7-1 secondo parziale 15.30-18.30

8-1 sera → correzione

9-1/10-1 9-13 orali

11-1 partenza per Dallas

SCRITTI GENERALI: 25-1

14-2

$$\int_{-2}^{-1} \underset{\overset{\downarrow}{g'}}{(x+2)}\ \underset{\overset{\uparrow}{f}}{\ln|x|}\, dx = \left[\underset{g}{\left(\tfrac{1}{2}x^2+2x\right)} \ln|x|\right]_{-2}^{-1} - \int_{-2}^{-1} \overbrace{\underset{g}{\left(\tfrac{1}{2}x^2+2x\right)}\ \underset{f'}{\frac{1}{x}}}^{\frac{1}{2}x+2}\, dx$$

$$= \left[\left(\tfrac{1}{2}x^2+2x\right)\ln|x|\right]_{-2}^{-1} - \int_{-2}^{-1} \left(\tfrac{1}{2}x+2\right) dx$$

$$= \left[\left(\tfrac{1}{2}x^2+2\right)\ln|x| - \left(\tfrac{1}{4}x^2+2x\right)\right]_{-1}^{-2} = \left\{4\ln 2 - (-3)\right\} - \left\{\emptyset - \left(-\frac{7}{4}\right)\right\}$$

$$= 4\ln 2 + \frac{5}{4}$$

$$\int_0^{\pi/2} \frac{dx}{-5+4\cos x} \cancel{dx} = \int_0^1 \frac{1}{-5+4\frac{1-t^2}{1+t^2}} \frac{2\,dt}{1+t^2}$$

$t = \operatorname{tg} \frac{1}{2}x$

$x = 2 \operatorname{arctg} t \longrightarrow dx = \frac{2\,dt}{1+t^2}$

$$= \int_0^1 \frac{2\,dt}{-5(1+t^2)+4(1-t^2)}$$

$$= \int_0^1 \frac{-2\,dt}{1+9t^2}$$

$$\cos x = 2 \underbrace{\cos^2 \tfrac{1}{2}x}_{1/(1+t^2)} - 1 = \frac{2}{1+\operatorname{tg}^2\frac{1}{2}x} - 1 = \frac{1-t^2}{1+t^2}$$

$u = 3t \qquad du = 3\,dt$

$$\operatorname{sen} x = 2 \operatorname{tg}\tfrac{1}{2}x \cos^2 \tfrac{1}{2}x = \frac{2t}{1+t^2}$$

$2\,dt = \frac{2}{3}\,du$

$$\Rightarrow \int_{\emptyset}^{3} \frac{-\frac{2}{3}\,du}{1+u^2} = \left[-\frac{2}{3}\operatorname{arctg} u\right]_{\emptyset}^{3} = -\frac{2}{3}\operatorname{arctg} 3$$

$$\int_{\emptyset} \frac{dx}{-5+4\cos x} = \dots\dots = \int \frac{-\frac{2}{3}\,du}{1+u^2} = -\frac{2}{3}\operatorname{arctg} u + \text{cost.}$$

$$= -\frac{2}{3}\operatorname{arctg}(3t) + \text{const.} = -\frac{2}{3}\operatorname{arctg}\left(3\,\operatorname{tg}\frac{1}{2}x\right) + \text{cost.}$$

$$\int \frac{3t+8}{t^2+6t+8}\,dt = \int \frac{3t+8}{(t+2)(t+4)}\,dt = \int \left(\frac{A}{t+2} + \frac{B}{t+4}\right) dt$$

$$A(t+4) + B(t+2) = 3t+8$$

$$A + B = 3$$

$$4A + 2B = 8 \quad \rightarrow 2A + B = 4$$

$$\rightarrow A = 1, \quad B = 2$$

$$= \int \left(\frac{1}{t+2} + \frac{2}{t+4}\right) dt = \ln|t+2| + 2\ln|t+4| + \text{const.}$$

$$\cancel{= \ln|(t+2)(t+4)^2| + \text{const.}}$$

$$\int \frac{3t+8}{t^2+6t+9}\,dt$$

$$\int \frac{3t+8}{t^2+6t+10}\,dt$$

$$\int \frac{3t+8}{t^2+6t+9}\,dt = \int \frac{3(t+3)-1}{(t+3)^2}\,dt = \int\left(\frac{3}{t+3} - \frac{1}{(t+3)^2}\right)dt$$

$$= 3\ln|t+3| + \frac{1}{t+3} + \text{cost.}$$

$$\int \frac{3t+8}{t^2+6t+10}\,dt = \int \frac{3(t+3)-1}{(t+3)^2+1}\,dt = \int\left(3\frac{t+3}{(t+3)^2+1} - \frac{1}{1+(t+3)^2}\right)dt$$

$$= \frac{3}{2}\ln|(t+3)^2+1| - \operatorname{arctg}(t+3) + \text{cost.}$$

cancellare

$$\int \frac{3t+8}{t^2+6t+13}\,dt = \int \frac{3(t+3)-1}{(t+3)^2+4}\,dt = \frac{3}{2}\ln\{(t+3)^2+4\}$$

$$-\int \frac{\frac{1}{4}\,dt}{\left(\frac{t+3}{2}\right)^2+1} = \frac{3}{2}\ln\{(t+3)^2+4\} \cancel{-\frac{1}{2}\operatorname{arctg}} -\frac{1}{2}\int\frac{du}{1+u^2}$$

$$= \frac{3}{2}\ln\{\dots\} - \frac{1}{2}\operatorname{arctg} u + \text{cost.}$$

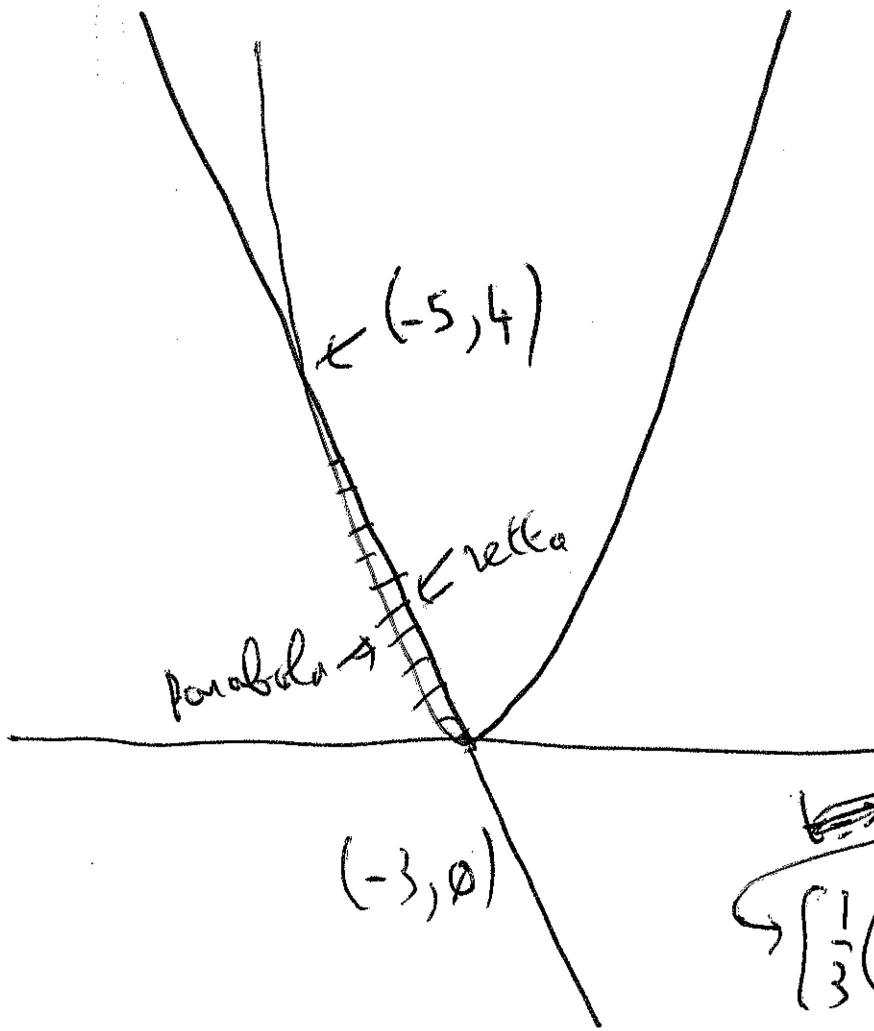
$u = \frac{t+3}{2}$, $dt = 2\,du$

$$= \frac{3}{2}\ln\{\dots\} - \frac{1}{2}\operatorname{arctg}\left(\frac{t+3}{2}\right) + \text{cost.}$$

$$\left.\begin{array}{l} y=(x+3)^2 \\ y=-2x-6 \end{array}\right\}$$ passano per il punto $(-3,0)$

$$(x+3)^2=\underbrace{-2x-6}_{-2(x+3)}$$

$$(x+3)(x+5)=0$$



$$=\int_{-5}^{-3}(x+3)^2\,dx$$

$$\cancel{t=x+3}$$

$$\left[\frac{1}{3}(x+3)^3\right]_{-5}^{-3}=0-\left(\frac{-8}{3}\right)=\frac{8}{3}$$

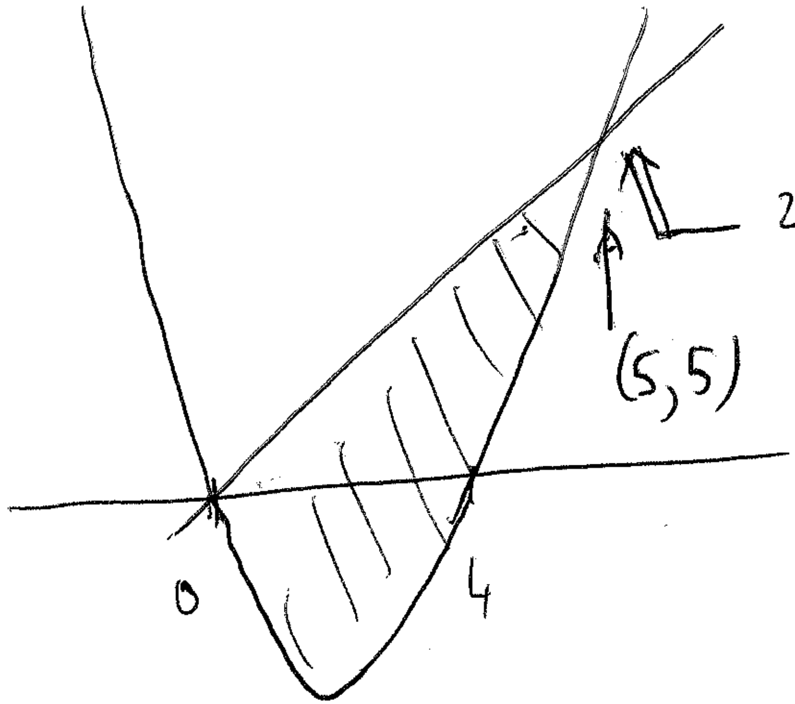
$$=\int_{-5}^{-3}(-2x-6)\,dx$$

$$\left[-x^2-6x\right]_{-5}^{-3}=4$$

$$=4$$

$$\text{area}=4-\frac{8}{3}=\frac{4}{3}$$

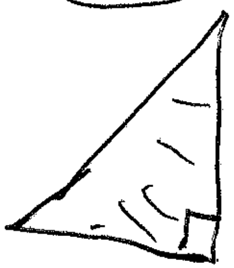
$$\left.\begin{aligned} y &= x^2 - 4x \\ y &= x \end{aligned}\right\}$$

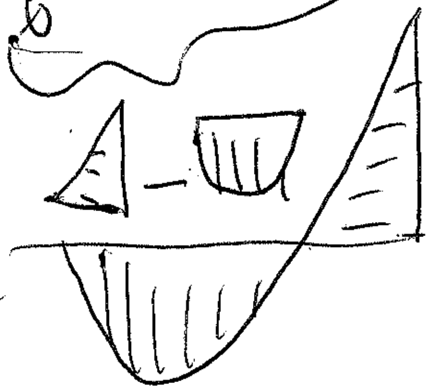


risolvere $x^2 - 4x = x$

$x(x-5) = 0$

$$\text{Area} = \int_0^5 x\,dx - \int_0^5 (x^2 - 4x)\,dx = \left[\frac{1}{2}x^2 - \left(\frac{1}{3}x^3 - 2x^2\right)\right]_0^5$$





$$= \left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^5$$

$$= \frac{125}{2} - \frac{125}{3} = \frac{125}{6}$$

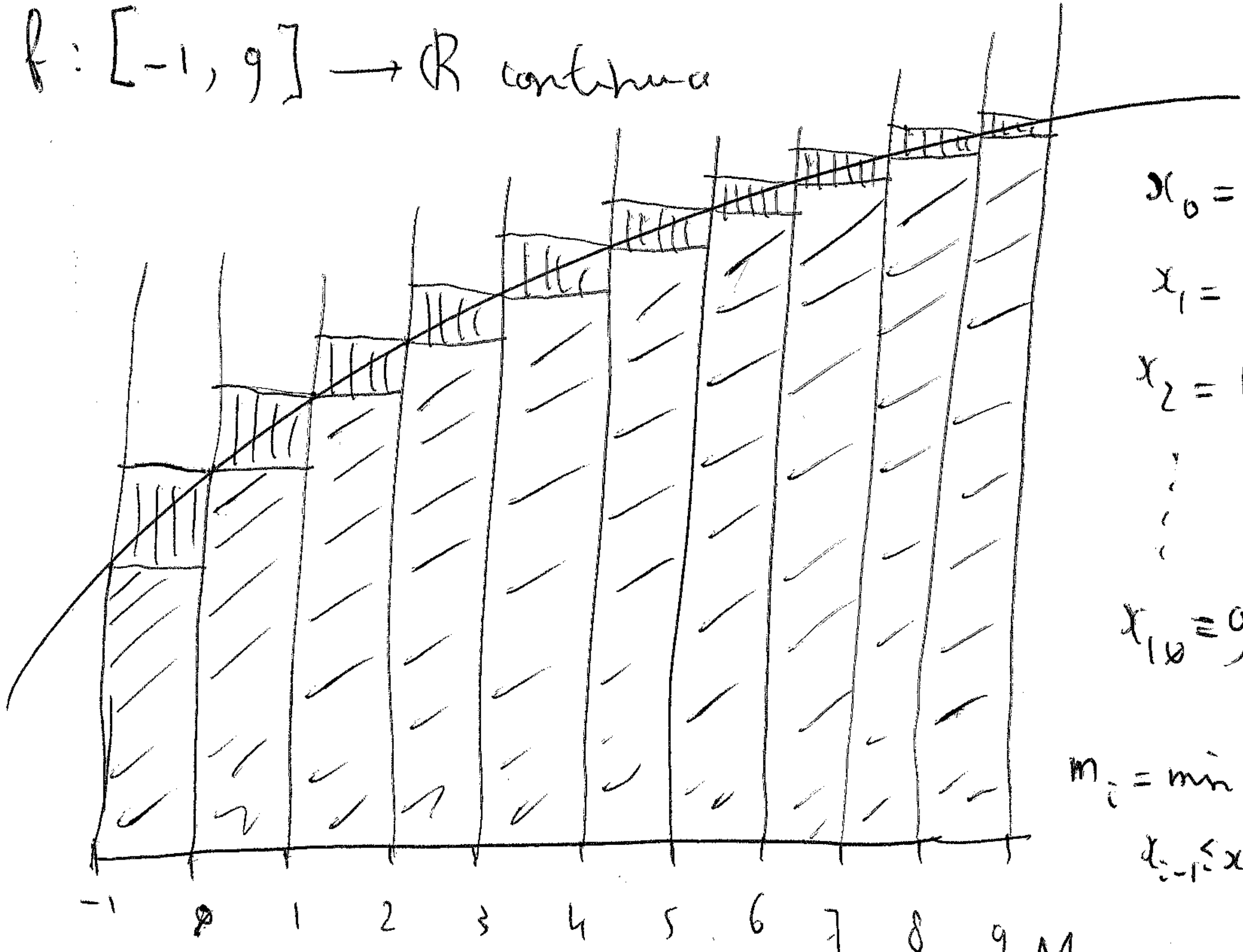
$$\int \frac{5}{\sqrt[3]{7-6x}}\,dx = \int 5(7-6x)^{-1/3}\,dx = -\frac{5}{4}(7-6x)^{2/3} + \text{cost.}$$

$$(7-6x)^{2/3} \xrightarrow{d/dx} \frac{2}{3}(7-6x)^{-1/3}\underbrace{\frac{d}{dx}(7-6x)}_{-6} = -4(7-6x)^{-1/3}$$

$$t = 7-6x \qquad dt = -6\,dx$$

$$\int \frac{-5/6}{\sqrt[3]{t}}\,dt = -\frac{5}{6}\,\frac{3}{2}\,t^{2/3} + \text{cost} = -\frac{5}{4}(7-6x)^{2/3} + \text{cost.}$$

$f : [-1, 9] \to \mathbb{R}$ continua



$x_0 = -1$

$x_1 = 0$ $\quad m_i$

$x_2 = 1$

$\vdots$

$x_{10} = 9$

$i = 1, \dots, 10$

$$m_i = \min_{x_{i-1} \le x \le x_i} f(x) \overset{\uparrow}{=} f(x_{i-1})$$

f crescente

$$M_i = \max_{x_{i-1} \le x \le x_i} f(x) \overset{\downarrow}{=} f(x_i)$$

Sia $f : [a,b] \to \mathbb{R}$ continua. Sia

- $F(x) = \int_a^x f(t)\,dt$, $a \le x \le b$.

allora F è derivabil e $F'(x) = f(x)$ per $a \le x \le b$. Se ci sono ~~due~~ F, G derivabili con $F' = G' = f$, allora $F - G$ è costante.

Somma inferiore $= \sum_{i=1}^{10} m_i \underbrace{(x_i - x_{i-1})}_{=1} = \sum_{i=1}^{10} m_i$

Somma superiore $= \sum_{i=1}^{10} M_i \underbrace{(x_i - x_{i-1})}_{=1} = \sum_{i=1}^{10} M_i$

$$\int_a^b f(t)\,dt = F(b) - F(a) = G(b) - G(a) \left(= \left[F(t) \right]_a^b \right)$$

$$\int_{\sqrt{3}}^{\infty} \frac{(3\operatorname{arctg} x)^{2008}}{1+x^2}\,dx \qquad \operatorname{arctg}(\sqrt{3}) = \frac{\pi}{3}$$

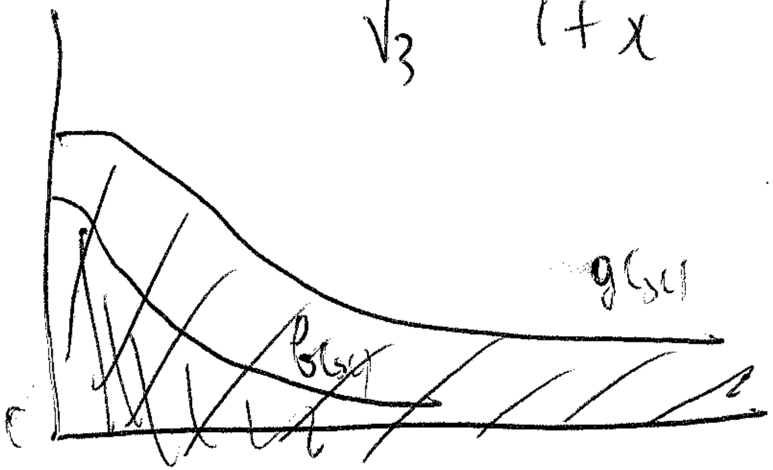
$$\pi \leq 3\operatorname{arctg} x \leq \frac{3\pi}{2}$$

$$= \lim_{a \to +\infty} \underbrace{\int_{\sqrt{3}}^{a} \frac{(3\operatorname{arctg} x)^{2008}}{1+x^2}\,dx}_{\text{crescente in } a,\ \text{limitato}} \qquad \frac{\pi^{2008}}{1+x^2} \leq \frac{3(\operatorname{arctg} x)^{2008}}{1+x^2} \leq \frac{(3\pi/2)^{2008}}{1+x^2}$$

superiore da
$$\int_{\sqrt{3}}^{a} \frac{(3\pi/2)^{2008}}{1+x^2}\,dx = \left(\frac{3\pi}{2}\right)^{2008} \Big[\operatorname{arctg} x\Big]_{\sqrt{3}}^{a} \leq \left(\frac{3\pi}{2}\right)^{2008} \left(\frac{\pi}{2} - \frac{\pi}{3}\right)$$

$$\int_{\sqrt{3}}^{\infty} \underbrace{\frac{(3\,\mathrm{arctg}\,x)^{2008}}{1+x}}_{\geq \frac{\pi^{2008}}{1+x}} dx$$

$$\underline{\text{mentre}} \quad \int_{\sqrt{3}}^{\infty} \frac{\pi^{2008}}{1+x}\,dx = \lim_{a \to +\infty} \left[\pi^{2008} \ln(1+x)\right]_{\sqrt{3}}^{a} = +\infty$$



Sia $0 \leq f(x) \leq g(x)$ per $x \geq c$, dove $f, g : [c, +\infty) \to \mathbb{R}$ sono continue.

Se è convergente $\int_c^{\infty} g(x)\,dx$, allora è convergente $\int_c^{\infty} f(x)\,dx$.

Se è divergente $\int_c^{\infty} f(x)\,dx$, allora è divergente $\int_c^{\infty} g(x)\,dx$

$$\int_{-3/2}^{\infty} (2x+3)e^{-2x}\,dx = \lim_{a \to +\infty} \int_{-3/2}^{a} \underset{f}{\underset{\uparrow}{(2x+3)}}\,\underset{g'}{\underset{\uparrow}{e^{-2x}}}\,dx$$

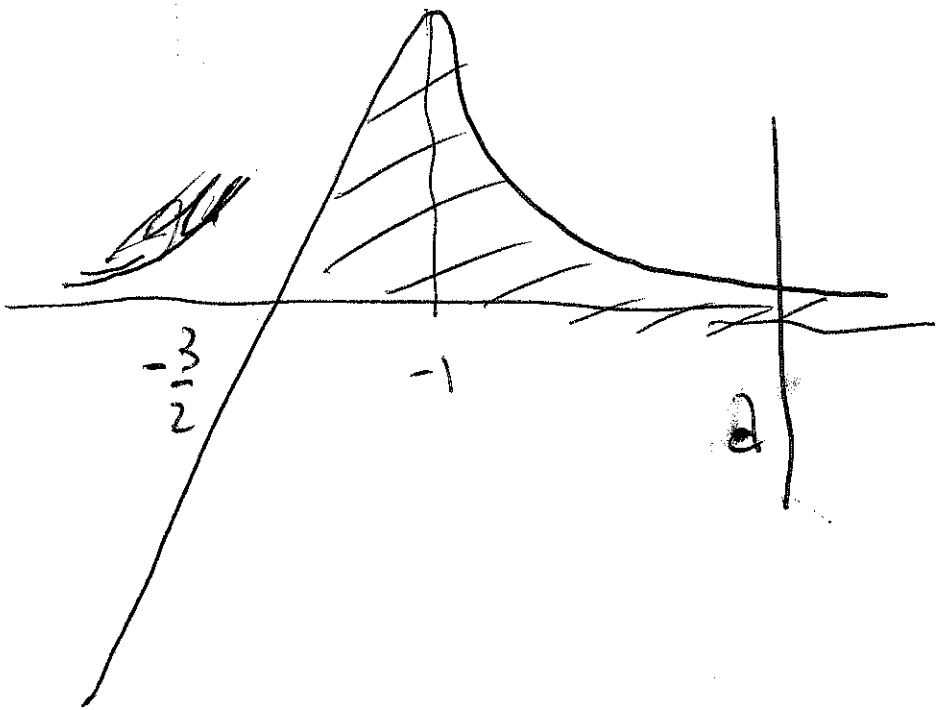
$$\left[\underset{f}{\underset{\uparrow}{(2x+3)}} \cdot \underset{g}{\underset{\uparrow}{-\tfrac{1}{2}e^{-2x}}}\right]_{-3/2}^{a} - \int_{-3/2}^{a} \underset{f'}{\underset{\uparrow}{2}} \cdot -\tfrac{1}{2}e^{-2x}\,dx$$

$$= \left[(2x+3) \cdot -\tfrac{1}{2}e^{-2x} + \left(-\tfrac{1}{2}\right)e^{-2x}\right]_{-3/2}^{a} = \underbrace{-\tfrac{1}{2}(2a+4)e^{-2a}}_{\to 0,\ \text{se } a \to +\infty} + \tfrac{1}{2}e^{3} \to \tfrac{1}{2}e^{3}$$

$$f(x) = (2x+3)e^{-2x} \longrightarrow f'(x) = 2e^{-2x} + (2x+3)\cdot -2e^{-2x}$$

$$= 2(1-2x-3)e^{-2x} = -4(x+1)e^{-2x}$$

		-1		
	+		−	$f'(x)$
	crescente	max	decrescente	$f(x)$



$$\int_{5/2}^{3} \frac{4}{\sqrt[3]{2x-5}}\,dx$$

$$f(x) = \frac{4}{\sqrt[3]{2x-5}}$$

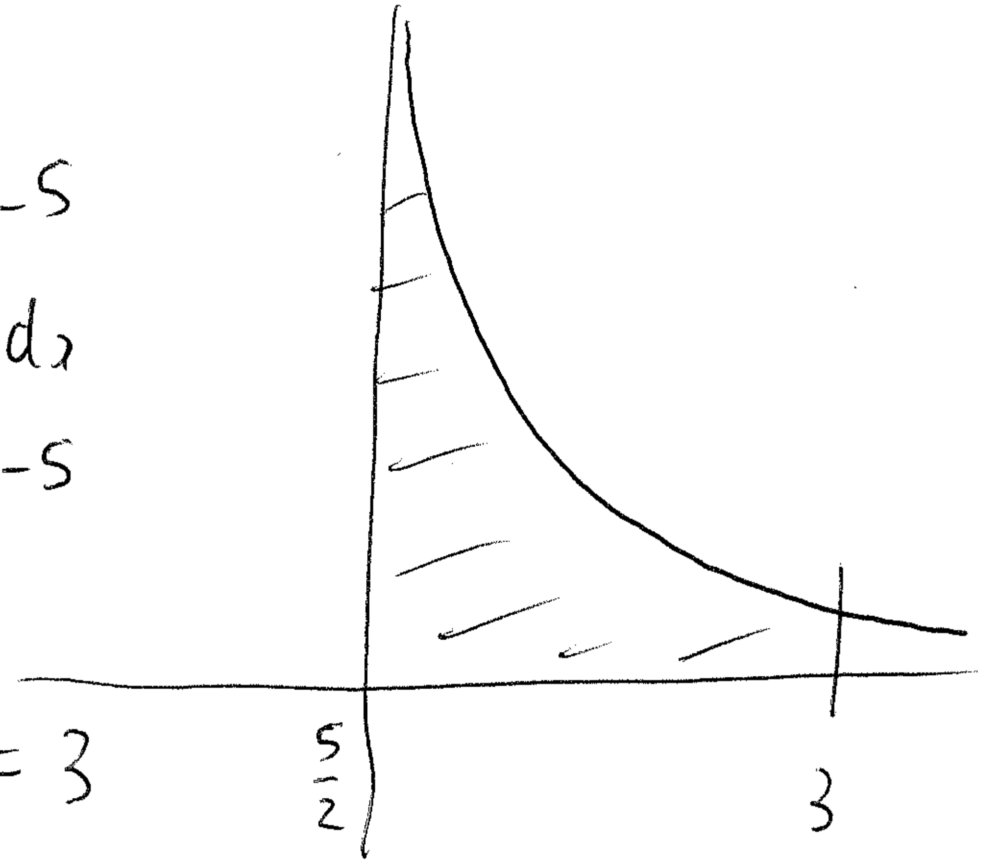
$$= \lim_{a\to\left(\frac{5}{2}\right)^+} \int_a^3 \frac{4}{\sqrt[3]{2x-5}}\,dx$$

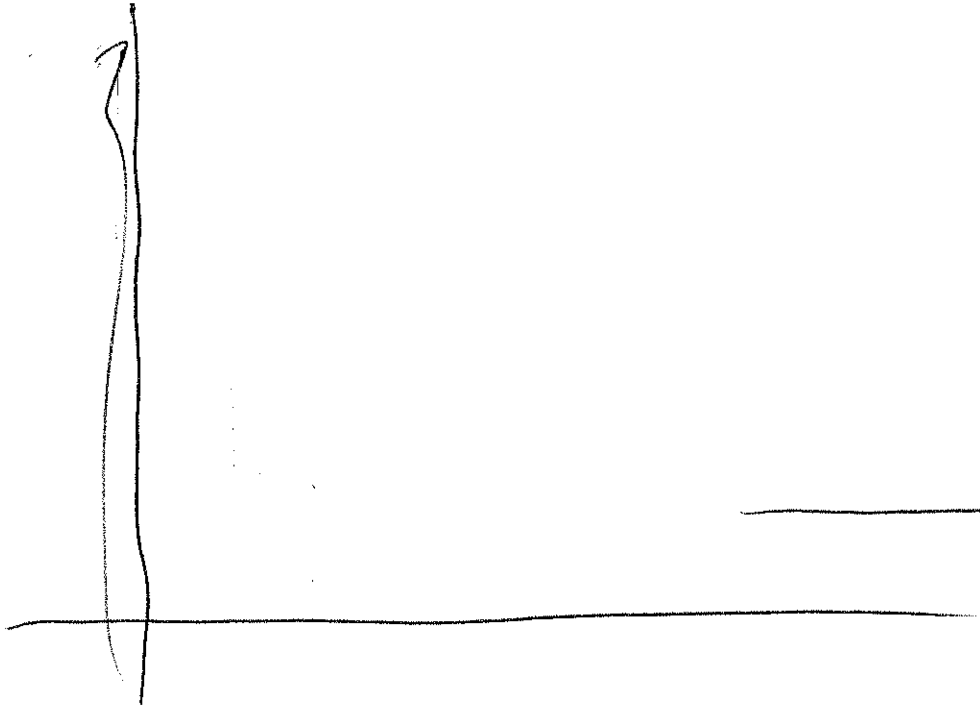
$t = 2x-5$

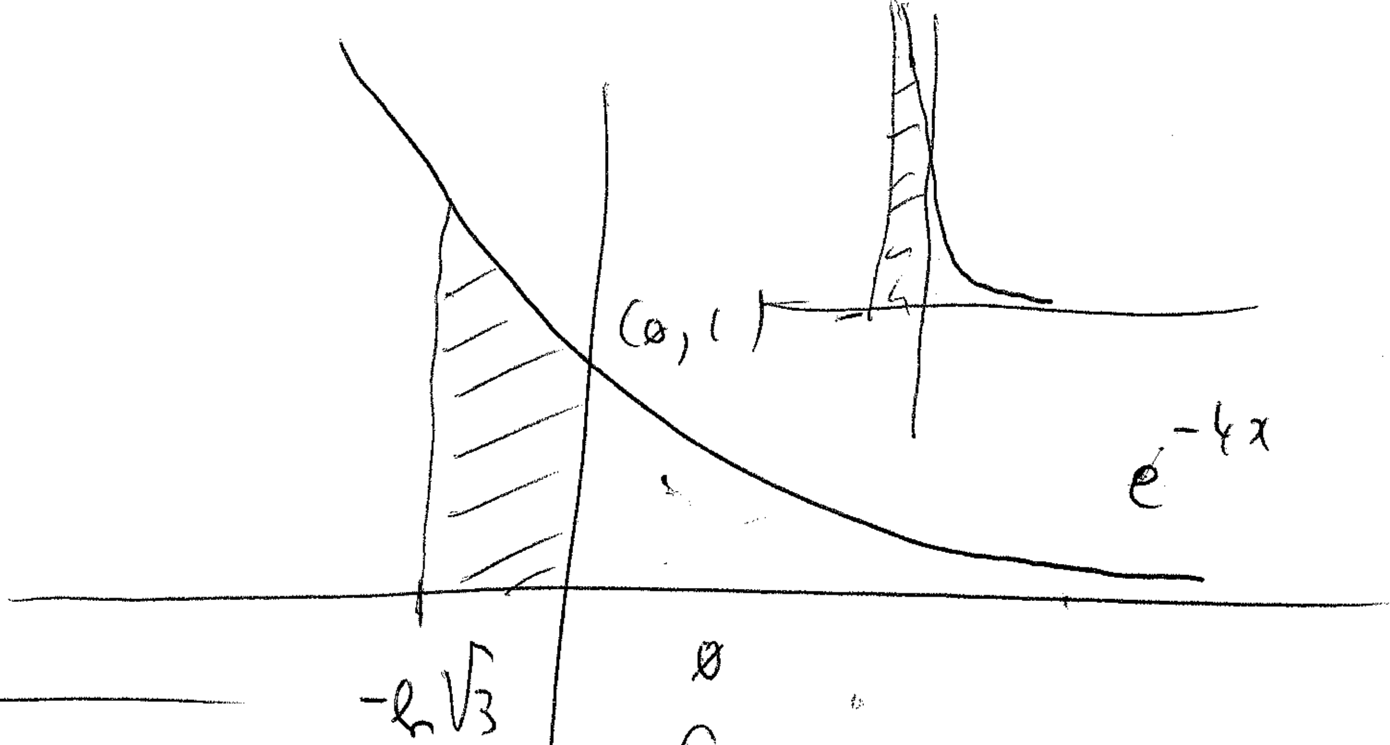
$dt = 2\,dx$

$b = 2a-5$

$$= \lim_{b\to 0^+} \int_b^1 \frac{2\,dt}{\sqrt[3]{t}} = \lim_{b\to 0^+} \left[3\,t^{2/3}\right]_b^1 = 3$$







$$\int_{-\ln\sqrt{3}}^{0} e^{-4x}\,dx$$

$$=$$

$$\left[-\frac{1}{4}e^{-4x}\right]_{-\ln\sqrt{3}}^{0}$$

$$= -\frac{1}{4} + \frac{1}{4}e^{4\ln\sqrt{3}} = 2$$

$$e^{\ln(\sqrt{3})^4} = e^{\ln 9} = 9$$

$$\int_{-\ln\sqrt{3}}^{a} e^{-4x}\,dx = \lim_{a\to+\infty}\left[-\frac{1}{4}e^{-4x}\right]_{-\ln\sqrt{3}}^{a} = \frac{9}{4}$$

$$f(x) = \operatorname{arctg}(5x-6) = f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \cdots$$

$$f(1) = \operatorname{arctg}(-1) = -\frac{\pi}{4}$$

$$f'(x) = \frac{1}{1+(5x-6)^2}\frac{d}{dx}(5x-6) = \frac{5}{1+(5x-6)^2} \to f'(1) = \frac{5}{1+(1)^2} = \frac{5}{2}$$

$$f''(x) = \frac{-5}{[1+(5x-6)^2]^2}\underbrace{\frac{d}{dx}\{1+(5x-6)^2\}}_{2(5x-6)\cdot 5} = \frac{-50(5x-6)}{[1+(5x-6)^2]^2}$$

$$\hookrightarrow f''(1) = \frac{50}{(1+(1)^2)^2} = \frac{25}{2}$$

$$\operatorname{arctg}(5x-6) = -\frac{\pi}{4} + \frac{5}{2}(x-1) + \frac{25}{4}(x-1)^2 + \cdots$$

$$\cancel{f(x)} = \int_1^{\infty} \frac{\left[2+\cos(x^2)\right]^{2008!}}{1+x^2}\,dx \qquad 1 \leq 2+\cos(x^2) \leq 3$$

$$\int_1^{\infty} \frac{dx}{1+x^2} = \lim_{a\to+\infty} \int_1^{a} \frac{dx}{1+x^2} = \lim_{a\to+\infty} \left\{ \operatorname{arctg} a - \frac{\pi}{4} \right\} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} \text{ convergente}$$

$$0 \leq \frac{\left[2+\cos(x^2)\right]^{2008!}}{1+x^2} \leq \frac{3^{2008!}}{1+x^2} \longrightarrow$$ È convergente l'integrale originale