Prossima Lezione: Martedi 18-12, 11-13

Sessione Tutoriale: Lunedi 17-12, 15-17

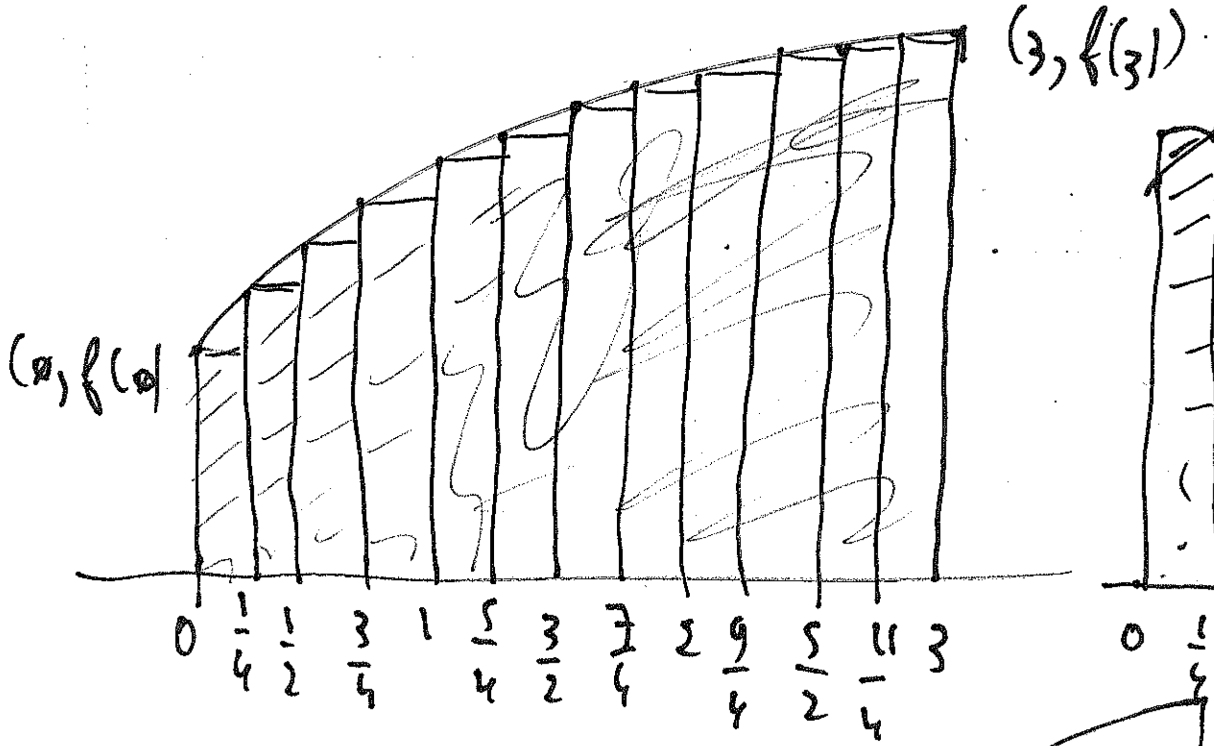
Secondo Parziale: ~~Mercoledi~~ Lunedi 7-1, $15^{30}-18^{30}$

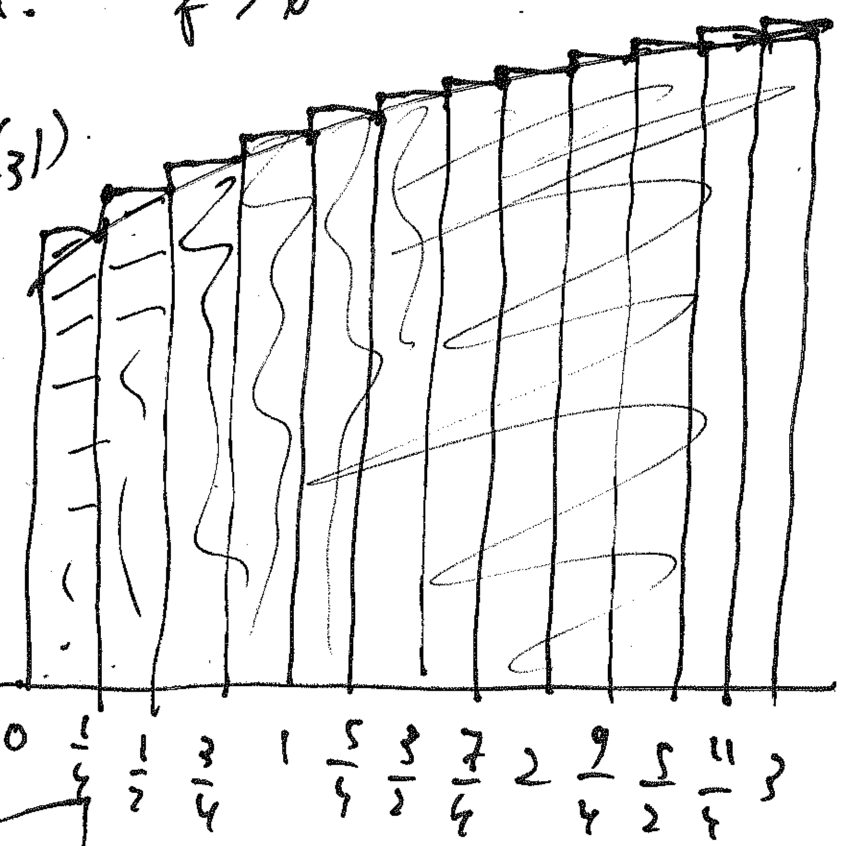
Orali: Mercoledi: 9-1, 9-13

Magari Giovedi 10-1

$n=12,\ a=0,\ b=3 \qquad x_i = a + i\,\frac{b-a}{n} = \frac{i}{4} \quad i=0,1,2,\dots,12$

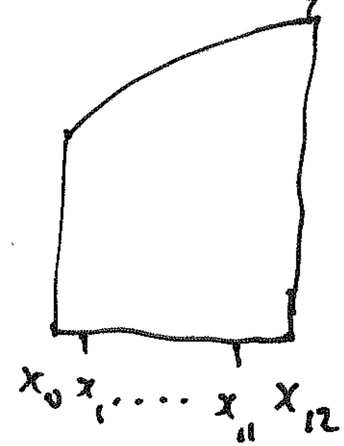
Sia $f:[0,3] \to \mathbb{R}$ continua. $f > 0$



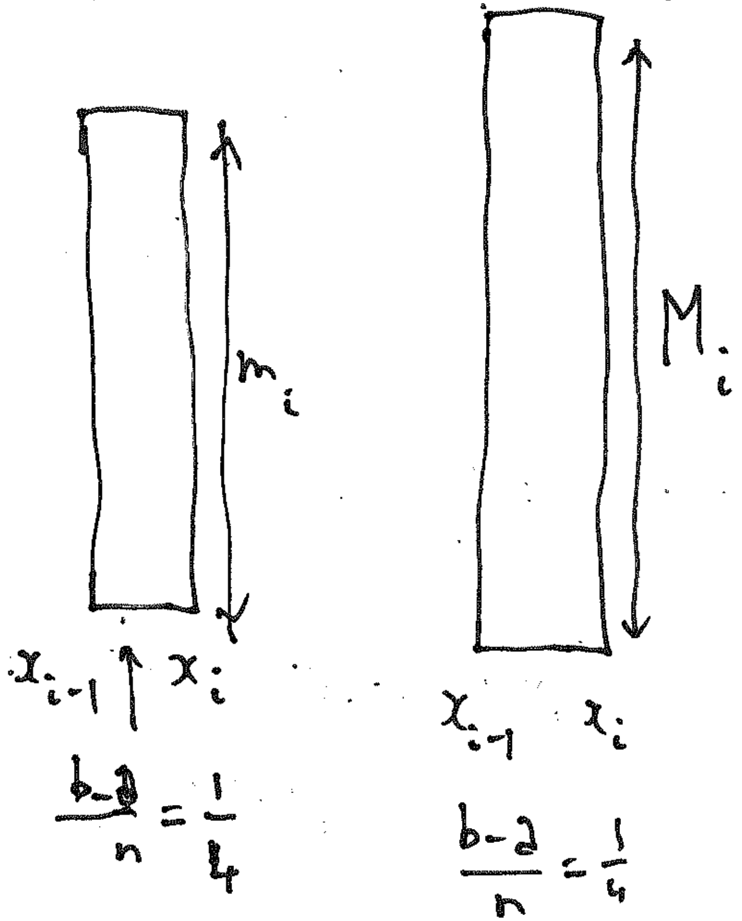


$m_i = \min\{f(x) : x_{i-1} \le x \le x_i\}$

$M_i = \max\{f(x) : x_{i-1} \le x \le x_i\}$



Somma (di Riemann) inferiore: $\frac{1}{4}\sum_{i=1}^{12} m_i \Longleftarrow \sum_{i=1}^{n} m_i (x_i - x_{i-1})$



Somma (di Riemann) superiore:

$\frac{1}{4}\sum_{i=1}^{12} M_i \longleftarrow \sum_{i=1}^{n} M_i (x_i - x_{i-1})$

Approssimazioni dell'integrale

$$\int_0^3 f(x)\,dx$$

Teorema fondamentale:

Sia $f:[a,b] \to \mathbb{R}$ continua. Allora $F:[a,b] \to \mathbb{R}$ definita da

$$F(x) = \int_a^x f(t)\, dt$$

è derivabile e

$$F'(x) = f(x)$$

<u>QUINDI</u>: Sia f continua. Trovare F tale che $F'(x) = f(x)$, $a \le x \le b$.

Allora $\int_a^b f(t)\, dt = F(b) - F(a)$.

$$\int \frac{dx}{5+3\cos x} = \int \frac{1}{5+3\frac{1-t^2}{1+t^2}} \frac{2\,dt}{1+t^2} = \int \frac{2\,dt}{5(1+t^2)+3(1-t^2)}$$

$$= \int \frac{2\,dt}{8+2t^2} = \int \frac{dt}{4+t^2}$$

$$t = \operatorname{tg}\tfrac{1}{2}x \longrightarrow x = 2\operatorname{arctg} t \rightarrow dx = \frac{2\,dt}{1+t^2}$$

$$1+t^2 = 1+\operatorname{tg}^2\tfrac{1}{2}x = \frac{\cos^2\frac{1}{2}x + \operatorname{sen}^2\frac{1}{2}x}{\cos^2\frac{1}{2}x} = \frac{1}{\cos^2\frac{1}{2}x}$$

$t = 2u \quad dt = 2\,du$

$$\cos^2\tfrac{1}{2}x = \frac{1}{1+t^2}$$

$$\Rightarrow \int \frac{2\,du}{4(1+u^2)} = \frac{1}{2}\operatorname{arctg} u + \text{cost}.$$

$$\cos x = 2\cos^2\tfrac{1}{2}x - 1 = \frac{2}{1+t^2} - 1 = \frac{1-t^2}{1+t^2}$$

$$= \frac{1}{2}\operatorname{arctg}(2t) + \text{cost}.$$

$$= \frac{1}{2}\operatorname{arctg}(2\operatorname{tg}\tfrac{1}{2}x) + \text{cost}.$$

$$\operatorname{sen} x = \cancel{\cos x} = 2\operatorname{sen}\tfrac{1}{2}x\cos\tfrac{1}{2}x = 2\operatorname{tg}\tfrac{1}{2}x\cos^2\tfrac{1}{2}x = \frac{2t}{1+t^2}$$

$$\int_{-3}^{-1} \underset{g'}{(x+3)}\, \underset{f}{\ln|x|}\; dx = \Big[\underset{g}{\left(\tfrac{1}{2}x^2+3x\right)} \underset{f}{\ln|x|}\Big]_{-3}^{-1} - \int_{-3}^{-1} \underbrace{\overset{g}{\left(\tfrac{1}{2}x^2+3x\right)} \overset{f'}{\frac{1}{x}}}_{\frac{1}{2}x+3}\, dx$$

$$= \Big[\left(\tfrac{1}{2}x^2+3x\right)\ln|x| - \tfrac{1}{4}x^2 - 3x\Big]_{-3}^{-1} \qquad \ln|-1| = 0$$

$$= \left(0 - \tfrac{1}{4} + 3\right) - \left(\tfrac{9}{2} - 9\right)\ln 3 + \tfrac{9}{4} - 9 = -4 + \tfrac{9}{2}\ln 3$$

$$\int_{\emptyset}^{\pi/3} \underset{\substack{\updownarrow \\ f}}{x} \; \underset{\substack{\uparrow \\ g'}}{\cos(3x)}\, dx = \left[x \cdot \frac{1}{3}\operatorname{sen}(3x)\right]_0^{\pi/3} - \int_0^{\pi/3} 1 \cdot \frac{1}{3}\operatorname{sen}(3x)\, dx$$

$$= \left[\frac{1}{3}x\operatorname{sen}(3x) + \frac{1}{9}\cos(3x)\right]_0^{\pi/3}$$

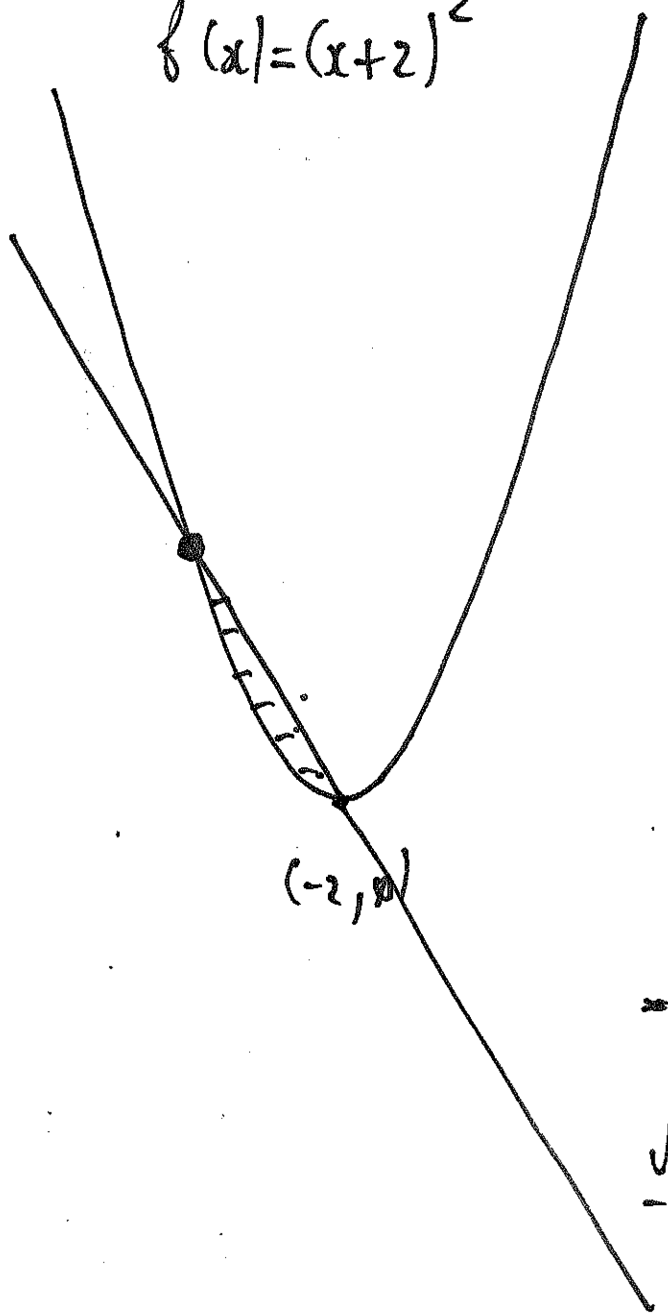
$\operatorname{sen}\pi = \emptyset$

$\cos\pi = -1$

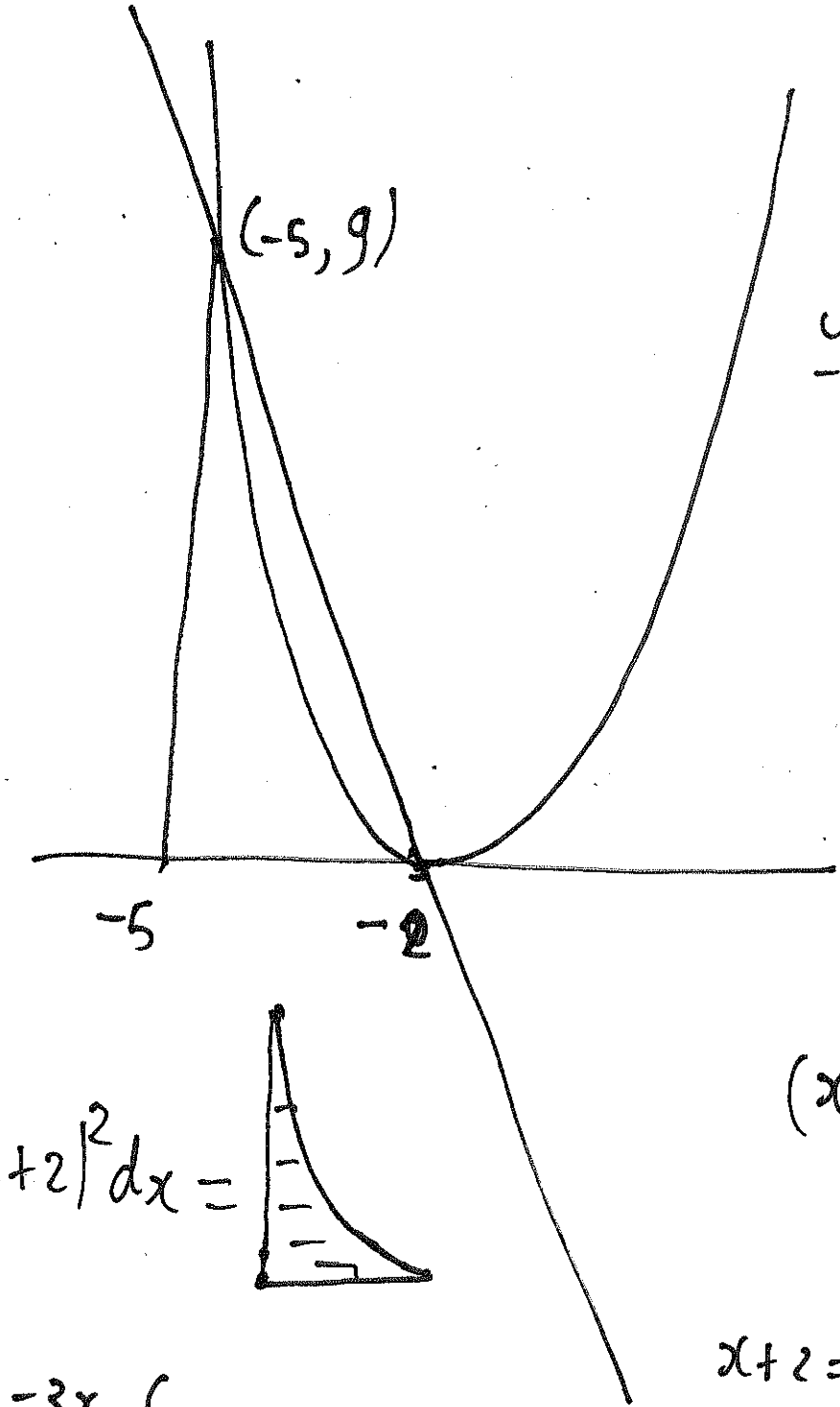
$$= \left[\frac{1}{9}\cos(3x)\right]_0^{\pi/3} = \frac{1}{9}\left\{(-1) - 1\right\} = -\frac{2}{9}$$

α	$\operatorname{sen}\alpha$	$\cos\alpha$
0°	$\frac{1}{2}\sqrt{\emptyset}$	$\frac{1}{2}\sqrt{4}$
30°	$\frac{1}{2}\sqrt{1}$	$\frac{1}{2}\sqrt{3}$
45°	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{2}$
60°	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{1}$
90°	$\frac{1}{2}\sqrt{4}$	$\frac{1}{2}\sqrt{\emptyset}$

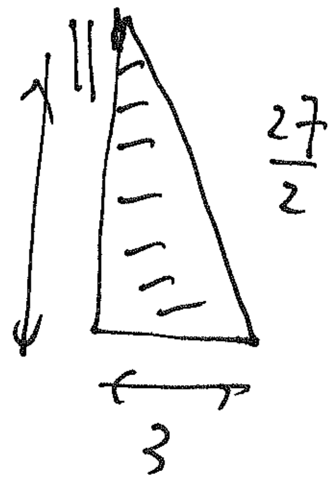
$f(x)=(x+2)^2$



$y=-3x-6$



$\int_{-5}^{-2}(-3x-6)\,dx =$



$\frac{27}{2}$

$\int_{-5}^{-2}(x+2)^2\,dx =$

$(x+2)^2 = -3x-6$

$\qquad\qquad\quad \shortparallel$

$\qquad\quad -3(x+2)$

$x+2=-3$

$$\text{Area} = \int_{-5}^{-2} (-3x-6)\,dx - \int_{-5}^{-2} (x+2)^2\,dx$$

$$= \int_{-5}^{-2} \left\{ -3(x+2) - (x+2)^2 \right\} dx$$

$$= \int_{-5}^{-2} \underbrace{(x+2)(-x-5)}_{-x^2-3x-10}\,dx = \left[-\frac{1}{3}x^3 - \frac{3}{2}x^2 - 10x \right]_{-5}^{-2}$$

$$= \left(\frac{8}{3} - \frac{125}{3}\right) - \frac{3}{2}(4-25) - 30$$

$$= -39 + \frac{63}{2} - 30$$

$dx = dt$

$x+5=t$

$$\int_{-5}^{-2} (x+2)^2\,dx = \int_0^3 t^2\,dt$$

$$= \left[\frac{1}{3}t^3\right]_0^3 = 9$$

$$\text{Area} = \frac{27}{2} - 9 = \frac{9}{2}$$

$$\int_{-1/2}^{+\infty} (2x+1)e^{-2x}\,dx = \lim_{a\to+\infty} \int_{-1/2}^{a} (2x+1)e^{-2x}\,dx$$

$$\left[(2x+1)\cdot -\frac{1}{2}e^{-2x}\right]_{-\frac{1}{2}}^{a} - \int_{-\frac{1}{2}}^{a} 2\cdot\left(-\frac{1}{2}e^{-2x}\right)dx = \left[-\frac{1}{2}(2x+1)e^{-2x}\right]_{-\frac{1}{2}}^{a} + \left[-\frac{1}{2}e^{-2x}\right]_{-\frac{1}{2}}^{a}$$

$$= \left[(-x-1)e^{-2x}\right]_{-\frac{1}{2}}^{a} = (-a-1)e^{-2a} + \frac{e}{\cancel{2e}\,2} \longrightarrow \frac{e}{2}, \text{ se } a\to+\infty$$

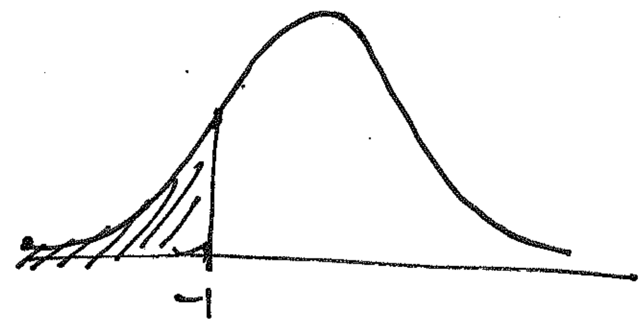
L'integrale è convergente con valore $\frac{e}{2}$

$\int_{-\infty}^{-1} f(x)\,dx$ si dice convergente se

① f è integrabile in $(a,-1)$ per ogni $a<-1$

② Esiste finito il limite

$$\lim_{a\to-\infty}\int_a^{-1} f(x)\,dx$$



$\int_{-\infty}^{-1} \frac{dx}{1+x^2}$

① $f:(-\infty,-1]\to\mathbb{R}$, $f(x)=\frac{1}{1+x^2}$, è continua.

② $\lim_{a\to-\infty}\int_a^{-1}\frac{dx}{1+x^2}=\lim_{a\to-\infty}\left[\operatorname{arctg} x\right]_a^{-1}=\lim_{a\to-\infty}\left(\left(-\frac{\pi}{4}\right)-\operatorname{arctg} a\right)=\frac{\pi}{4}$

sia $\overbrace{\text{convergente}}^{\text{di}}$ $\displaystyle\int_{-\infty}^{-1} \overbrace{g(x)}^{f(x)}\, dx$.

sia $0 \leq f(x) \leq g(x)$ per $x \leq -1$

Allora è $\overbrace{\text{convergente}}^{\text{di}}$ $\displaystyle\int_{-\infty}^{-1} \overbrace{f(x)}^{g(x)}\, dx$

$|2 \operatorname{arctg} x| \leq \pi$

$$\int_{-\infty}^{-1} \frac{(-2 \operatorname{arctg} x)^{2005}}{1+x^2}\, dx \quad (*)$$

$$0 \leq \left| \frac{(-2 \operatorname{arctg} x)^{2005}}{1+x^2} \right| \leq \frac{\pi^{2005}}{1+x^2}$$

è convergente $(*)$

$\displaystyle\int_{-\infty}^{-1} \frac{\pi^{2005}}{1+x^2}\, dx$ è convergente

$$\int_{-\infty}^{-1} |x|^{-p}\,dx = \int_{1}^{\infty} x^{-p}\,dx = \lim_{a\to+\infty}\int_{1}^{a} x^{-p}\,dx = \begin{cases} +\infty, & p\le 1 \\ \dfrac{1}{p-1}, & p>1 \end{cases}$$

$$\int_{-\infty}^{-1} \underbrace{\frac{(-2\,\mathrm{arctg}\,x)^{207}}{\sqrt[2005]{1+x^2}}}_{f(x)}\,dx$$

$$(-2\,\mathrm{arctg}\,x)^{207} \to \pi^{207}, \; x\to-\infty$$

$$\lim_{x\to-\infty}\frac{|x|^{2/2005}}{\sqrt[2005]{1+x^2}} = \lim_{x\to-\infty}\sqrt[2005]{\frac{x^2}{1+x^2}} = 1$$

$$f(x) \sim \frac{\pi^{207}}{|x|^{2/207}}, \; x\to-\infty$$

$$p = \frac{2}{207} < 1$$

$$\int_1^{\infty} \underbrace{\frac{(2\arctan x)^{207}}{\sqrt[2005]{1+x^2}}}_{f(x)} dx$$

$$\frac{(\pi/2)^{2007}}{\sqrt[2005]{1+x^2}} \leq f(x) \leq \frac{\pi^{207}}{\sqrt[2005]{1+x^2}}$$

$$\frac{(\pi/2)^{207}}{2^{1/2005}\, x^{2/2005}} \leq \frac{(\pi/2)^{2007}}{\sqrt[2005]{1+x^2}} \qquad \frac{\pi^{207}}{\sqrt[2005]{1+x^2}} \leq \frac{\pi^{207}}{x^{2/2005}}$$

$$\underbrace{\int_1^{\infty} f(x)\, dx}_{\text{diverge}} \text{ converge} \Leftrightarrow \underbrace{\int_1^{\infty} x^{-2/2005}\, dx}_{\text{Diverge}} \text{ converge}$$

$$f(x) = \operatorname{arctg}(3x-2) = f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \dots$$

$$f'(x) = \frac{1}{1+(3x-2)^2} \cdot 3 = \frac{3}{1+(3x-2)^2}$$

$$\frac{d}{dx}(3x-2)^2 = 2(3x-2) \cdot \frac{d}{dx}(3x-2)$$

$$= 6(3x-2)$$

$$f''(x) = 3 \cdot \frac{-1}{\left(1+(3x-2)^2\right)^2} \, 6(3x-2)$$

$$f(x) = \frac{\pi}{4} + \frac{3}{2}(x-1) - \frac{9}{4}(x-1)^2 + \dots$$

$$f(1) = \operatorname{arctg} 1 = \frac{\pi}{4}$$

$$f'(1) = \frac{3}{1+1} = \frac{3}{2}$$

$$f''(1) = \frac{-18}{(1+1)^2} = -\frac{9}{2}$$

$$\int_{3/2}^{2} \underbrace{\frac{4}{\sqrt[3]{2x-3}}}_{f(x)}\,dx = \lim_{a\to\left(\frac{3}{2}\right)^+} \int_a^2 \frac{4}{\sqrt[3]{2x-3}}\,dx$$

$$= \lim_{a\to\left(\frac{3}{2}\right)^+} \left[3(2x-3)^{2/3}\right]_a^2 = \left[3(2x-3)^{2/3}\right]_{3/2}^{2} = 3$$

Quindi: convergente

$$f(x) = 4(2x-3)^{-1/3}$$

$$\frac{d}{dx}(2x-3)^{2/3} = \frac{2}{3}(2x-3)^{-1/3}\frac{d}{dx}(2x-3) = \frac{4}{3}(2x-3)^{-1/3}$$