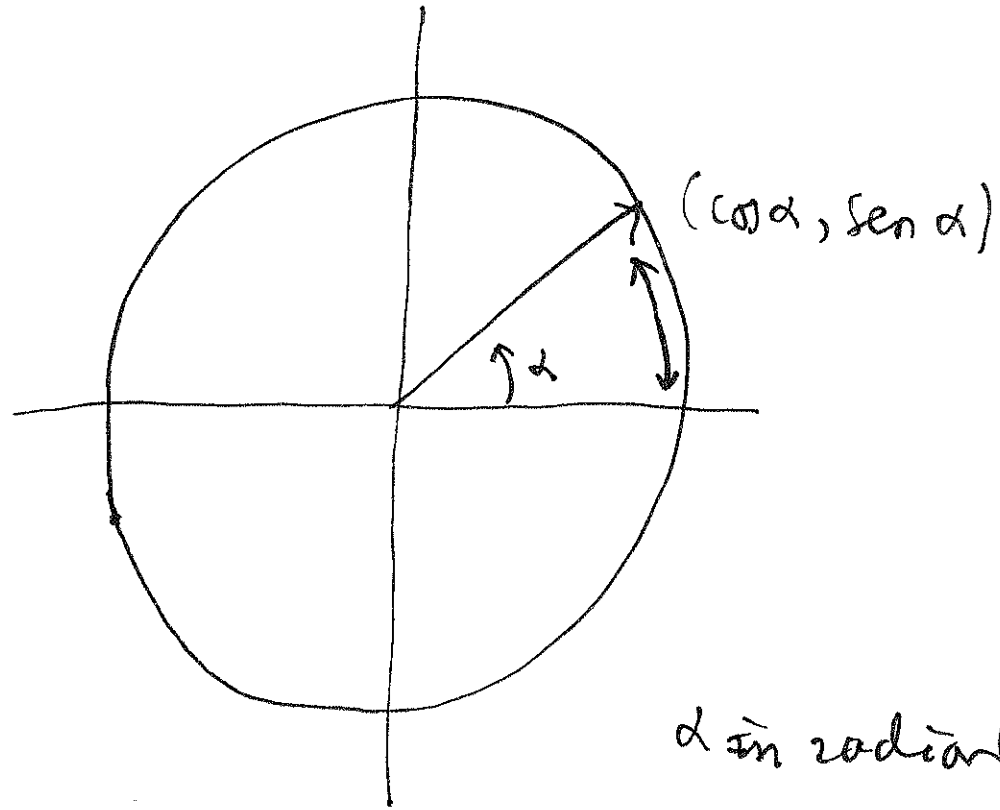


$$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} \qquad \operatorname{cotg}\alpha = \frac{\cos\alpha}{\operatorname{sen}\alpha}$$

NON definita se $\alpha = \frac{\pi}{2}, \frac{3\pi}{2}$

NON definita se $\alpha = \varnothing, \pi$

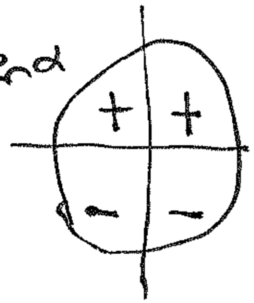
α in radianti $\Longleftrightarrow$ lunghezza dell'arco $= \alpha$

30°	$\pi/6$	180°	π
45°	$\pi/4$	360°	2π
60°	$\pi/3$	270°	$3\pi/2$
90°	$\pi/2$		

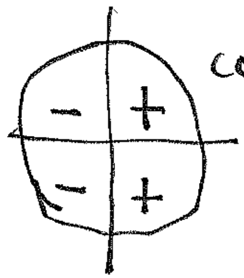
$$x^2 + y^2 = 1$$

$$\cos^2\alpha + \operatorname{sen}^2\alpha = 1$$

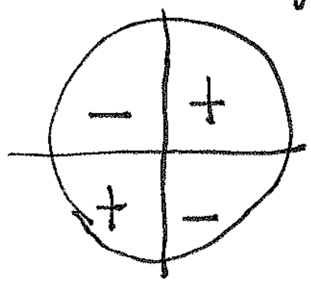
sen α

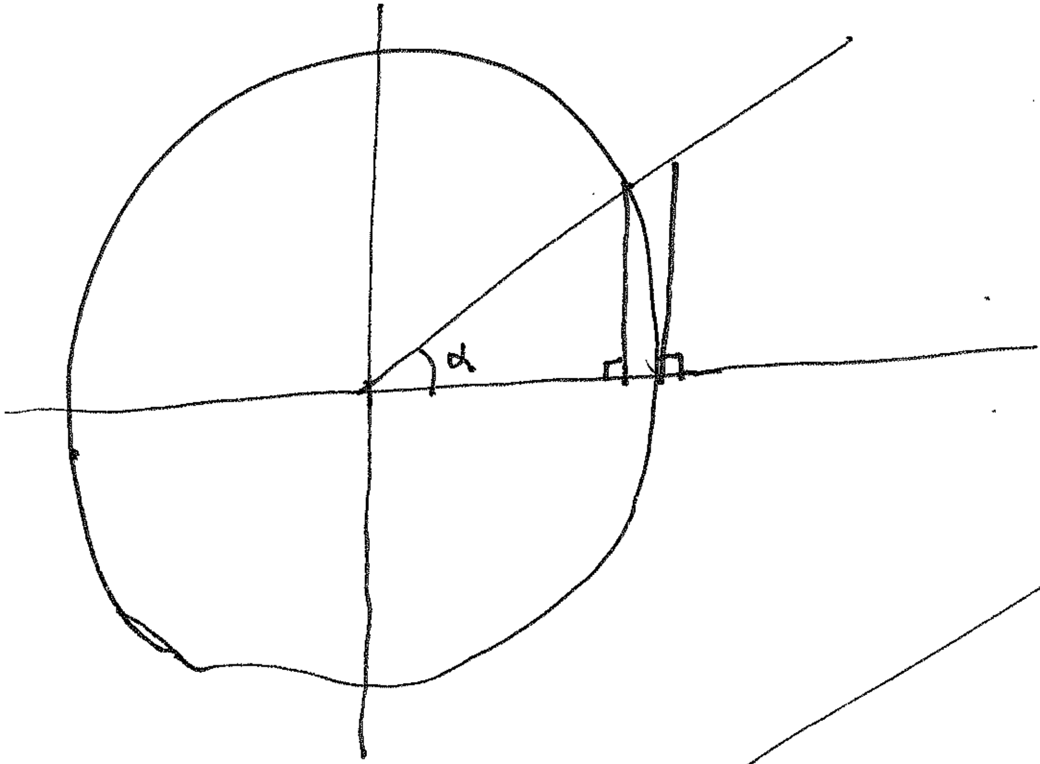


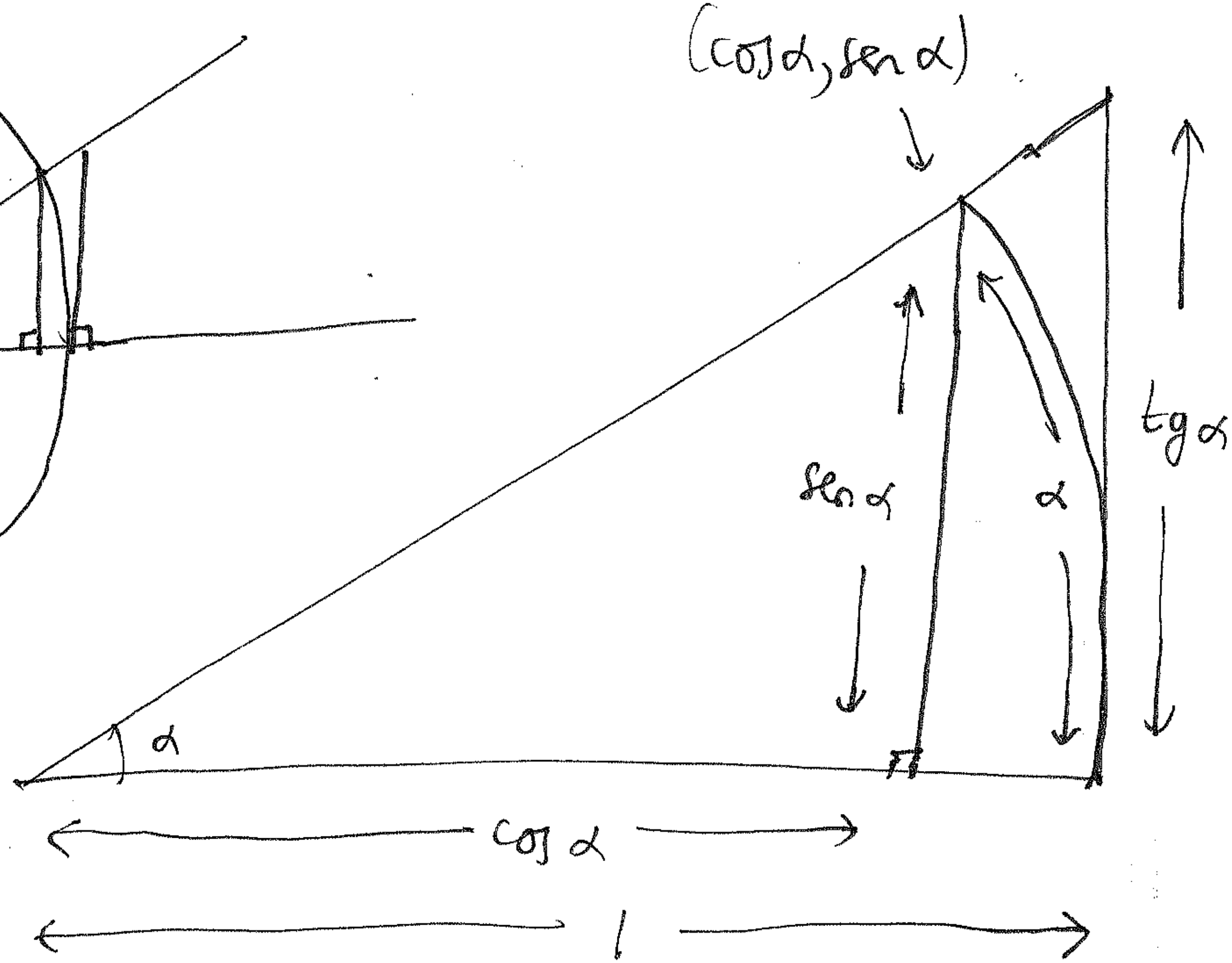
cos α



tgα/cotgα







$$\frac{\operatorname{tg}\alpha}{\operatorname{sen}\alpha} = \frac{1}{\cos\alpha}$$

$$\emptyset < \operatorname{sen}\alpha < \alpha < \operatorname{tg}\alpha$$

$$0 < \operatorname{sen}\alpha < \alpha < \operatorname{tg}\alpha$$

$$\left.\begin{array}{l} 0 < 1 < \dfrac{\alpha}{\operatorname{sen}\alpha} < \dfrac{1}{\cos\alpha} \\[2em] 0 < \cos\alpha < \dfrac{\alpha}{\operatorname{tg}\alpha} < 1 \end{array}\right\} \qquad \begin{array}{l} \lim\limits_{\alpha \to \emptyset} \dfrac{\operatorname{sen}\alpha}{\alpha} = 1 \\[2em] \lim\limits_{\alpha \to \emptyset} \dfrac{\operatorname{tg}\alpha}{\alpha} = 1 \end{array}$$

$$0 < \underset{\substack{\downarrow \\ 1}}{\cos\alpha} < \frac{\alpha}{\operatorname{tg}\alpha} < \underset{\substack{\downarrow \\ 1}}{1} < \frac{\alpha}{\operatorname{sen}\alpha} < \underset{\substack{\downarrow \\ 1}}{\frac{1}{\cos\alpha}}$$

$$\frac{\pi}{180^\circ} \rightarrow 1 \text{ grado}$$

$$\hookrightarrow \simeq 0{,}01745$$

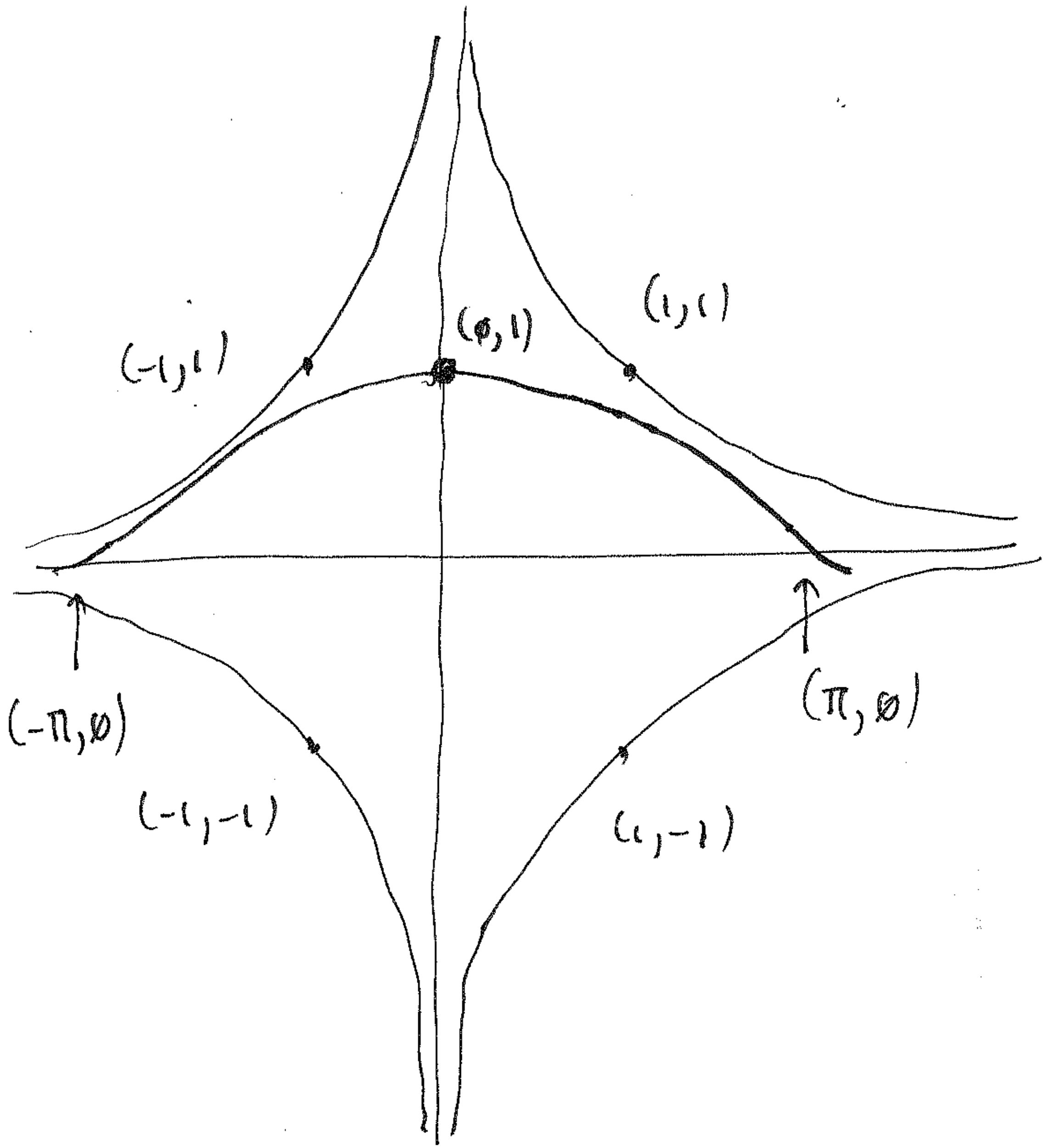
$$0 < \cos\alpha < \frac{\operatorname{sen}\alpha}{\alpha} < 1 < \frac{\operatorname{tg}\alpha}{\alpha} < \frac{1}{\cos\alpha}$$

$$\cos\alpha \to 1 \qquad 1 \to 1 \qquad \frac{1}{\cos\alpha} \to 1$$

$$\frac{\operatorname{sen}\alpha}{\alpha} \to 1 \qquad \frac{\operatorname{tg}\alpha}{\alpha} \to 1$$

$f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} \dfrac{\operatorname{sen} x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$



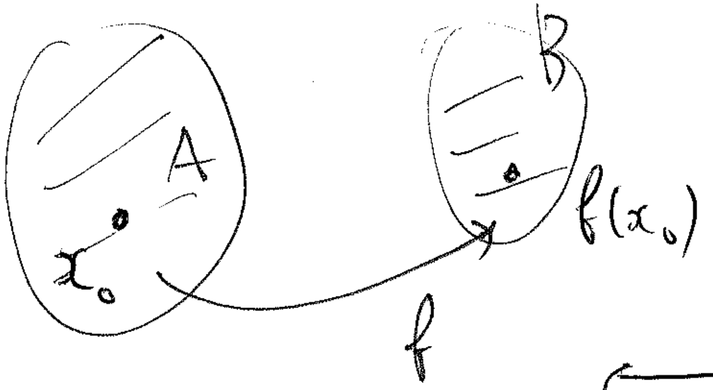
$$-\frac{1}{x} \leq \frac{\operatorname{sen} x}{x} \leq \frac{1}{x}$$

per $x > 0$

$$1 \text{ rad} = \frac{180}{3,14\ldots} \text{ gradi} = 57^\circ 17' 44,92''$$

$f : A \to B$

~~f si dice~~



$$\lim_{x \to x_0} f(x) = y_0$$

$\Longleftrightarrow$ Per ogni successione (x_n) in A, $x_n \neq x_0$, ~~abbiamo~~ per cui $\lim_{n\to\infty} x_n = x_0$, abbiamo

$$\lim_{n \to \infty} f(x_n) = y_n$$

$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1 \Longleftrightarrow$ se $(x_n)_{n=1}^{\infty}$ è una successione con limite 0, $x_n \neq 0$,

allora $\lim_{n \to \infty} \frac{\operatorname{sen} x_n}{x_n} = 1$

$\lim_{x \to x_0} f(x) = y_0 \iff$ Per ogni successione x_n in A di punti $x_n \neq x_0$ che converge ad x_0, abbiamo

$$\lim_{n \to \infty} f(x_n) = y_0$$

f è continua in $x_0 \iff \lim_{x \to x_0} f(x) = f(x_0)$

$f : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = x^2$

Se $\lim_{n \to \infty} x_n = x_0$, allora $\lim_{n \to \infty} (x_n)^2 = (x_0)^2$

$\Longleftrightarrow \lim_{n \to \infty} f(x_n) = f(x_0) \Longleftrightarrow$ f è continua in x_0

f è continua $\Longleftrightarrow$ f è continua in tutti i punti del suo dominio

$f: \mathbb{R} \to \mathbb{R}$ $\qquad \lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$

$$f(x) = \begin{cases} \frac{\operatorname{sen} x}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

f è discontinua in 0

$f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^3$

Se $\lim_{n \to \infty} x_n = x_0$, allora $\lim_{n \to \infty} x_n^3 = x_0^3$

$\cancel{x^{p}}$

$f: \mathbb{R}^+ \to \mathbb{R}^+$, $f(x) = \sqrt{x}$

Se $\lim_{n \to \infty} x_n = x_0$ e $x_n > 0$, allora $\lim_{n \to \infty} \sqrt{x_n} = \sqrt{x_0}$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$

$$\lim_{x\to 0}(1+x)^{1/x} = e$$

$$\Updownarrow$$

se $\lim_{n\to\infty} x_n = 0$, $x_n \neq 0$, allora

$$\lim_{n\to\infty}(1+x_n)^{1/x_n} = e$$

$f : \mathbb{R} \to \mathbb{R}$, $f(x) = \operatorname{sen} x$

$f(x) = \cos x$

$a > 0$ $\quad f(x) = a^x$

$g > 0$, $g \neq 1$ $\quad f(x) = \log_g(x)$

$\lim\limits_{x \to +\infty} f(x) = y_0 \iff$ per ogni successione $(x_n)_{n=1}^{\infty}$ per cui $\lim\limits_{n \to \infty} x_n = +\infty$, abbiamo

$$\lim_{n \to \infty} f(x_n) = y_0$$

$\lim\limits_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x = e \iff$ per ogni successione $(x_n)_{n=1}^{\infty}$ con limite $+\infty$ abbiamo

$$\lim_{n \to \infty} \left(1 + \frac{1}{x_n}\right)^{x_n} = e$$

$$x_n = n^2 - 1$$

$$\lim_{n \to \infty} \left(\frac{n^2}{n^2 - 1}\right)^{n^2 - 1} = \lim_{n \to \infty} \left(1 + \frac{1}{n^2 - 1}\right)^{n^2 - 1} = e$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x^2 \cdot (1+\cos x)}$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)} = \lim_{x\to 0} \frac{\operatorname{sen}^2 x}{x^2(1+\cos x)} = \lim_{x\to 0} \left(\frac{\operatorname{sen} x}{x}\right)^2 \frac{1}{1+\cos x}$$

$$= (1)^2 \frac{1}{1+1} = \frac{1}{2}$$

$$\cos 1^\circ \simeq 1 - \frac{(0{,}01745)^2}{2}$$

$$\lim_{x\to 0} \cos x = \cos 0 = 1$$

$$\operatorname{sen}^2 x + \cos^2 x = 1$$

continuità del coseno

$$\cos 2\alpha = 1 - 2\,\text{sen}^2\alpha \qquad \cos x = 1 - 2\,\text{sen}^2\frac{x}{2}$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\frac{2\,\text{sen}^2\frac{x}{2}}{x^2} = \lim_{x\to 0}\left(\frac{\text{sen}\frac{x}{2}}{x/2}\right)^2\frac{1}{2} = (1)^2\frac{1}{2} = \frac{1}{2}$$

Se $\lim_{n\to\infty} x_n = 0$, $x_n \neq 0$, allora $\lim_{n\to\infty}\frac{x_n}{2} = 0$, $\frac{x_n}{2} \neq 0$

$$\lim_{n\to\infty}\frac{\text{sen}\frac{x_n}{2}}{x_n/2} = 1$$

$$\lim_{x\to+\infty} a^x = \begin{cases} +\infty, & a>1 \\ 1, & a=1 \\ 0, & 0<a<1 \end{cases}$$

$\underline{a>0}$

$$\lim_{n\to\infty} a^n = \begin{cases} +\infty, & a>1 \\ 1, & a=1 \\ 0, & 0<a<1 \end{cases} \qquad \lim_{x\to+\infty}\log_{10} x = +\infty$$

$$\lim_{x \to \emptyset} \frac{x}{|x|} \text{ NON esiste}$$

$$\frac{x}{|x|} = \begin{cases} 1, & x > 0 \\ -1, & x < \emptyset \end{cases}$$

$$x_n = \frac{1}{n} \longrightarrow \emptyset, \quad \frac{x_n}{|x_n|} = 1 \to 1$$

$$x_n = -\frac{1}{n} \to \emptyset, \quad \frac{\cancel{x_n}}{|x_n|} = -1 \to -1$$

$$\lim_{x \to \emptyset^+} \frac{x}{|x|} = 1$$

$$\lim_{x \to \emptyset^-} \frac{x}{|x|} = -1$$

$$\lim_{x \to x_0^+} f(x) = y_0$$

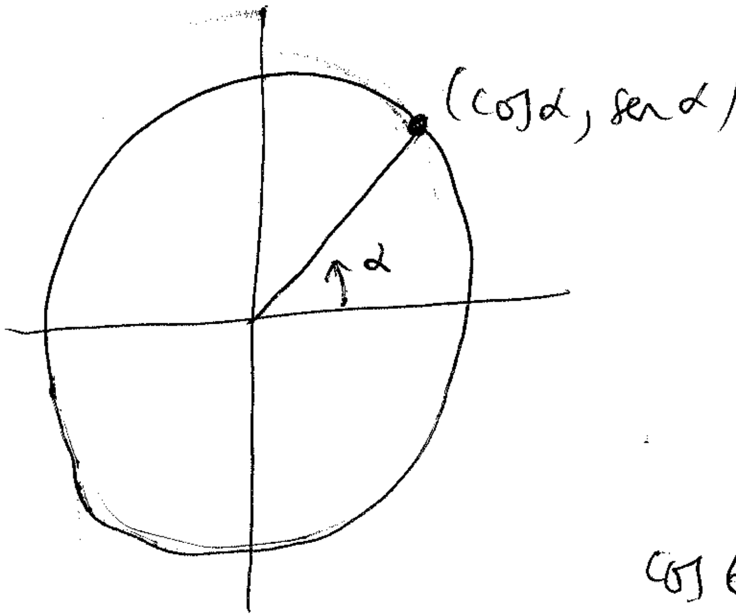
Se $\lim_{n \to \infty} x_n = x_0$ (mentre $x_n > x_0$), allora $\lim_{n \to \infty} f(x_n) = y_0$

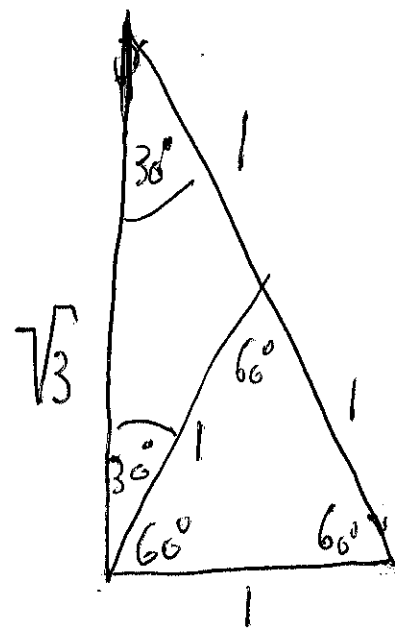
$$\lim_{x \to x_0^-} f(x) = y_0$$

$$\lim_{x \to 0} \operatorname{sen}\left(\frac{1}{x}\right)$$

$$x_n = \frac{1}{\frac{\pi}{3} + 2\pi n}$$

$$\operatorname{sen} \frac{1}{x_n} = \frac{1}{2}\sqrt{3}$$





$$\frac{1}{x_n} = \frac{\pi}{6} + 2\pi n$$

$$x_n = \frac{1}{\frac{\pi}{6} + 2\pi n}$$

$$\operatorname{sen} \frac{1}{x_n} = \operatorname{sen} \frac{\pi}{6} = \frac{1}{2}$$

$$\cos 60^\circ$$

$$\shortparallel$$

$$\operatorname{sen} 30^\circ = \operatorname{sen} \frac{\pi}{6} = \frac{1}{2}$$

$$\cos(\alpha + 2\pi) = \cos \alpha$$

$$\operatorname{sen}(\alpha + 2\pi) = \operatorname{sen} \alpha$$

$$\operatorname{sen} 60^\circ = \cos 30^\circ = \cos \frac{\pi}{6} = \frac{1}{2}\sqrt{3}$$

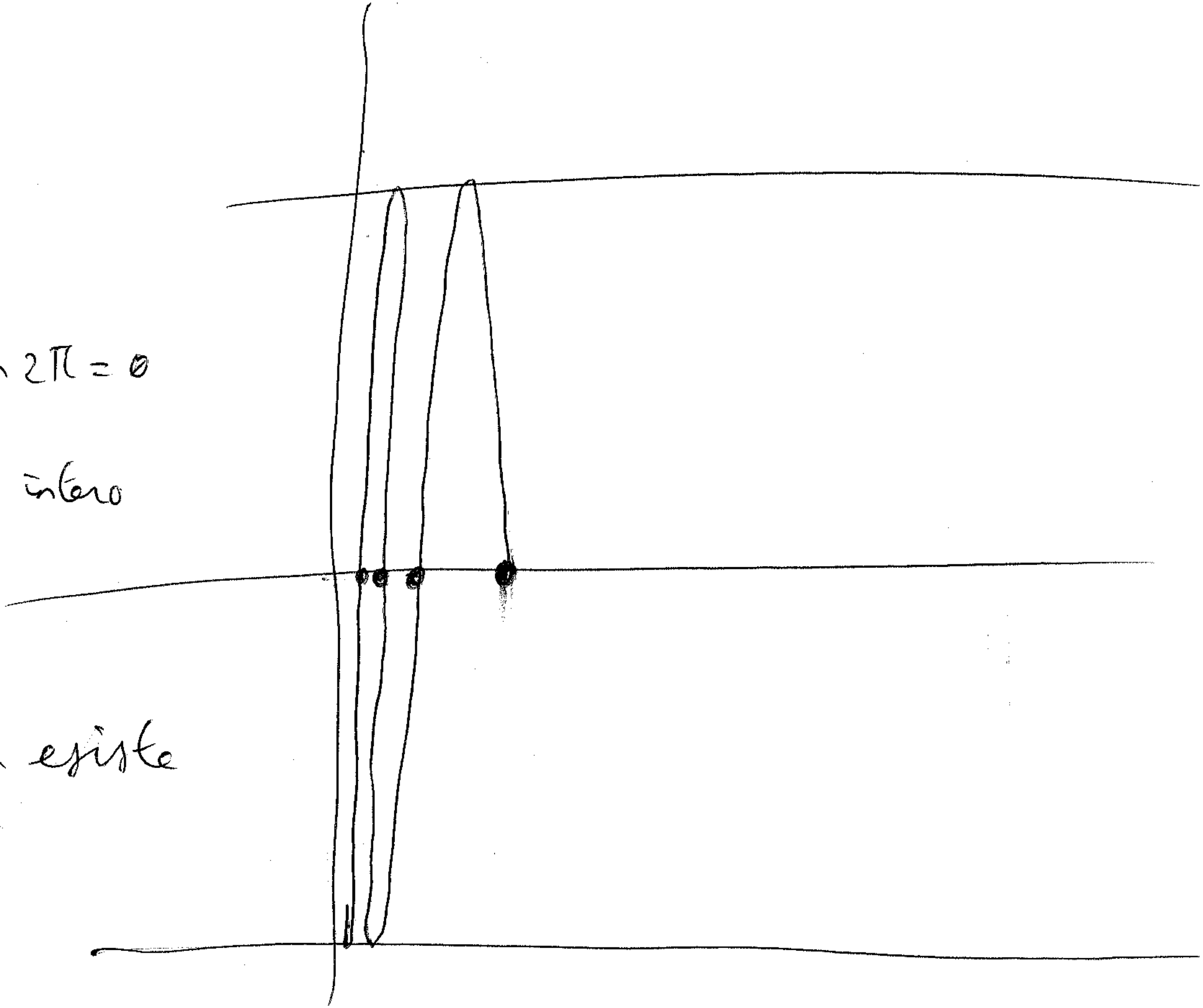
$$\operatorname{sen}\frac{1}{x} = 0$$

$$\Downarrow$$

$$\operatorname{sen} 0 = \operatorname{sen}\pi = \operatorname{sen} 2\pi = 0$$

$\operatorname{sen}(m\pi) = 0$, m intero

$$\lim_{x\to 0} \operatorname{sen}\frac{1}{x} \text{ non esiste}$$



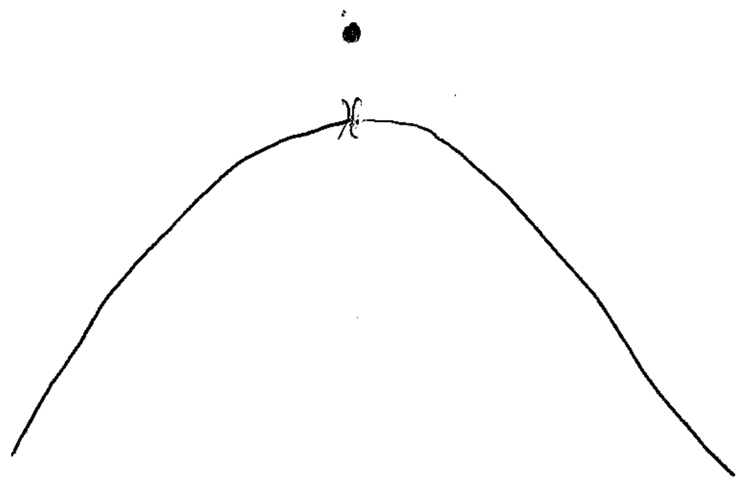
$$f(x) = \begin{cases} \dfrac{\operatorname{sen} x}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$$

f è discontinua in 0

poiché $\lim_{x \to 0} \dfrac{\operatorname{sen} x}{x} = 1 \neq f(0)$

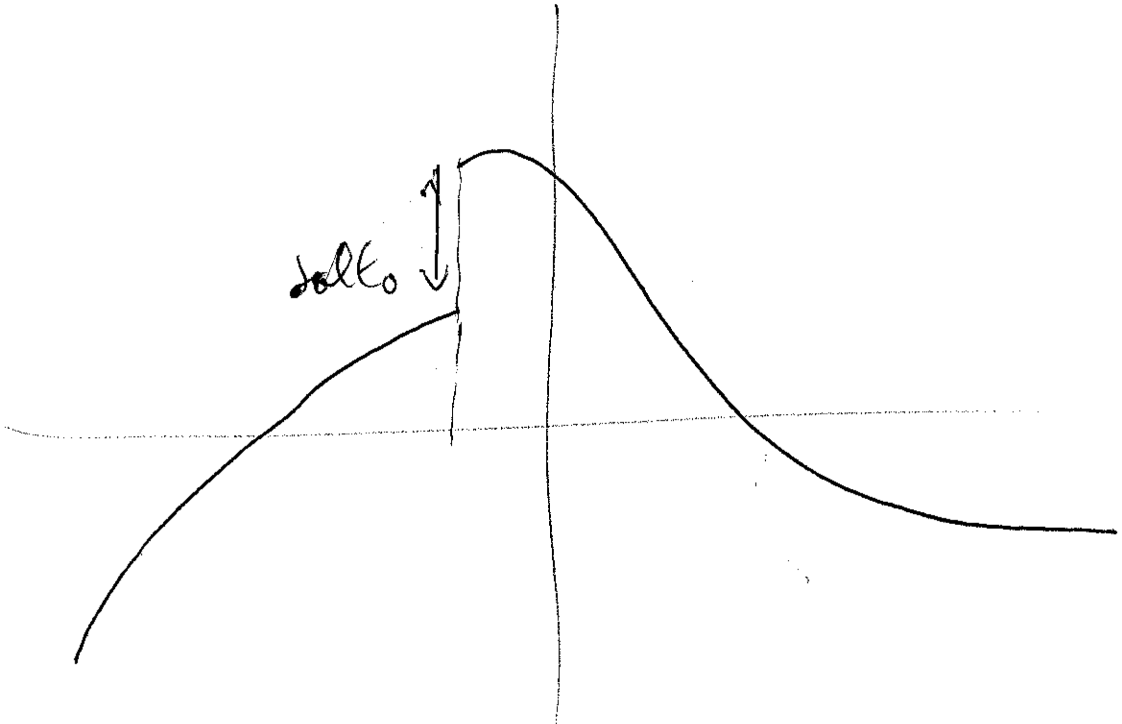
Discontinuità <u>eliminabile</u>

Cambiare la f in: $f(x) = \begin{cases} \dfrac{\operatorname{sen} x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$



$$\lim_{x \to x_0^-} f(x) \neq \lim_{x \to x_0^+} f(x)$$

Quindi non esiste $\lim_{x \to x_0} f(x)$



Discontinuità di prima specie

$$f(x) = \begin{cases} \dfrac{x}{|x|}, & x \neq 0 \\ \text{qualsiasi valore}, & x = 0 \end{cases}$$

$$f(x) = \begin{cases} \operatorname{sen}\left(\frac{1}{x}\right), & x \neq 0 \\ \text{qualsiasi valore}, & x = 0 \end{cases}$$

Discontinuità di seconda specie

$$\lim_{x \to 0} \frac{\operatorname{sen}(2x)}{3x} = \lim_{x \to 0} \frac{\operatorname{sen}(2x)}{2x} \frac{2}{3} = 1 \cdot \frac{2}{3} = \frac{2}{3}$$

Se $\lim\limits_{\substack{n \to \infty \\ (\text{con } x_n \neq 0)}} x_n = 0$ allora $\lim\limits_{\substack{n \to \infty \\ (\text{con } 2x_n \neq 0)}} 2x_n = 0$, $\lim\limits_{x_n \to 0} \frac{\operatorname{sen}(2x_n)}{2x_n} = 1$

$$\lim_{x\to 0}(1+2x)^{1/3x} = \lim_{x\to 0}\left[(1+2x)^{\frac{1}{2x}}\right]^{\frac{2}{3}} = \left[\lim_{x\to 0}(1+2x)^{\frac{1}{2x}}\right]^{\frac{2}{3}} = e^{2/3}$$

$$\left.\begin{aligned} &f:\mathbb{R}^+\to\mathbb{R}^+\\ &f(x)=x^{2/3}\end{aligned}\right\}\ \text{è continua}$$

$$\lim_{x\to 0}(1+\operatorname{sen}x)^{1/x} = \lim_{x\to 0}\Bigg[\underbrace{(1+\operatorname{sen}x)^{\frac{1}{\operatorname{sen}x}}}_{\to e}\Bigg]^{\overbrace{\frac{\operatorname{sen}x}{x}}^{\to 1}} = e^1 = e$$

$$\lim_{x\to 0}\left(\cos x\right)^{\frac{1}{1-\cos x}} = \lim_{x\to 0}\left[\left(1-(1-\cos x)\right)^{-\frac{1}{1-\cos x}}\right]^{-1} = e^{-1} = \frac{1}{e}$$

$$\lim_{x\to 0}\cos x = \cos \emptyset = 1 \qquad \lim_{x\to \emptyset}\left\{-(1-\cos x)\right\} = \emptyset$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}\left(\frac{x}{\operatorname{sen} x}\right)^2 = \frac{1}{2}(1)^2 = \frac{1}{2}$$

$$\lim_{x\to 0}\frac{1-\cos x}{\operatorname{sen}^2 x}\times\frac{1+\cos x}{1+\cos x} = \lim_{x\to 0}\frac{1-\cos^2 x}{\operatorname{sen}^2 x\,(1+\cos x)}$$

$$= \lim_{x\to 0}\frac{1}{1+\cos x} = \frac{1}{1+\cos \emptyset} = \frac{1}{1+1} = \frac{1}{2}$$