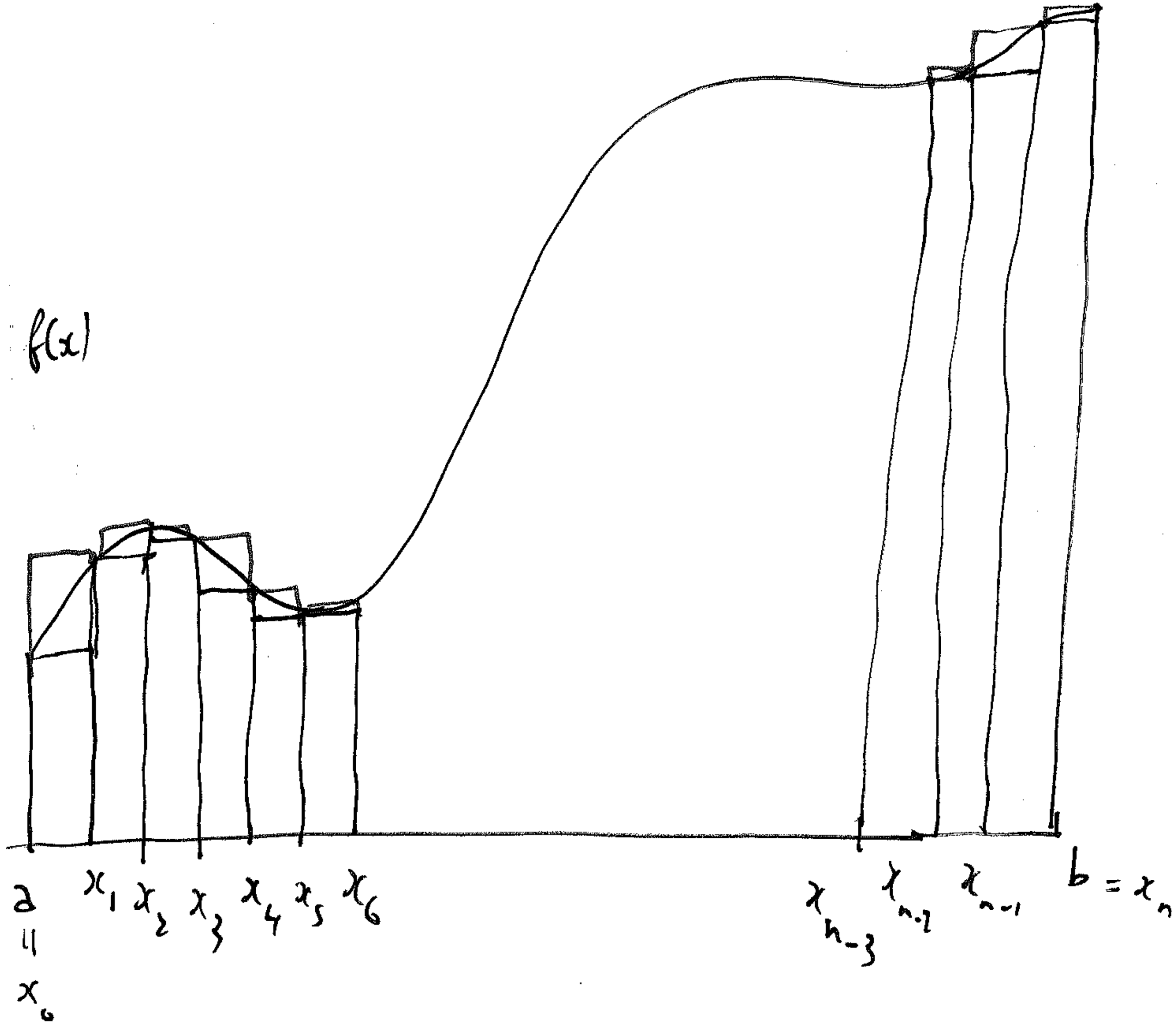
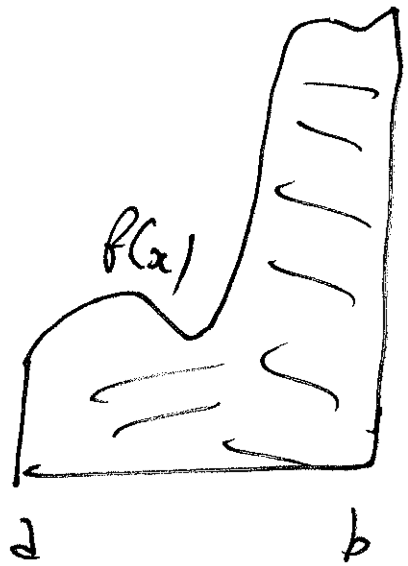
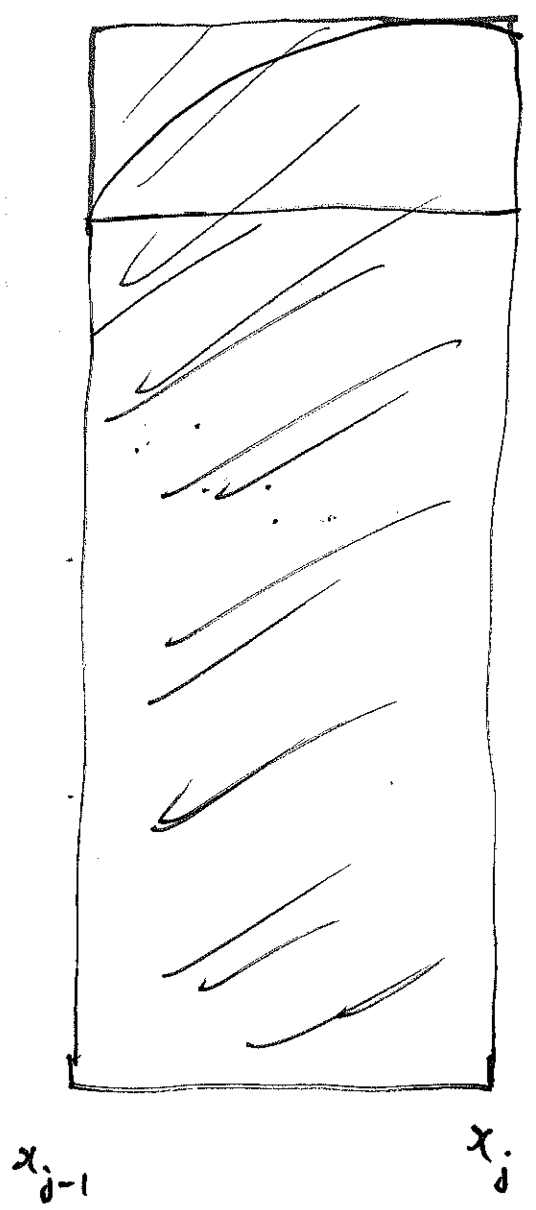


f(x)
a = x0
x1 x2 x3 x4 x5 x6
xn-3 xn-2 xn-1
b = xn

f(x)
a
b

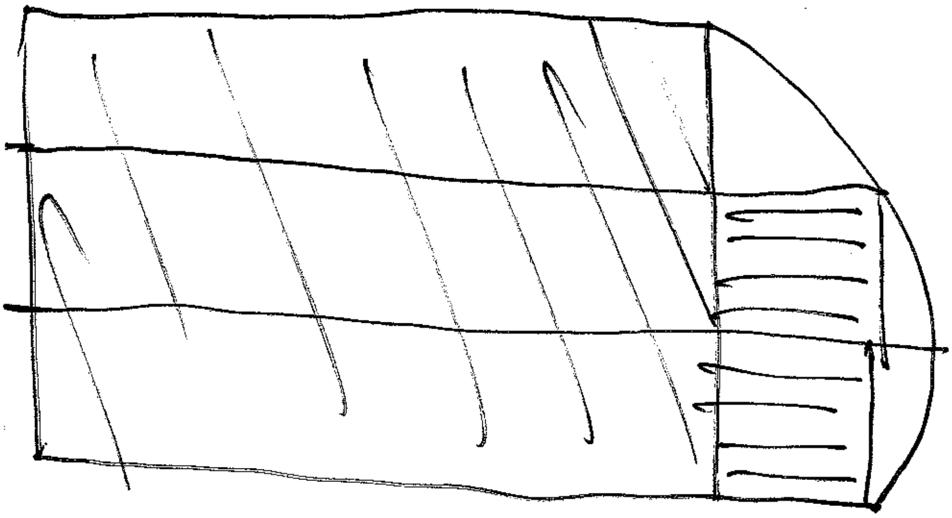


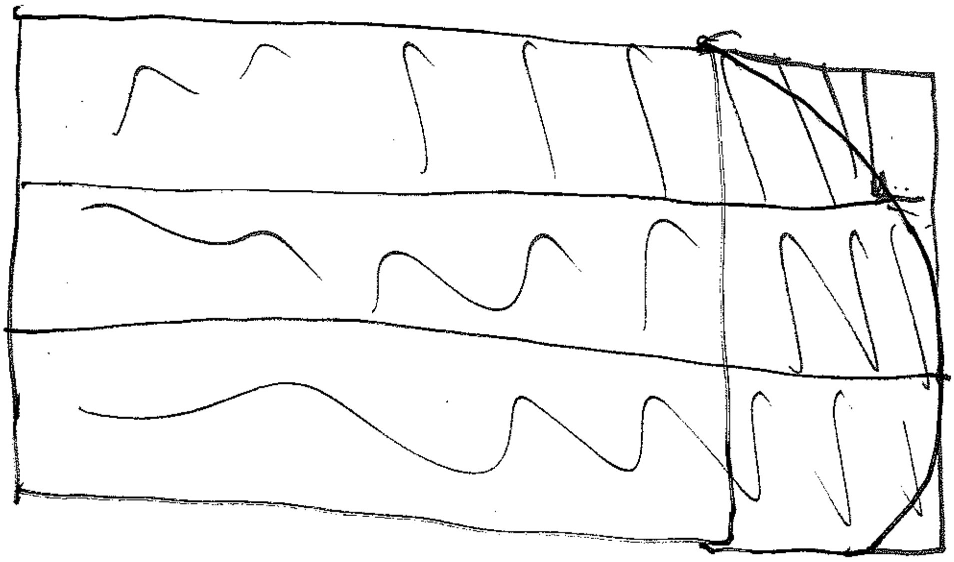

$$m_j = \min\{f(x) : x_{j-1} \le x \le x_j\}$$

$$M_j = \max\{f(x) : x_{j-1} \le x \le x_j\}$$

$$m_j(x_j - x_{j-1}) \le \text{Area}_j \le M_j(x_j - x_{j-1})$$

$$\underbrace{\sum_{j=1}^{n} m_j(x_j - x_{j-1})}_{\text{somma di Riemann inferiore}} \le \text{Area} \le \underbrace{\sum_{j=1}^{n} M_j(x_j - x_{j-1})}_{\text{somma di Riemann superiore}}$$





$$\underline{P} = \{ a = x_0 < x_1 < \dots < x_{n-1} < x_n = b \} \quad \text{partizione}$$

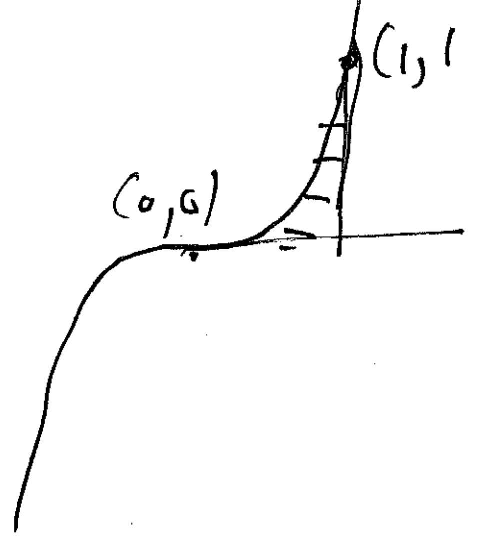
$\hookrightarrow$ $s(\underline{P})$ = somma inferiore

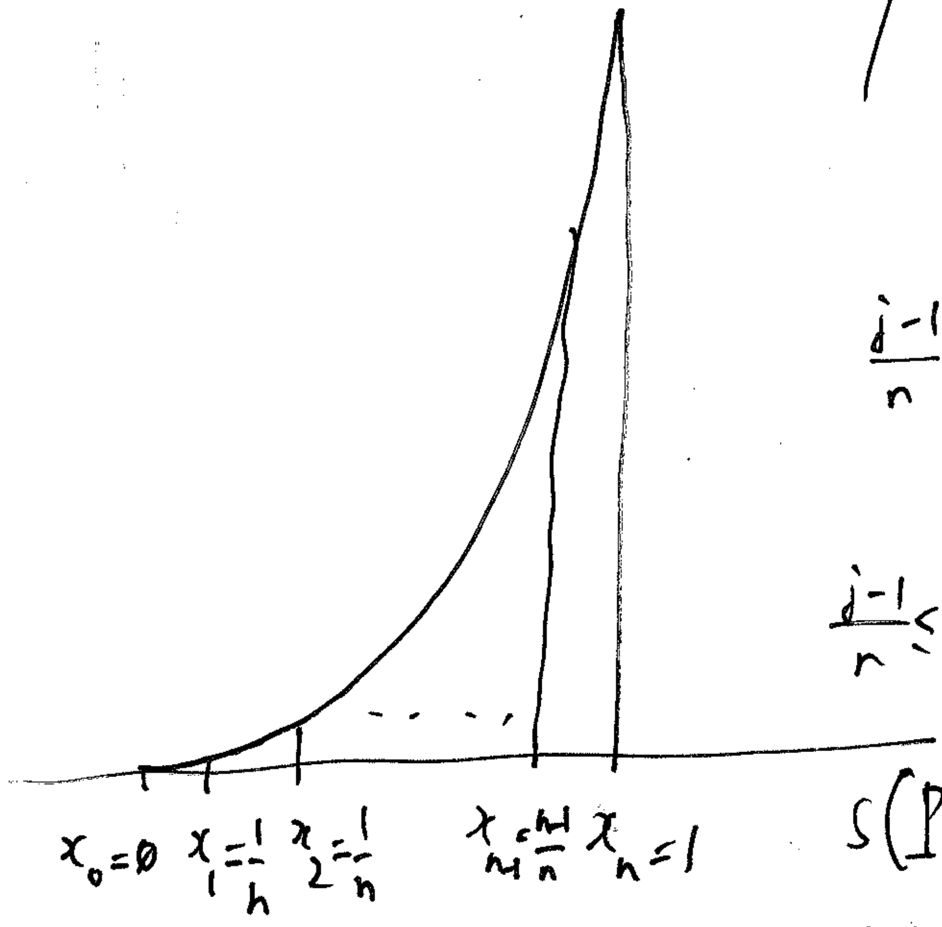
$S(\underline{P})$ = somma superiore

$$r(\underline{P}) = \max_j (x_j - x_{j-1})$$

$$\lim_{r(\underline{P}) \to 0} s(\underline{P}) = \underbrace{\int_a^b f(x)\, dx}_{\text{integrale di Riemann}} = \lim_{r(\underline{P}) \to 0} S(\underline{P})$$

$$f(x) = x^3 \qquad \int_0^1 x^3\,dx$$




$$\min f(x) = f\left(\frac{j-1}{n}\right) = \left(\frac{j-1}{n}\right)^3$$

$$\frac{j-1}{n} \le x \le \frac{j}{n}$$

$$\max f(x) = f\left(\frac{j}{n}\right) = \left(\frac{j}{n}\right)^3$$

$$\frac{j-1}{n} \le x \le \frac{j}{n}$$

$$s(P) = \sum_{j=1}^{n} \left(\frac{j-1}{n}\right)^3 \frac{1}{n}$$

$$S(P) = \sum_{j=1}^{n} \left(\frac{j}{n}\right)^3 \frac{1}{n}$$

$$\frac{1}{n^4}\sum_{j=1}^{n}(j-1)^3 \leq \int_0^1 x^3\,dx \leq \frac{1}{n^4}\sum_{j=1}^{n} j^3$$

| $k$ | $k^3$ | | |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 2 | 8 | 9 | 3 |
| 3 | 27 | 36 | 6 |
| 4 | 64 | 100 | 10 |
| 5 | 125 | 225 | 15 |
| 6 | 216 | 441 | 21 |
| 7 | 343 | 784 | 28 |
| 8 | 512 | 1296 | 36 |

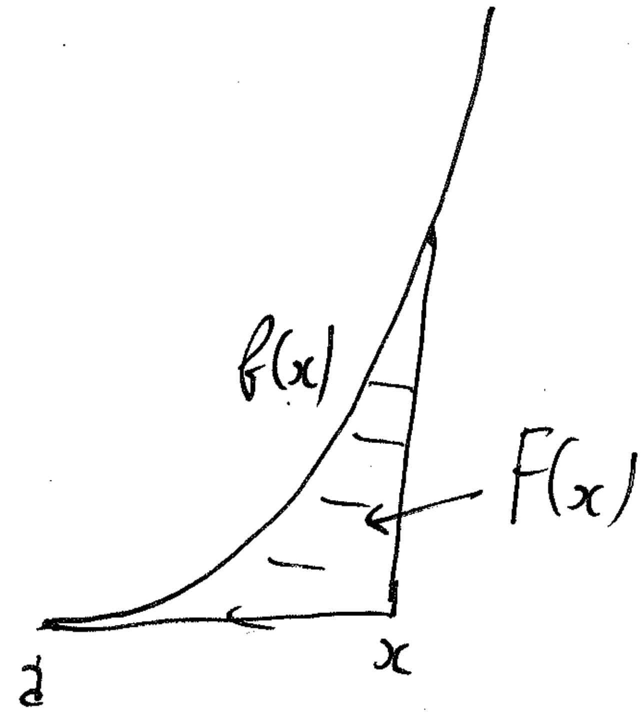
$$\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2 = \left[\frac{1}{2}n(n+1)\right]^2$$

$$\frac{1}{n^4}\left[\frac{1}{2}(n-1)n\right]^2 \leq \int_0^1 x^3\,dx \leq \frac{1}{n^4}\left[\frac{1}{2}n(n+1)\right]^2$$

$$\frac{(n-1)^2 n^2}{4n^4} \leq \int_0^1 x^3\,dx \leq \frac{n^2(n+1)^2}{4n^4}$$

$$\frac{1}{4}\left(\frac{n-1}{n}\right)^2 \qquad \frac{1}{4} \qquad \frac{1}{4}\left(\frac{n+1}{n}\right)^2$$

$$\left.\begin{aligned} F'(x) &= f(x) \\ F(a) &= 0 \end{aligned}\right\}$$



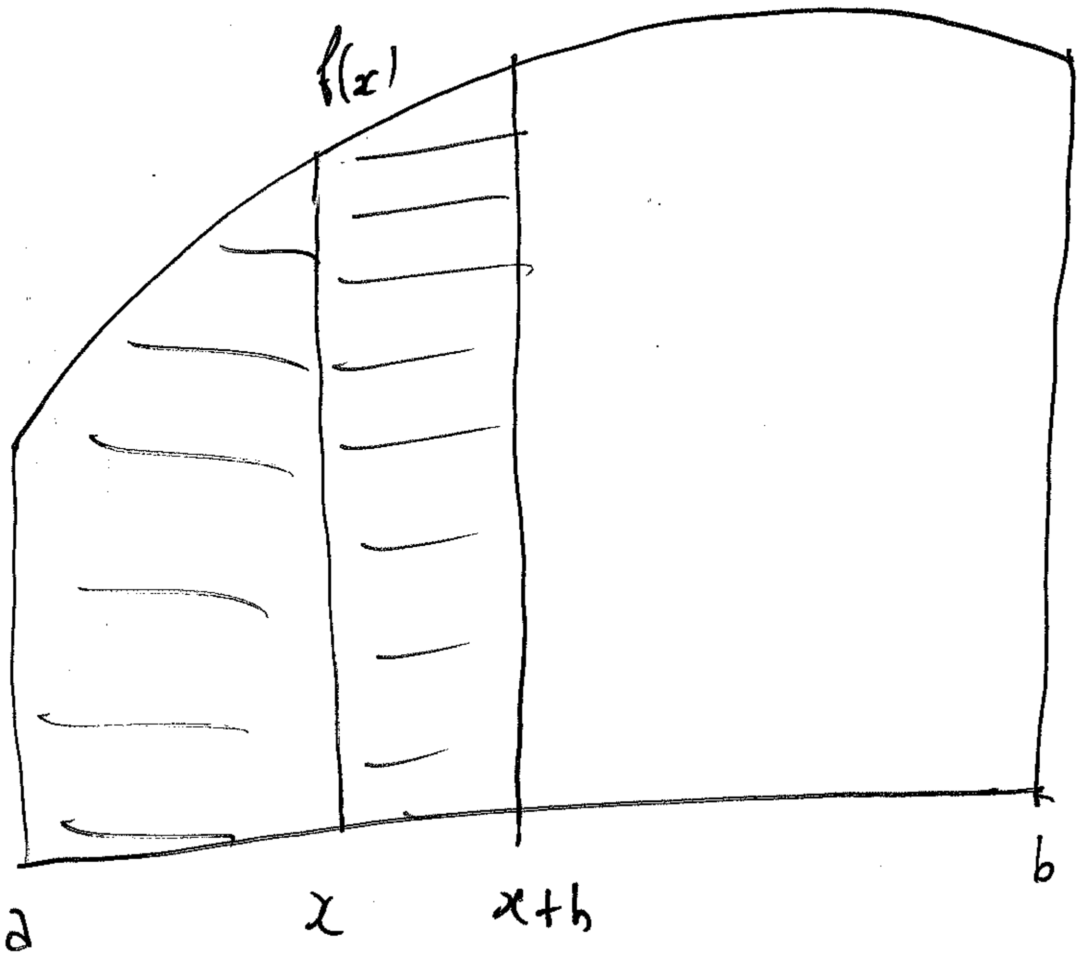

$$F'(x) = x^3$$

$$F(0) = 0 \qquad F(x) = \frac{1}{4}x^4$$

Calcolare $F(1)$

$$\Downarrow$$

$$F(1) = \frac{1}{4}$$




$$F(x)=\int_a^x f(t)\,dt$$

$$F(x+h)=\int_a^{x+h} f(t)\,dt$$

$$F(x+h)-F(x)=\int_x^{x+h} f(t)\,dt$$

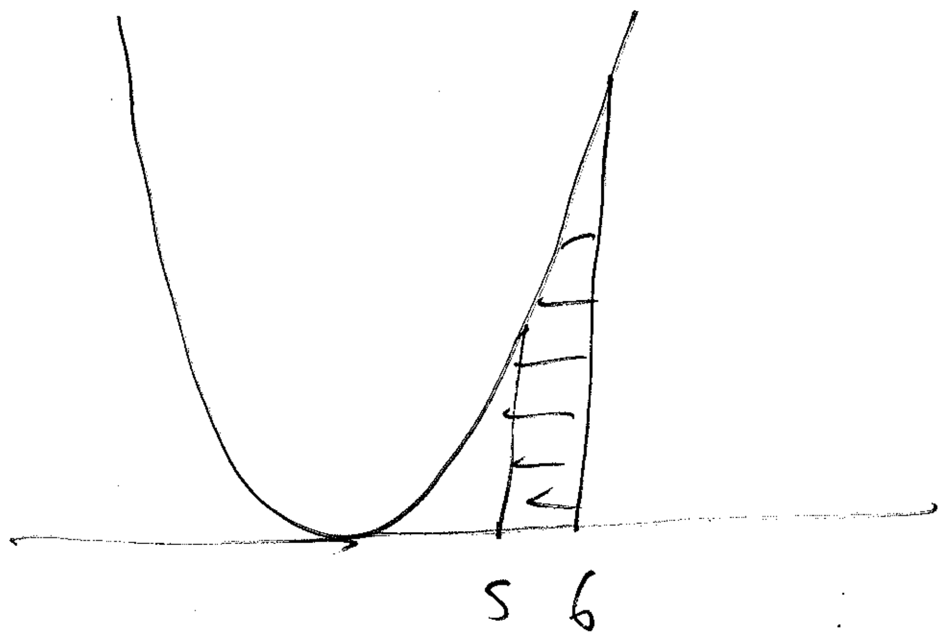
$$h\min\{f(t): x\le t\le x+h\}\le F(x+h)-F(x)\le h\max\{f(t): x\le t\le x+h\}$$

$$\min\{f(t): x \le t \le x+h\} \le \frac{F(x+h)-F(x)}{h} \le \max\{f(t): x \le t \le x+h\}$$

$$f(x) \le F'(x) \le f(x)$$

$$\boxed{\begin{array}{l} F'(x) = f(x) \\ F(a) = \varnothing \end{array}}$$

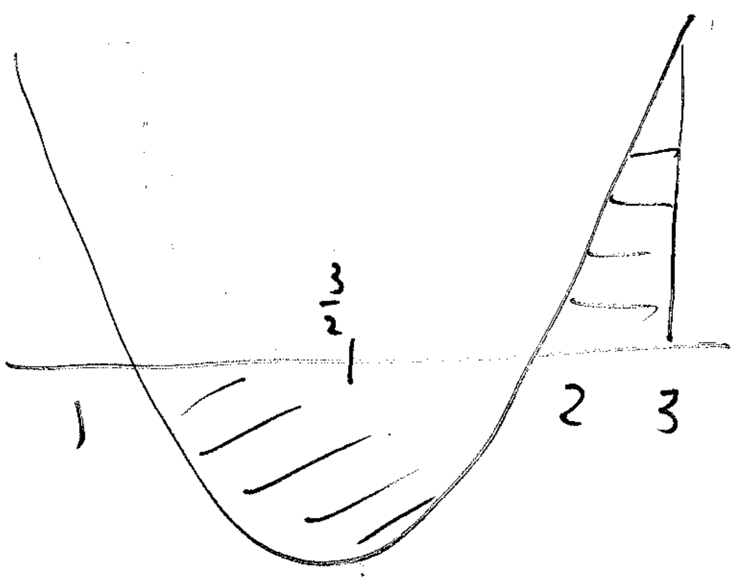
$$\int_5^6 x^2\,dx$$




$$= \left[\frac{1}{3}x^3\right]_5^6$$

$$= \frac{1}{3}(6^3 - 5^3) = \frac{1}{3}(216 - 125) = \frac{1}{3}\,91 = \frac{91}{3}$$

$$f(x) = x^2 - 3x + 2 = (x-1)(x-2)$$




$$\int_1^2 f(x)\,dx = \left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x\right]_1^2$$

$$= \left(\frac{8}{3} - 6 + 4\right) - \left(\frac{1}{3} - \frac{3}{2} + 2\right)$$

$$= \frac{7}{3} - \frac{9}{2} + 2 = \frac{14 - 27 + 12}{6} = -\frac{1}{6}$$

$$\int_1^3 f(x)\,dx = \left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x\right]_1^3 = \left(9 - \frac{27}{2} + 6\right) - \left(\frac{1}{3} - \frac{3}{2} + 2\right) = \frac{3}{2} - \frac{5}{6} = \frac{2}{3}$$

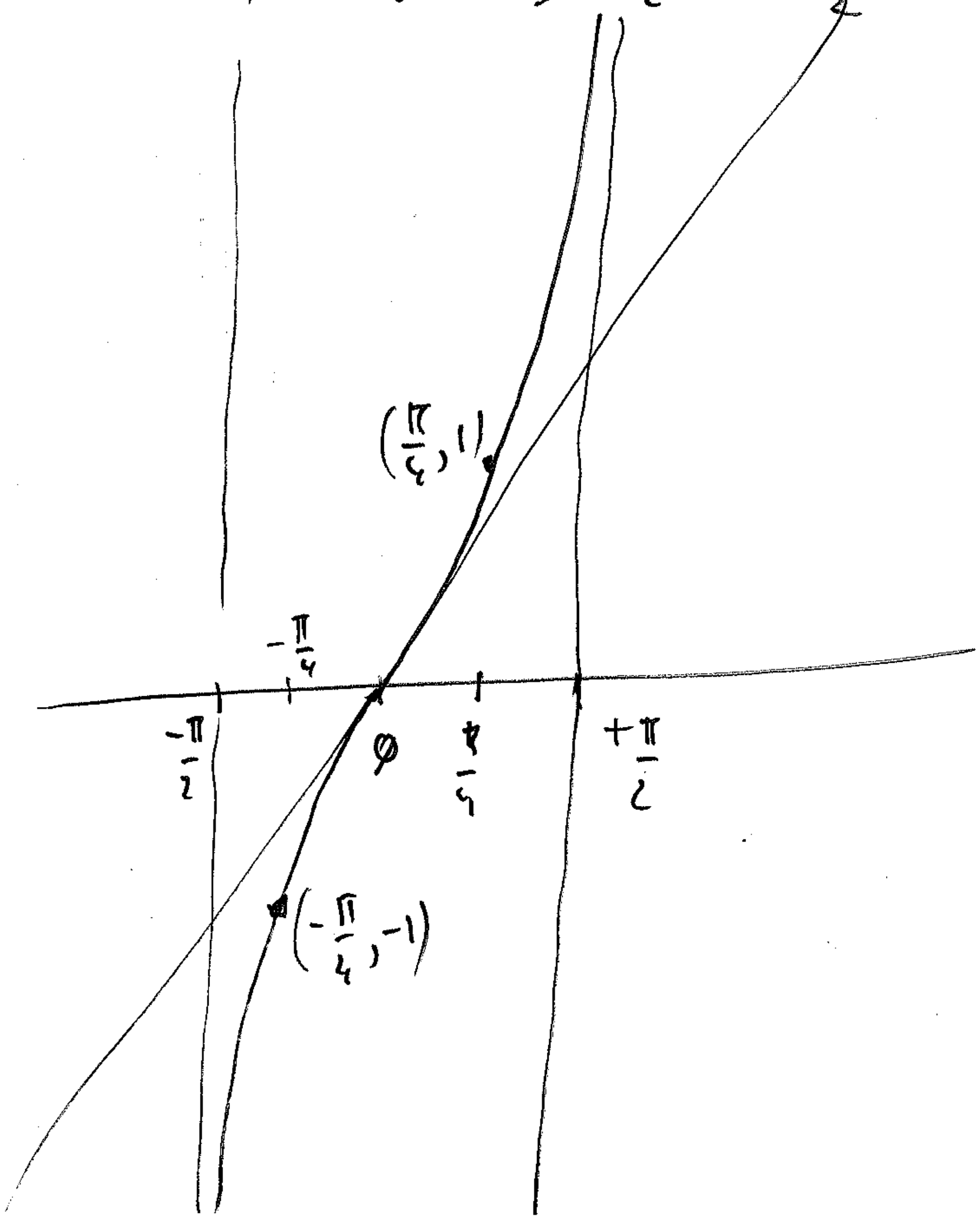
| $f(x)$ | $F(x)$ |
|---|---|
| $1$ | $x$ |
| $x$ | $\frac{1}{2}x^2$ |
| $x^2$ | $\frac{1}{3}x^3$ |
| $x^n$, $n \neq -1$ | $\frac{x^{n+1}}{n+1}$ |
| $\frac{1}{x}$ | $\ln\|x\|$ |
| $e^x$ | $e^x$ |

<u>$x<0$</u> $\frac{d}{dx}\ln(-x) = \frac{1}{-x}\frac{d}{dx}(-x) = \frac{1}{x}$

| $f(x)$ | $F(x)$ |
|---|---|
| $\cos x$ | $\operatorname{sen} x$ |
| $\operatorname{sen} x$ | $-\cos x$ |
| $\frac{1}{\cos^2 x}$ | $\operatorname{tg} x$ |
| $e^{-x^2}$ | (?) |

$$\frac{1}{x^2+1}$$

$f(x) = tg\ x \ , \ -\frac{\pi}{2} < x < \frac{\pi}{2}$




$f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R} \ , \ f(x) = tg\ x$

$$f'(x) = \frac{1}{\cos^2(x)}$$

$$f'(x) = \frac{\cos^2(x) + sen^2(x)}{\cos^2(x)}$$

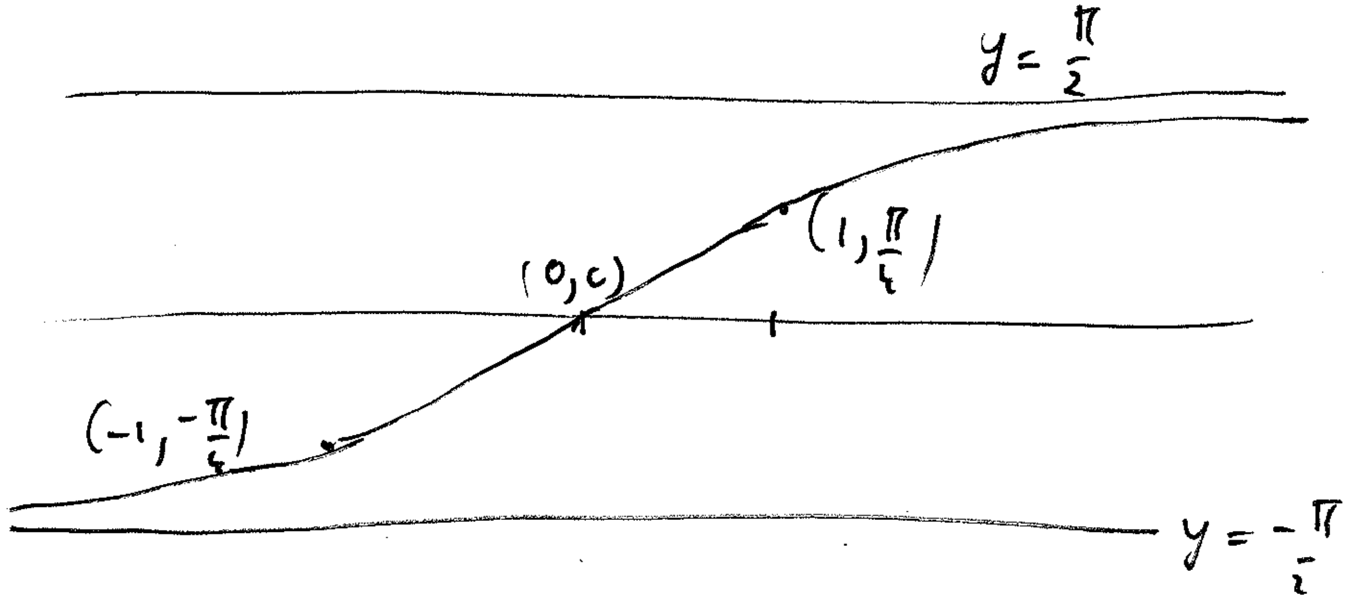
$$= 1 + tg^2(x) = 1 + f(x)^2$$

$g : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$g(\operatorname{tg} x) = x$

$\parallel$

$g(f(x))$




$g(t) = \operatorname{arc\,tg}(t)$

$x = \operatorname{arctg} t$

$\Updownarrow$

$\operatorname{tg} x = t$

$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$g'(f(x))\, f'(x) = 1$

$g'(f(x))\left[1 + f(x)^2\right] = 1 \quad \to \quad g'(f(x)) = \dfrac{1}{1 + f(x)^2}$

$t = f(x) \in \mathbb{R}$

$$g'(t) = \frac{1}{1+t^2}$$

$$\int_0^1 \frac{dx}{1+x^2} = \left[ \operatorname{arctg} x \right]_0^1$$

$$= \operatorname{arctg} 1 - \operatorname{arctg} \emptyset$$

$$= \frac{\pi}{4} - \emptyset = \frac{\pi}{4}$$