LIMITI NOTEVOLI

17.10.2007

$$\lim_{n \to \infty} \frac{1}{n} = 0$$

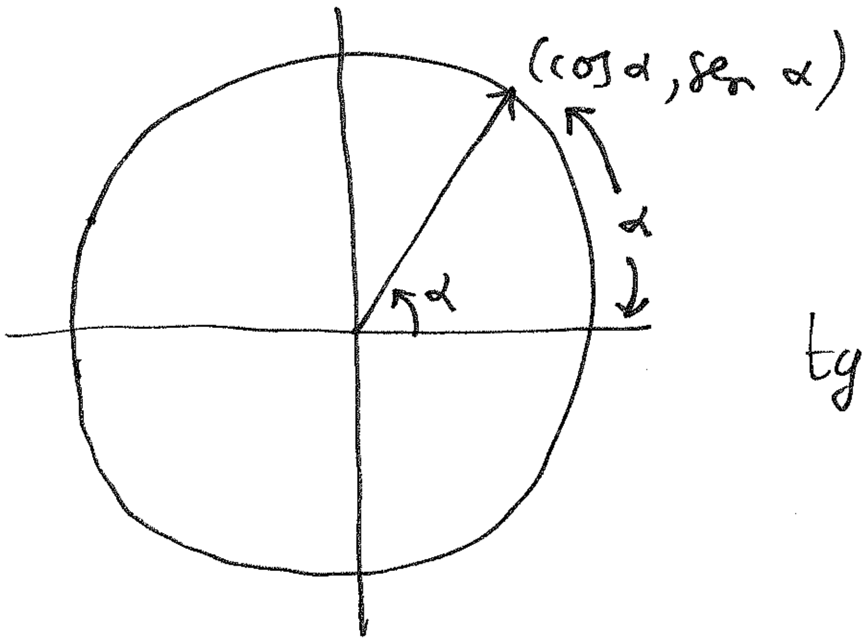
$$\lim_{n \to \infty} a^n = \begin{cases} +\infty, & a > 1 \\ 1, & a = 1 \\ 0, & -1 < a < 1 \\ \text{non esiste}, & a \leq -1 \end{cases}$$

$$\lim_{n \to \infty} \sqrt[n]{a} = 1, \quad a > 0$$

$$\lim_{n \to \infty} \sqrt[n]{n} = 1$$

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$$

$$\lim_{n \to \infty} \left(1 - \frac{1}{n}\right)^n = \frac{1}{e}$$



$$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha}$$

$$(\cos\alpha)^2 + (\operatorname{sen}\alpha)^2 = 1$$

$$-1 \leq \operatorname{sen}\alpha \leq 1$$

$$-1 \leq \cos\alpha \leq 1$$

TEOREMA DEI CARABINIERI

Siano a_n, b_n, c_n tre successioni tali che

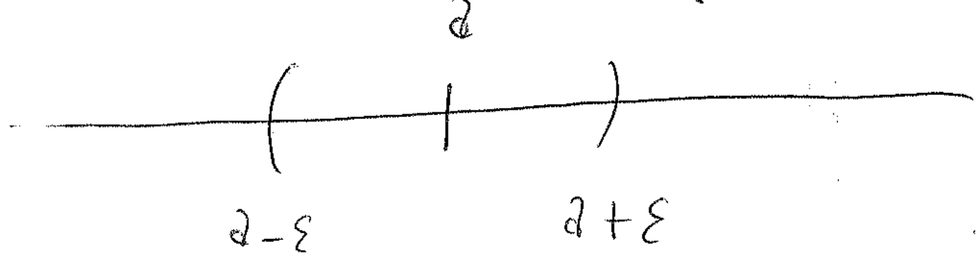
$$a_n \le c_n \le b_n$$

Supponiamo che

$$\lim_{n\to\infty} a_n = a = \lim_{n\to\infty} b_n$$

allora

$$\lim_{n\to\infty} c_n = a$$



$$\exists N: \left.\begin{array}{l} a-\varepsilon < a_n < a+\varepsilon \\ a-\varepsilon < b_n < a+\varepsilon \end{array}\right\} \text{per } n>N$$

Quindi

$a-\varepsilon < c_n$

Oppure: $\exists N: \ a-\varepsilon < a_n \leq c_n \leq b_n < a+\varepsilon$

per $n \geq N$

Quindi $\lim_{n\to\infty} c_n = a$

$\cancel{\lim_{n\to\infty}} \quad \cancel{\operatorname{sen}(n\pi)}$

$$\lim_{n\to\infty} \frac{\operatorname{sen}(n! + n^n + n^2 + 1)}{n^2+1} = \emptyset$$

$$-\frac{1}{n^2+1} \leq \frac{\operatorname{sen}(\ldots)}{n^2+1} \leq \frac{1}{n^2+1}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$\emptyset \qquad\qquad \emptyset \qquad\qquad \emptyset$$

$$\frac{1}{n^2+1} = \frac{\frac{1}{n^2}}{1+\frac{1}{n^2}} \longrightarrow \emptyset$$

$$\lim_{n\to\infty} \frac{n-4}{3n+2} = \lim_{n\to\infty} \frac{1-\frac{4}{n}}{3+\frac{2}{n}} = \frac{1-4\cdot 0}{3+2\cdot 0} = \frac{1}{3}$$

$$\lim_{n\to\infty} \frac{n^2+2n}{n+1} = \lim_{n\to\infty} \frac{n(n+2)}{n+1} = \lim_{n\to\infty} n \cdot \underbrace{\frac{1+\frac{2}{n}}{1+\frac{1}{n}}}_{\to \frac{1+0}{1+0} = 1} = +\infty \cdot 1 = +\infty$$

$$\frac{n^2}{n+n} \leq \frac{n^2+2n}{n+1} \leq \frac{n^2+2n+1}{n+1}$$

$$\frac{n}{2} \leq \frac{n^2+2n}{n+1} \leq n+1 \qquad \lim_{n\to\infty} \frac{n^2+2n}{n+1} = +\infty$$

$$\lim_{n\to\infty} \frac{3+n-2n^2}{5n^2+7} = \lim_{n\to\infty} \frac{\frac{3}{n^2}+\frac{1}{n}-2}{5+\frac{7}{n^2}} = \frac{\emptyset+\emptyset-2}{5+\emptyset} = \frac{-2}{5}$$

$$\frac{-2n^2}{\quad} \leq \frac{3+n-2n^2}{5n^2+7} \leq \frac{\quad}{\quad}$$

$$\lim_{n\to\infty} \frac{n+(-1)^n}{n-(-1)^n} = \lim_{n\to\infty} \frac{1+\frac{(-1)^n}{n}}{1-\frac{(-1)^n}{n}} = \frac{1+0}{1-0} = 1$$

$$\begin{array}{ccccc} -\frac{1}{n} & \leq & \frac{(-1)^n}{n} & \leq & \frac{1}{n} \\ \downarrow & & \downarrow & & \downarrow \\ 0 & & 0 & & 0 \end{array}$$

$$\lim_{n\to\infty}\left(\sqrt{n^2+1}-n\right)=\emptyset$$

$$\frac{\left(\sqrt{n^2+1}-n\right)\left(\sqrt{n^2+1}+n\right)}{\sqrt{n^2+1}+n}=\frac{(n^2+1)-n^2}{\sqrt{n^2+1}+n}=\frac{1}{\sqrt{n^2+1}+n}$$

$$\lim_{n\to\infty}\left(\sqrt{n^2+2n}-n\right)$$

$$\emptyset<\frac{1}{\sqrt{n^2+1}+n}<\frac{1}{n}$$

$$\underbrace{\emptyset \qquad\qquad \emptyset}_{\emptyset}$$

$$\frac{\left(\sqrt{n^2+2n}-n\right)\left(\sqrt{n^2+2n}+n\right)}{\sqrt{n^2+2n}+n} = \frac{(n^2+2n)-n^2}{\sqrt{n^2+2n}+n} = \frac{2n}{\sqrt{n^2+2n}+n}$$

$$= \frac{2}{\sqrt{1+\frac{2}{n}}+1} \longrightarrow \frac{2}{\sqrt{1+\emptyset}+1} = 1$$

$$\lim_{n\to\infty} n\sqrt{\frac{1}{n+1}} = \lim_{n\to\infty}\sqrt{\frac{n^2}{n+1}} = +\infty$$

$$\begin{array}{ccccc}
\dfrac{n^2-1}{n+1} & \leq & \dfrac{n^2}{n+1} & \leq & \dfrac{n^2}{n} \\
\| & & & & \| \\
n-1 & & & & n \\
\downarrow & & & & \downarrow \\
+\infty & & & & +\infty
\end{array}$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{\downarrow\, +\infty}$$

$$\lim_{n\to\infty} \sqrt[n]{n^2+1} = 1$$

$$\sqrt[n]{n^2} \leq \sqrt[n]{n^2+1} \leq \sqrt[n]{n^2+n^2}$$

$$\sqrt[n]{n^2} = \left(\sqrt[n]{n}\right)^2 \to (1)^2 = 1$$

$$\sqrt[n]{n^2+n^2} = \sqrt[n]{2}\cdot\left(\sqrt[n]{n}\right)^2 \to 1\cdot(1)^2 = 1$$

$$\underbrace{(1)^2 = 1 \qquad\qquad 1\cdot(1)^2 = 1}_{\downarrow\, 1}$$

$$\sqrt[n]{3^n} < \sqrt[n]{2^n+3^n} < \sqrt[n]{3^n+3^n}$$

$$\sqrt[n]{3^n} = 3 \to 3$$

$$\sqrt[n]{3^n+3^n} = \sqrt[n]{2}\,\sqrt[n]{3^n} = \sqrt[n]{2}\cdot 3 \to 3$$

$$\sqrt[n]{2^n+3^n} \to 3$$

$$\underbrace{\underset{\substack{\downarrow\\ 1}}{1} < \underset{\substack{\downarrow\\ 1}}{\sqrt[n]{1+\left(\frac{2}{3}\right)^n}} < \sqrt[n]{1+1} = \underset{\substack{\downarrow\\ 1}}{\sqrt[n]{2}}}$$

$$\lim_{n\to\infty} \frac{2^n+3^n}{2^{n+1}+3^{n+1}} = \lim_{n\to\infty} \frac{\left(\frac{2}{3}\right)^n + 1}{2\cdot\left(\frac{2}{3}\right)^n + 3} = \frac{\emptyset+1}{2\cdot\emptyset+3} = \frac{1}{3}$$

$$\lim_{n\to\infty} \left(\frac{2}{3}\right)^n = \emptyset$$

$$\lim_{n\to\infty} \sqrt[n]{2^n+3^n} = \lim_{n\to\infty} \sqrt[n]{3^n\left(\left(\frac{2}{3}\right)^n+1\right)} = 3 \lim_{n\to\infty} \sqrt[n]{1+\left(\frac{2}{3}\right)^n}$$

$$= 3\cdot 1 = 3$$

$$\frac{2n^2}{n^2+n+2} = \frac{2}{1+\frac{1}{n}+\frac{2}{n^2}} \longrightarrow \frac{2}{1+\emptyset+\emptyset} = 2$$

$$\left(\frac{n^2+n}{n^2+n+2}\right)^{n^2} = \left(1-\frac{2}{n^2+n+2}\right)^{n^2}$$

$$= \boxed{\left(1-\frac{2}{n^2+n+2}\right)^{\frac{n^2+n+2}{2}}}^{\cdot\frac{2n^2}{n^2+n+2}} = \left(\frac{1}{e}\right)^2 = e^{-2}$$

$$K = n^2+n+2$$

$$\lim_{K\to\infty}\left(1-\frac{1}{K}\right)^K = \frac{1}{e}$$

$$= \frac{2 + \frac{4}{n}}{\sqrt{1 + \frac{4}{n} + \frac{7}{n^2}} + \sqrt{1 + \frac{2}{n} + \frac{3}{n^2}}} \Longrightarrow \frac{2 + 0}{\sqrt{1 + 0 + 0} + \sqrt{1 + 0 + 0}} = 1$$

$$a_n = \sqrt{n^2+4n+7} - \sqrt{n^2+2n+3}$$

$$\frac{\left(\sqrt{n^2+4n+7} - \sqrt{n^2+2n+3}\right)\left(\sqrt{n^2+4n+7} + \sqrt{n^2+2n+3}\right)}{\sqrt{n^2+4n+7} + \sqrt{n^2+2n+3}}$$

$$= \frac{(n^2+4n+7) - (n^2+2n+3)}{\sqrt{n^2+4n+7} + \sqrt{n^2+2n+3}} = \frac{2n+4}{\sqrt{n^2+4n+7} + \sqrt{n^2+2n+3}}$$

$$\lim_{n\to\infty}\left(\frac{n+3}{n+2}\right)^{2n+1}$$

$$\left\{\left(1+\frac{1}{n+2}\right)^{p}\right\}^{q}=\left(1+\frac{1}{n+2}\right)^{pq}$$

$$=\lim_{n\to\infty}\left\{\left(1+\frac{1}{n+2}\right)^{(n+2)}\right\}^{\frac{2n+1}{n+2}}=e^{2}$$

$$\frac{2n+1}{n+2}=\frac{2+\frac{1}{n}}{1+\frac{2}{n}}\to\frac{2+0}{1+0}=2$$

$$\frac{n+3}{n+2}=1+\frac{1}{n+2}\to 1$$

$$K=n+2\qquad \lim_{K\to\infty}\left(1+\frac{1}{K}\right)^{K}=e$$

$$2n+1\longrightarrow+\infty$$