$$a_n = \frac{3n-7}{5n} = \frac{3}{5} - \frac{7}{5n} \qquad \lim_{n\to\infty} a_n = \frac{3}{5}$$

Cercare $n \in \mathbb{N}$ tali che $\left|a_n - \frac{3}{5}\right| < \varepsilon$, cioè $\frac{7}{5n} < \varepsilon$

Ciò è equivalente a $n > \frac{7}{5\varepsilon}$.

Sia $N(\varepsilon)$ il primo intero $> \frac{7}{5\varepsilon}$. Allora per $n \geq N(\varepsilon)$ abbiamo

$$\left|a_n - \frac{3}{5}\right| < \varepsilon$$

$$f(x) = \begin{cases} \mu x^2 + \partial x & \text{se } x \in [1,2] \\ x & \text{se } x \in [-2,1) \\ \mu x - 4 & \text{se } x \in [-3,-2) \end{cases}$$

$f : [-3,2] \longrightarrow \mathbb{R}$

f è continua in tutti i punti di $[-3,2]$ tranne, magari, -2 e 1.

$\lim\limits_{x \to (-2)^-} f(x) = -2\mu - 4$ $\qquad$ $\lim\limits_{x \to (-2)^+} f(x) = -2$

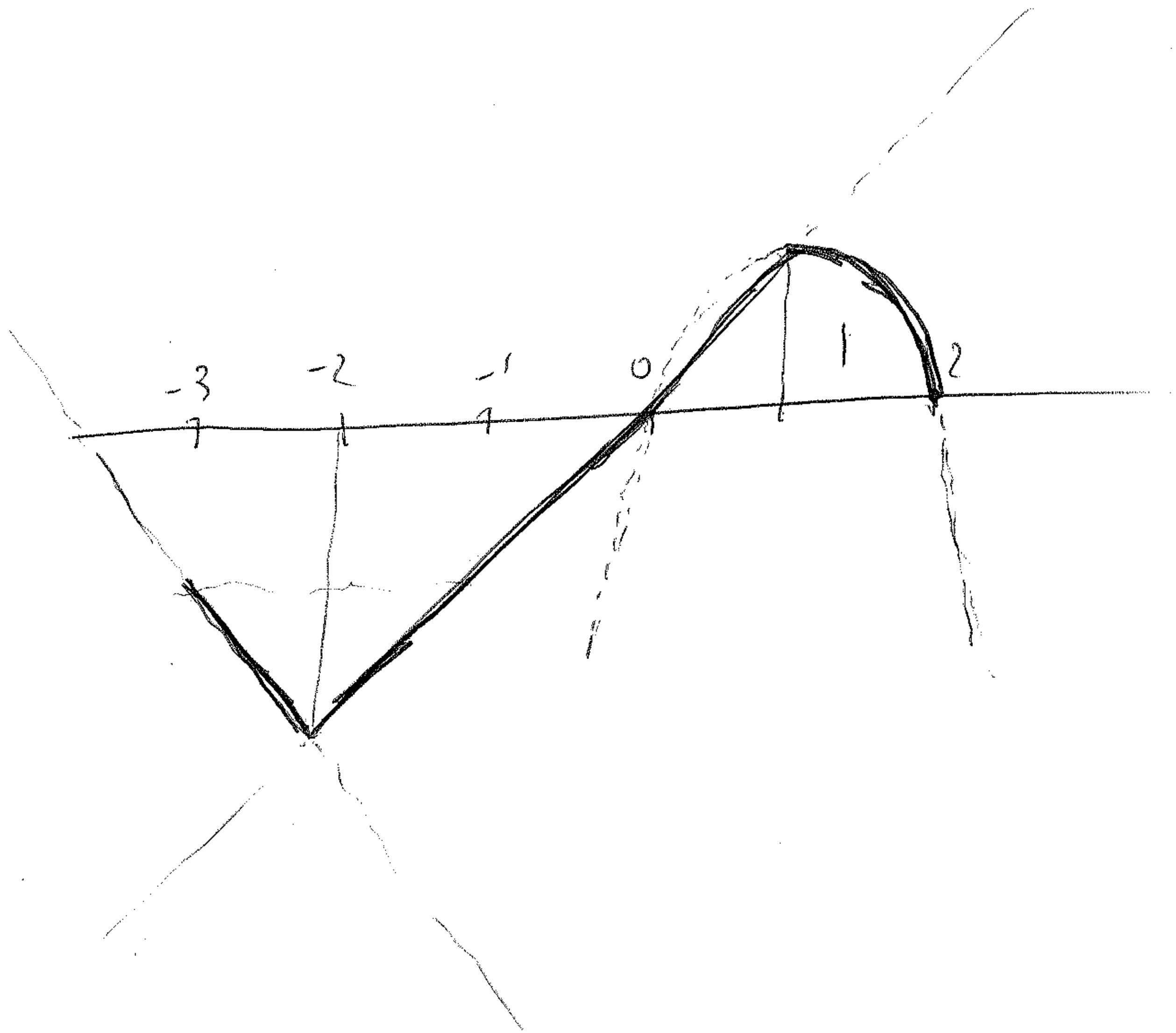
$\lim\limits_{x \to 1^-} f(x) = 1$ $\qquad$ $\lim\limits_{x \to 1^+} f(x) = \mu + \partial$

f è continua in $-2 \iff -2\mu - 4 = -2 \iff 2\mu = -2 \iff \mu = -1$

f è continua in $1 \iff 1 = \mu + \partial$ $\qquad$ f continua $\iff \mu = -1,\ \partial = 2$

$\underline{\mu = -1, \lambda = 2}$ $\qquad f(1) = 1, f(-2) = -2$

$$f(x) = \begin{cases} 2x - x^2, & 1 \le x \le 2 \\ x, & -2 \le x < 1 \\ -x-4, & -3 \le x < -2 \end{cases}$$



$f(x) = 7x^{3x}, \quad x_0 = 1$ $\qquad x = e^{\ln x} \to x^{3x} = e^{3x \ln x}$

$f(x) = 7e^{3x\ln x}$

$f'(x) = 7\underbrace{e^{3x\ln x}}_{=x^{3x}} \frac{d}{dx}(3x\ln x) = 7x^{3x} \cdot \left\{3x \cdot \frac{1}{x} + 3 \cdot 1 \cdot \ln x\right\}$

$= 21x^{3x}(1+\ln x), \quad x > \emptyset \to f'(1) = 21 \cdot 1 \, \overbrace{(1+\emptyset)}^{21} = 21$

$f(1) = 7$

Retta tangente: $\boxed{y - f(x_0) = f'(x_0)(x - x_0)}$

$y - 7 = 21(x-1) \qquad y = 21x - 14$

$$f'(x) = \underset{\substack{\uparrow \\ e^{-x\ln 5}}}{5^{-x}} (\ln 25)\{1 - x\ln 5\}$$

$$f''(x) = \left(e^{-x\ln 5} \cdot (-\ln 5)\right)(\ln 25)\{1 - x\ln 5\} + 5^{-x}(\ln 25) \times (-\ln 5)$$

$$= 5^{-x}(\ln 25)\left[-(\ln 5)\{1 - x\ln 5\} - \ln 5\right]$$

$$= 5^{-x}(\ln 25)(\ln 5)\left[-x\ln 5 + 1 + 1\right] = \underbrace{5^{-x} \cdot 2(\ln 5)^2}_{>0}\left[2 - x\ln 5\right]$$

$+ \quad \frac{2}{\ln 5} \quad - \quad f''(x)$

convessa $\quad \uparrow \quad$ concava $\quad f(x)$

flesso

$$f(x) = \underbrace{(\ln 25)}_{>\emptyset} x \cdot 5^{-x}$$

$\lim_{x\to+\infty} 5^{-x} = \emptyset$ $\quad \cancel{\lim_{x\to-\infty} 5^{-x} = +\infty}$

$$\begin{array}{ccccc} & - & & + & f(x) \\ \hline & & \emptyset & & \end{array}$$

$\lim_{x\to-\infty} 5^{-x} = +\infty \qquad 5 = e^{\ln 5}$

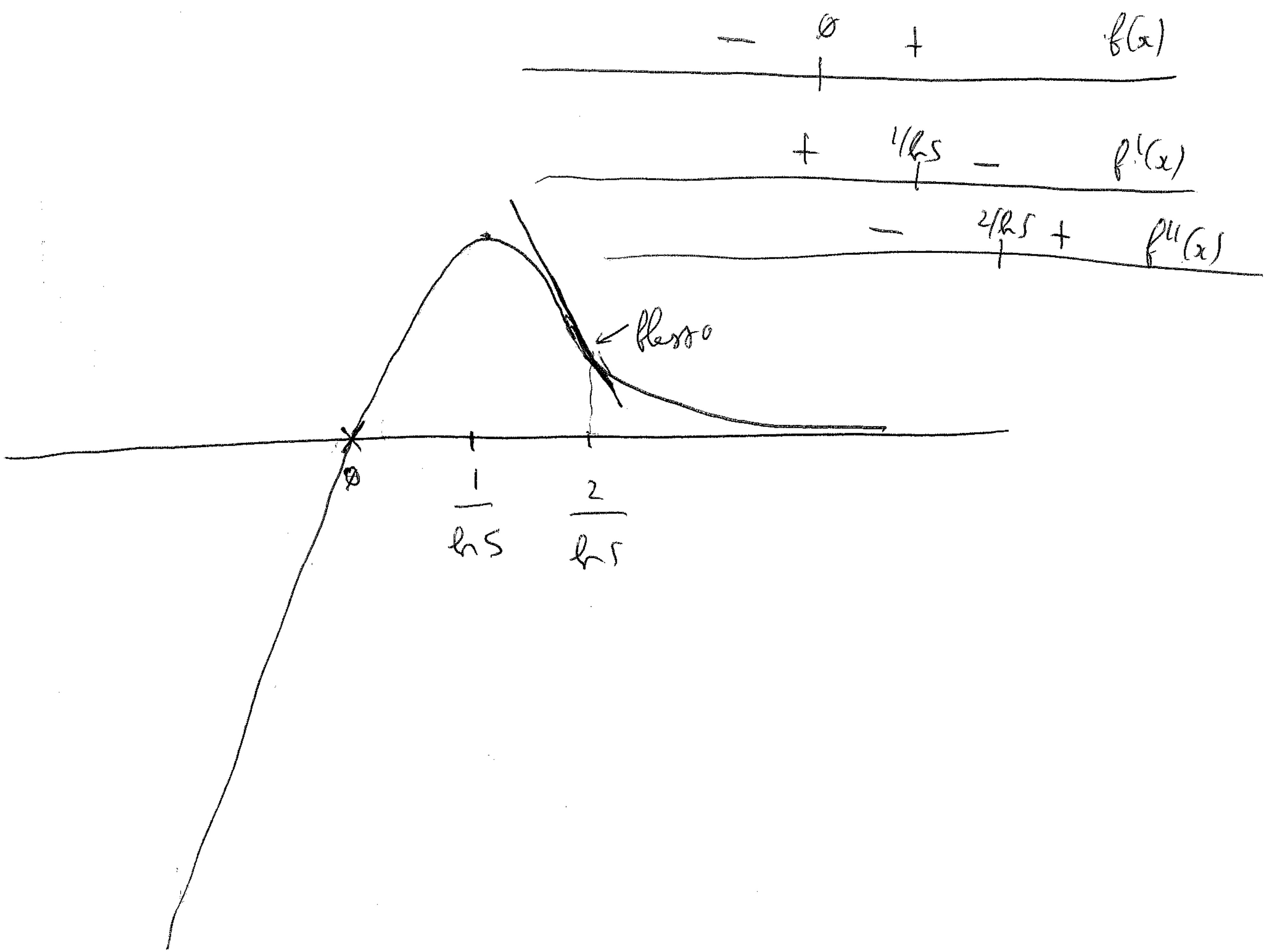
$\lim_{x\to+\infty} f(x) = \emptyset \qquad \lim_{x\to-\infty} f(x) = -\infty \qquad f(x) = (\ln 25)\, x\, e^{-x\ln 5}$

$$f'(x) = (\ln 25)\cdot 1 \cdot e^{-x\ln 5} + (\ln 25)\cdot x \cdot e^{-x\ln 5} \times (-\ln 5)$$

$$= 5^{-x}(\ln 25)\{1 - x\ln 5\}$$

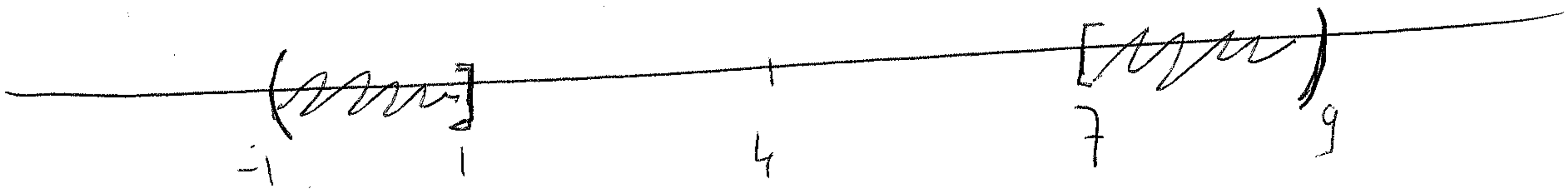
$$\begin{array}{ccccc} & + & \frac{1}{\ln 5} & - & f'(x) \\ \hline & \text{crescente} & \text{MAX} & \text{decrescente} & f(x) \end{array}$$

MASSIMO in $x = \frac{1}{\ln 5} \to f\left(\frac{1}{\ln 5}\right) = \frac{\ln 25}{\ln 5} 5^{-\ln 5} = 2 \cdot 5^{-\ln 5}$



– 0 + f(x)
+ 1/ln5 – f'(x)
– 2/ln5 + f''(x)
flesso
0
1/ln5
2/ln5

$$A = \{x \in \mathbb{R} : 3 \leq |x - 4| < 5\} = (-1, 1] \cup [7, 9)$$



$\inf A = -1$ NON c'è minimo

$\sup A = 9$ NON c'è massimo

$$\lim_{x \to -2} \frac{2^{x+2} - 1 + \operatorname{sen}(3x+6)}{\underbrace{(x^2+3x+2)}_{(x+1)(x+2)} \ln(6+x)}$$

$$\frac{1}{(x+1)\ln(6+x)} \left[\frac{2^{x+2}-1}{x+2} + \frac{\operatorname{sen}\{3(x+2)\}}{x+2} \right]$$

$$\longrightarrow \frac{1}{(-2+1)\ln(6-2)} \left[\ln 2 + 3 \right] = \frac{-[\ln 2 + 3]}{\ln 4}$$

$= \ln 2$

$$\lim_{t \to 0} \frac{2^t - 1}{t} = \lim_{t \to 0} \frac{e^{t\ln 2} - 1}{t} = \ln 2$$

$$\lim_{t \to 0} \frac{\operatorname{sen} 3t}{t} = \lim_{t \to 0} \frac{\operatorname{sen} 3t}{3t} 3$$

$$\lim_{x \to -2} \frac{2^{x+2} - 1 + \sin(3x+6)}{(x^2+3x+2)\ln(6+x)}$$

$$= \lim_{x \to -2} \frac{2^{x+2}\ln 2 + 3\cos(3x+6)}{(2x+3)\ln(6+x) + (x^2+3x+2)\frac{1}{6+x}}$$

$$= \frac{1 \cdot \ln 2 + 3 \cdot 1}{-1 \cdot \ln 4 + 0 \cdot \frac{1}{4}} = \frac{\ln 2 + 3}{-\ln 4}$$

$$\lim_{x\to -2} \frac{2^{x+2} - 1 + \operatorname{sen}(3x+6)}{(x^2+3x+2)\ln(6+x)}$$

$$= \lim_{x\to -2} \frac{1}{(x+1)\ln(6+x)} \lim_{x\to -2} \frac{2^{x+2} - 1 + \operatorname{sen}(3x+6)}{x+2}$$

$$= \frac{1}{(-1)\cdot \ln 4} \times \lim_{x\to -2} \frac{2^{x+2}\ln 2 + 3\cos(3x+6)}{1}$$

$$= \frac{\ln 2 + 3}{-\ln 4}$$

$$\lim_{x \to -\infty} \left(\sqrt{4x^2 + 9x} + 2x \right)$$

$$\frac{\left(\sqrt{4x^2+9x} + 2x\right)\left(\sqrt{4x^2+9x} - 2x\right)}{\sqrt{4x^2+9x} - 2x}$$

$$= \frac{\cancel{4x^2} + 9x - \cancel{4x^2}}{\sqrt{4x^2+9x} - 2x} = \frac{9}{-\sqrt{4 + \frac{9}{x}} - 2} \longrightarrow \frac{9}{-2-2} = -\frac{9}{4}$$

$$\cancel{= \lim_{x \to -\infty} \frac{9}{\frac{8x+9}{2\sqrt{4x^2+9x}} - 2}}$$

$$a_n = \frac{9^{n+1} + 36n^4 - 5}{9^n + 4n^4} = \frac{9[9^n + 4n^4] - 5}{9^n + 4n^4}$$

$$= 9 - \frac{5}{9^n + 4n^4} \longrightarrow 9$$

$9^n + 4n^4$ è crescente

$b_n = \dfrac{5}{9^n + 4n^4}$ è decrescente

a_n è crescente $(a_n = 9 - b_n)$

$\{a_n\}_{n=1}^{\infty}$ è crescente, $a_n < 9$

Una successione crescente e limitata superiormente ha limite finito.

3b)

$$a_n = \frac{3\sqrt[3]{n}\,(1+n^4) - 7n^{25/6}}{2006 + 12n^{13/3}} \qquad \frac{25}{6} - \frac{13}{3} = -\frac{1}{6}$$

$$a_n = \frac{3n^{1/3} + 3n^{13/3} - 7n^{25/6}}{2006 + 12n^{13/3}} = \frac{3n^{-4} + 3 - 7n^{-\cancel{1}/6}}{2006n^{-13/3} + 12} \to \frac{3}{12} = \frac{1}{4}$$

$$\frac{3}{12} = \frac{1}{4}$$

$$\lim_{t\to 0}\left(\cos 8t\right)^{\cot g^2(3t)}$$

$$\downarrow \ln$$

$$\lim_{t\to 0}\frac{\ln\cos 8t}{tg^2(3t)} = \lim_{t\to 0}\frac{\frac{1}{\cos 8t}\frac{d}{dt}\cos 8t}{2\,tg(3t)\,\frac{d}{dt}tg\,3t}$$

$$= \lim_{t\to 0}\frac{\overbrace{\frac{1}{\cos 8t}}^{\to +1}\cdot -8\,\mathrm{sen}\,8t}{2\,tg\,3t\,\underbrace{\frac{3}{\cos^2 3t}}_{\to 3}}$$

$$= \frac{-8}{6}\lim_{t\to 0}\frac{\mathrm{sen}\,8t}{tg\,3t} = -\frac{4}{3}\lim_{t\to 0}\frac{8\cos 8t}{3/\cos^2 3t} = -\frac{4}{3}\,\frac{8}{3} = \frac{-32}{9}$$

LIMITE ORIGINALE

$$e^{-32/9}$$

$$\lim_{x\to 0} \underbrace{(1+\operatorname{sen} x)^{\operatorname{cotg} x}} = e$$

$$\downarrow \ln$$

$$\lim_{x\to 0} \frac{\ln(1+\operatorname{sen} x)}{\operatorname{tg} x} = \lim_{x\to 0} \frac{\frac{1}{1+\operatorname{sen} x}\cos x}{1/\cos^2 x} = 1$$

$$\underbrace{\lim_{x\to 0} (1+x)^{1/x}}_{= e} \xrightarrow{\ln} \lim_{x\to 0} \frac{\ln(1+x)}{x} = \lim_{x\to 0} \frac{1/(1+x)}{1} = 1$$

$$f(x) = \ln\left(12x^2 + 2\cos(10\pi x)\right) \qquad x_0 = -\frac{1}{2}$$

$$f'(x) = \frac{1}{12x^2 + 2\cos(10\pi x)} \frac{d}{dx}\left\{12x^2 + 2\cos(10\pi x)\right\}$$

$$= \frac{24x - 2\,\mathrm{sen}(10\pi x) \cdot 10\pi}{12x^2 + 2\cos(10\pi x)}$$

$$y - f(x_0) = f'(x_0)(x - x_0)$$

$$f\left(-\frac{1}{2}\right) = \ln\left(3 + 2\cos(-5\pi)\right) = \ln(3-2) = 0$$

$$f'\left(-\frac{1}{2}\right) = \frac{-12 - 2\cancel{\mathrm{sen}(-5\pi)} \cdot 10\pi}{3 + 2\cos(-5\pi)} = -12$$

$$y - 0 = -12\left(x + \frac{1}{2}\right)$$

$$y = -12x - 6$$