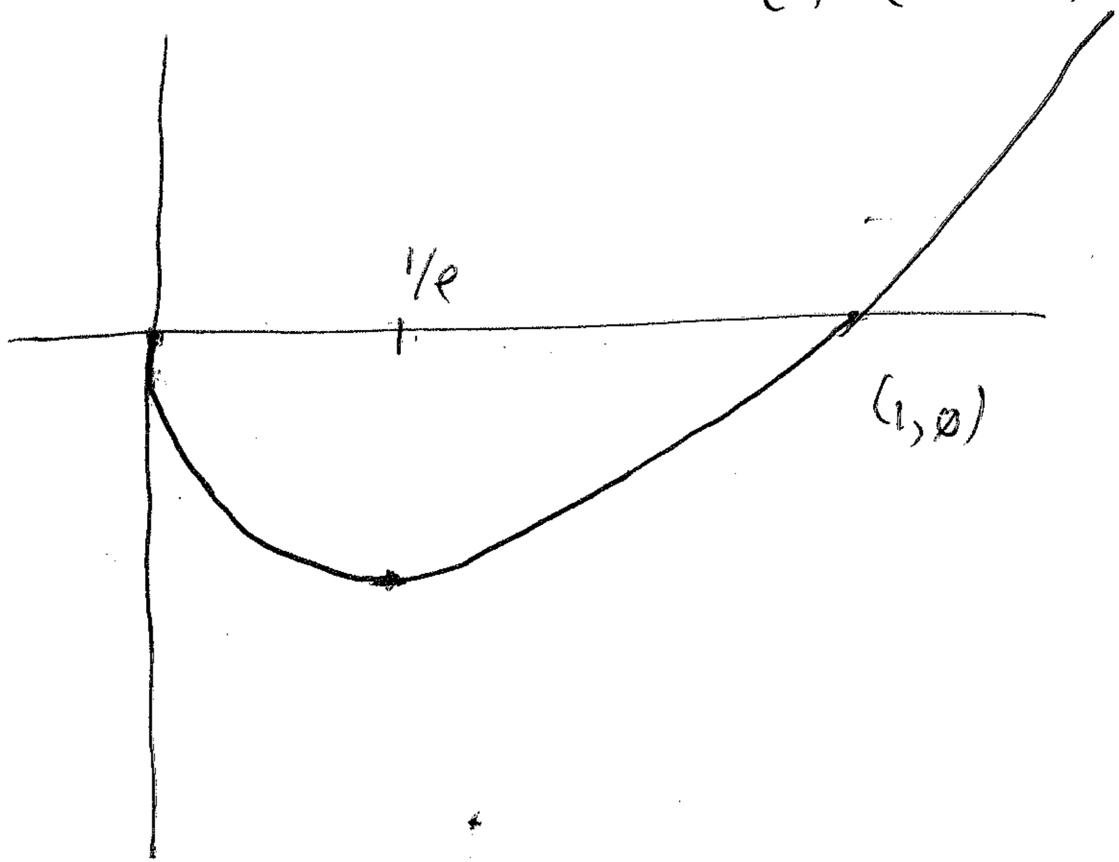
0	$-$	$1/e$	$+$	$f'(x)$
	decrescente	MIN	crescente	

$f''(x) = \frac{1}{x} > 0 \quad f$ è convessa

$f\left(\frac{1}{e}\right) = \frac{1}{e} \ln\left(\frac{1}{e}\right) = -\frac{1}{e}$

$\lim_{x \to 0^+} x \ln x = 0$

$\lim_{x \to 0^+} (1 + \ln x) = -\infty$



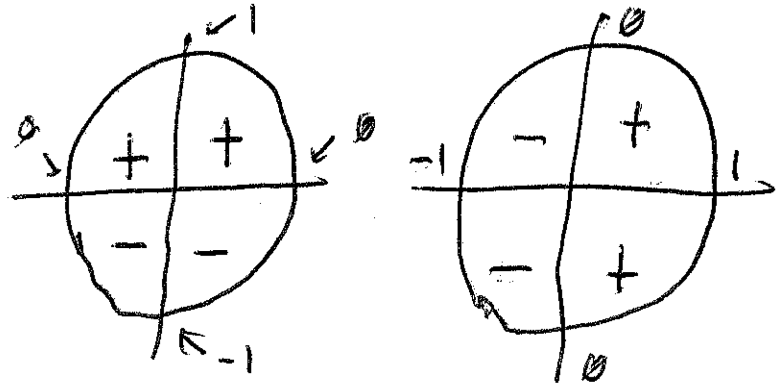
$$f(x) = \text{sen}(x)$$

$$f'(x) = \cos(x)$$

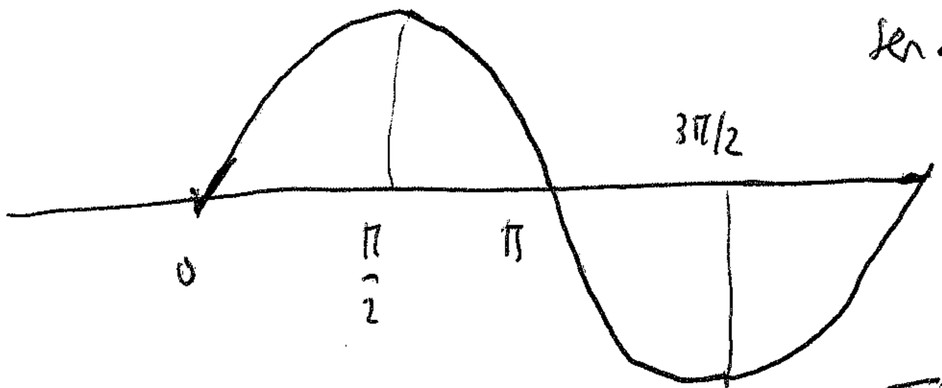
$$f''(x) = -\text{sen}(x)$$

$$-1 \leq f(x) \leq +1$$

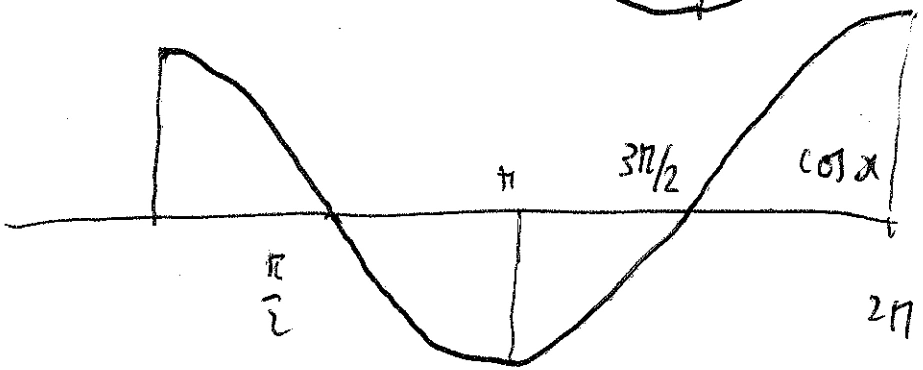
PERIODICITÀ : $f(x+2\pi) = f(x)$



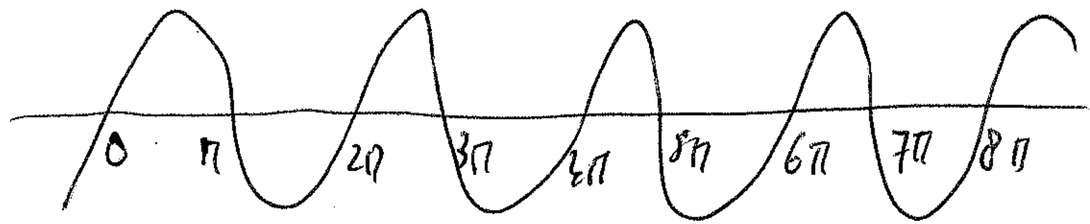
sen x   cos x



sen x

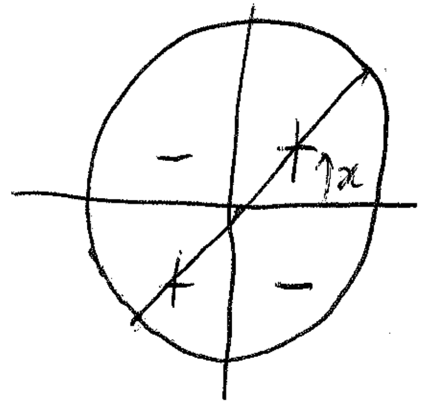


cos x



sinusoid

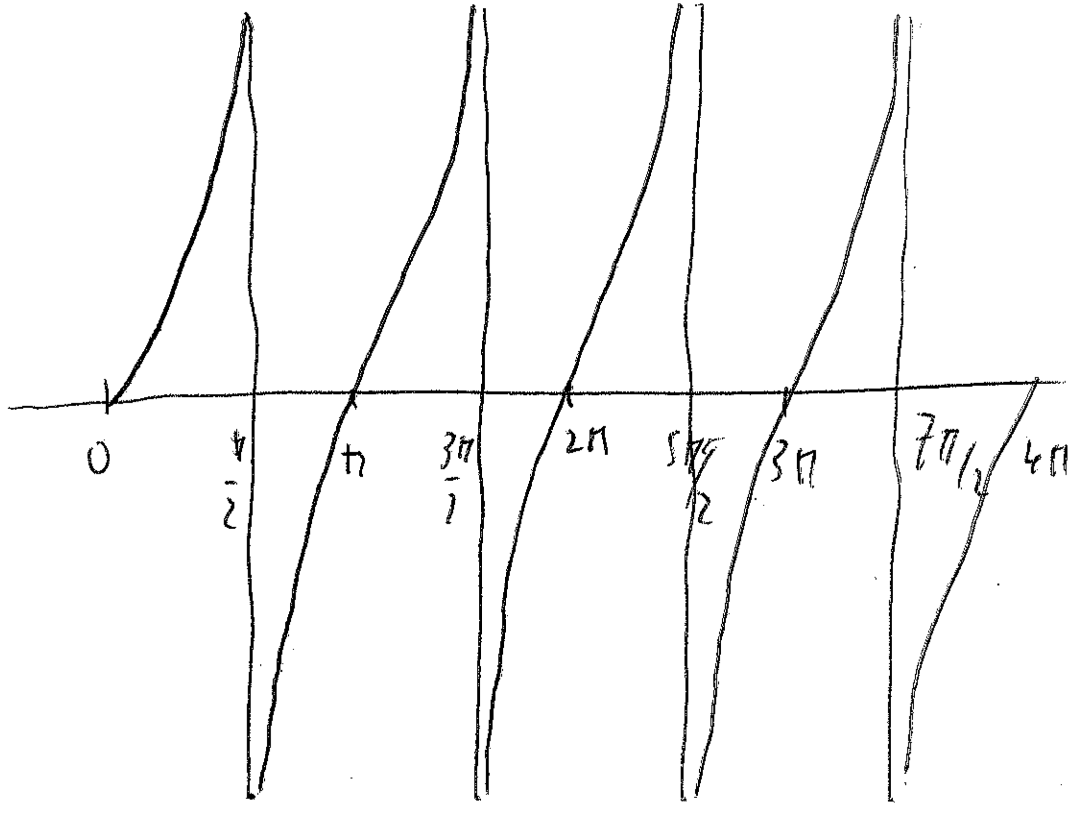
$$f(x) = tg\, x = \frac{sen\, x}{\cos x} \rightarrow f'(x) = \frac{1}{\cos^2(x)} > \emptyset$$



PERIODICITÀ : $tg(x+\pi) = tg\, x$

zeri se $sen\, x = 0 \quad x = k\pi,\ k \in \mathbb{Z}$

asintoti verticali se $\cos x = \emptyset \quad x = \frac{\pi}{2} + k\pi$



$\mapsto f(0) = 0$

$$f(x) = \frac{x}{x^2+1}$$

$$f'(x) = \frac{(x^2+1)\cdot 1 - x\cdot 2x}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$$\lim_{x \to \pm\infty} f(x) = 0$$

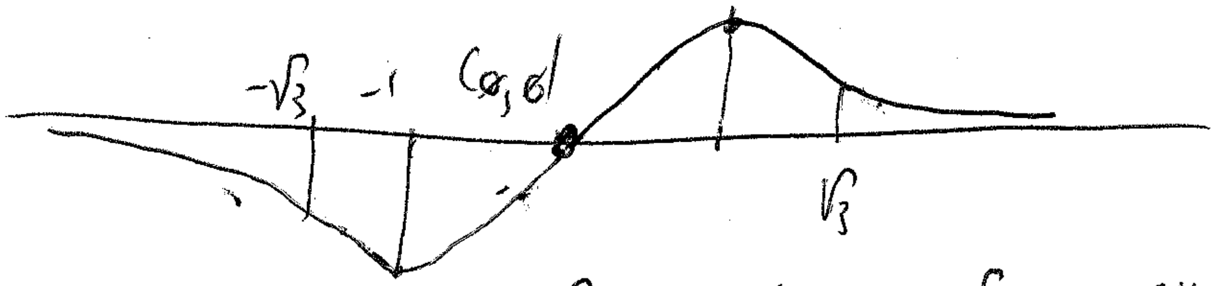
$y = 0$ asintoto orizzontale

− -1 + $+1$ −

decrescente MIN crescente MAX decrescente

$$f''(x) = \frac{(x^2+1)^{\cancel{2}}\cdot(-2x) - (1-x^2)\cdot 2\cancel{(x^2+1)}\,2x}{(x^2+1)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{-2x\{x^2+1+2(1-x^2)\}}{(x^2+1)^3} = \frac{-2x(3-x^2)}{(x^2+1)^3}$$



− $-\sqrt{3}$ + 0 − $\sqrt{3}$ + $f''(x)$

concava convessa concavo convessa