Secondo Parziale

Mercoledì ~~Martedi~~ 9 gennaio 9 – 12

$$F(x) = \int_a^x f(t)\,dt \qquad F'(x) = f(x), \quad F(a) = 0$$

$f(x)$	$F(x)$	$f(x)$	$F(x)$
x^n	$\dfrac{x^{n+1}}{n+1}$	$\dfrac{1}{1+x^2}$	$\operatorname{arctg} x$
$\dfrac{1}{x}$	$\ln\lvert x\rvert$	$\dfrac{1}{\cos^2 x}$	$\operatorname{tg} x$
$\operatorname{sen} x$	$-\cos x$	$\dfrac{-1}{\operatorname{sen}^2 x}$	$\operatorname{cotg} x$
$\cos x$	$\operatorname{sen} x$		
e^x	e^x		

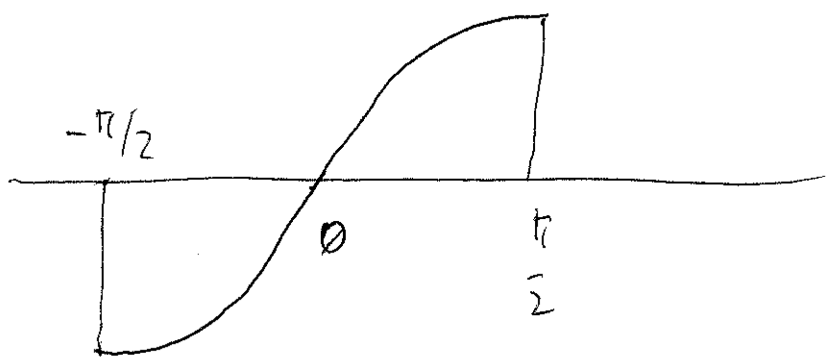
$g(f(x)) = x \qquad g'(f(x))\, \underbrace{f'(x)}_{\cos x} = 1, \quad -\frac{\pi}{2} < x < +\frac{\pi}{2}$

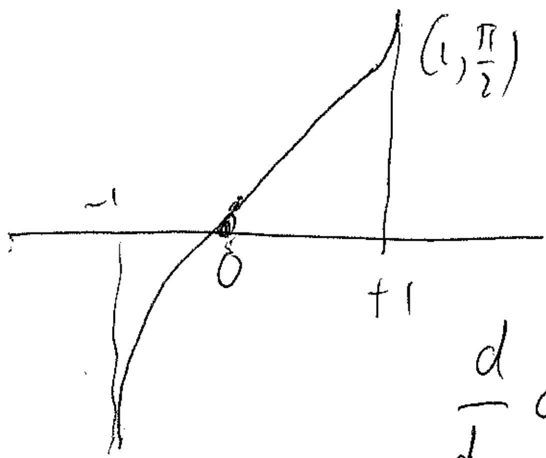
$t = f(x) \in (-1, 1)$

$$g'(f(x)) = \frac{1}{\cos x} = \frac{1}{\sqrt{1-(\operatorname{sen} x)^2}} = \frac{1}{\sqrt{1-f(x)^2}} \qquad g'(t) = \frac{1}{\sqrt{1-t^2}}$$

$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1], \quad f(x) = \operatorname{sen} x \to f'(x) = \cos x$

$g(x) = \operatorname{arcsen} x, \quad g : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

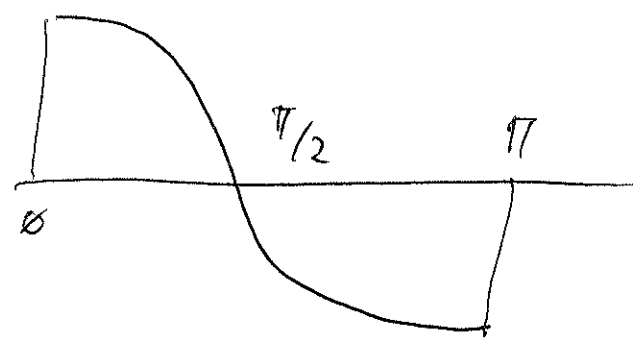




$$\frac{d}{dx} \operatorname{arcsen} x = \frac{1}{\sqrt{1-x^2}}$$

$f : [\emptyset, \pi] \to [-1, 1]$, $f(x) = \cos x$

$g : [-1, 1] \to [\emptyset, \pi]$, $g(t) = \arccos t$



$g(f(x)) = x$, $0 < x < \pi$

$$g'(f(x))\,\underbrace{f'(x)}_{-\operatorname{sen} x} = 1 \to g'(f(x)) = \frac{-1}{\operatorname{sen} x} = \frac{-1}{\sqrt{1-\cos^2 x}} = \frac{-1}{\sqrt{1-f(x)^2}}$$

$t = f(x) \in (-1, 1)$ $\quad g'(t) = \dfrac{-1}{\sqrt{1-t^2}}$

$$\frac{d}{dx} \arccos x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left\{\operatorname{arcsen} x + \arccos x\right\} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = \emptyset \quad , -1 < x < +1$$

x	$\operatorname{arcsen} x$	$\arccos x$
-1	$-\pi/2$	$\pi \longrightarrow \pi/2$
$-\frac{1}{2}\sqrt{3}$	$-\pi/3$	$5\pi/6 \longrightarrow \pi/2$
$-\frac{1}{2}\sqrt{2}$	$-\pi/4$	$3\pi/4 \longrightarrow \pi/2$
$-\frac{1}{2}$	$-\pi/6$	$2\pi/3 \longrightarrow \pi/2$
$\emptyset$	$\emptyset$	$\pi/2 \longrightarrow \pi/2$
$\frac{1}{2}$	$\pi/6$	$\pi/3 \longrightarrow \pi/2$
$\frac{1}{2}\sqrt{2}$	$\pi/4$	$\pi/4 \longrightarrow \pi/2$
$\frac{1}{2}\sqrt{3}$	$\pi/3$	$\pi/6 \longrightarrow \pi/2$
1	$\pi/2$	$\emptyset \longrightarrow \pi/2$

$$\operatorname{arcsen} x + \arccos x = \frac{\pi}{2}$$

$$-1 \le x \le +1$$

$$\int (x^2+5x+6)\,dx = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 6x + \text{cost}.$$

$$\int \left(x^4 - 3x^2 + \frac{7}{x} - \frac{3}{x^2}\right) dx = \frac{1}{5}x^5 - x^3 + 7\ln|x| + \frac{3}{x} + \text{cost}.$$

$$\int \left(e^{3x} - 2e^{5x} + 2^x\right) dx = \frac{1}{3}e^{3x} - \frac{2}{5}e^{5x} + \frac{e^{x\ln 2}}{\ln 2} + \text{cost}.$$

$$e^{x\ln 2} \to 2^x$$

$$2^x \downarrow \quad (e^{\ln 2})^x = e^{x\ln 2}$$

$$\int \frac{dx}{1+4x^2} = \int \frac{1}{1+4x^2}\,dx = \frac{1}{2}\operatorname{arctg}(2x) + \text{cost}.$$

$$\xrightarrow{d/dx} \frac{1}{1+(2x)^2}\frac{d}{dx}(2x)$$

$$\int \frac{dx}{x} = \ln|x| + \text{const.}$$

$$\int \frac{dx}{x-3} = \ln|x-3| + \text{const}$$

$$\xrightarrow{d/dx} \frac{1}{x-3} \frac{d}{dx}(x-3) = \frac{1}{x-3}$$

$$\int \frac{dx}{2x+5} = \frac{1}{2} \ln|2x+5| + \text{const.} \longrightarrow \frac{1}{2}\left(\ln\left|x+\frac{5}{2}\right| + \ln 2\right) + \text{const.}$$

$$\xrightarrow{d/dx} \frac{1}{2x+5} \frac{d}{dx}(2x+5) = \frac{2}{2x+5}$$

$$\|$$

$$\frac{1}{2}\int \frac{dx}{x+\frac{5}{2}} = \frac{1}{2} \ln\left|x+\frac{5}{2}\right| + \text{const.}$$

$$\int \frac{2x}{1+x^2}\,dx = \ln\left(1+x^2\right) + \text{cost}.$$

$$\xrightarrow{d/dx} \frac{1}{1+x^2}\frac{d}{dx}(1+x^2) = \frac{2x}{1+x^2}$$

$$\int \operatorname{tg} x\,dx = \int \frac{\operatorname{sen} x}{\cos x}\,dx = -\ln|\cos x| + \text{cost}.$$

$$\xrightarrow{d/dx} \frac{1}{\cos x}\frac{d}{dx}\cos x = \frac{-\operatorname{sen} x}{\cos x} = -\operatorname{tg} x$$

$$\int \operatorname{cotg} x\,dx = \int \frac{\cos x}{\operatorname{sen} x}\,dx = \ln|\operatorname{sen} x| + \text{cost}.$$

$$\int \frac{dx}{\operatorname{sen} x} = \int \frac{1}{2 \operatorname{sen}\frac{1}{2}x \cos\frac{1}{2}x}\, dx$$

$$\ln\left|\operatorname{tg}\tfrac{1}{2}x\right| + \text{cost}$$

$$= \int \frac{\cos\frac{1}{2}x}{\operatorname{sen}\frac{1}{2}x}\, \frac{1}{\cos^2\frac{1}{2}x}\, \frac{1}{2}\, dx = \int \frac{1}{\operatorname{tg}\frac{1}{2}x}\, \frac{1}{\cos^2\frac{1}{2}x}\, \frac{1}{2}\, dx$$

$$\frac{d}{dx} \ln\left|\operatorname{tg}\tfrac{1}{2}x\right| = \frac{1}{\operatorname{tg}\frac{1}{2}x} \frac{d}{dx} \operatorname{tg}\tfrac{1}{2}x = \frac{1}{\operatorname{tg}\frac{1}{2}x}\, \frac{1}{\cos^2\frac{1}{2}x}\, \frac{d}{dx}\left(\tfrac{1}{2}x\right)$$

$$= \frac{1}{\operatorname{tg}\frac{1}{2}x}\, \frac{1}{\cos^2\frac{1}{2}x}\, \frac{1}{2}$$

$$\int \operatorname{sen}^2 x \, dx$$

$$\int \cos^2 x \, dx$$

$$\int \cos^2 x \, dx + \int \operatorname{sen}^2 x \, dx = \int dx = x + \text{cost}.$$

$$\int \cos^2 x \, dx - \int \operatorname{sen}^2 x \, dx = \int \cos 2x \, dx$$

$$= \frac{1}{2} \operatorname{sen} 2x + \text{cost}.$$

$$\int \cos^2 x \, dx = \frac{x + \frac{1}{2}\operatorname{sen} 2x}{2} + \text{cost} = \frac{1}{2}x + \frac{1}{4}\operatorname{sen} 2x + \text{cost}.$$

$$\int \operatorname{sen}^2 x \, dx = \frac{x - \frac{1}{2}\operatorname{sen} 2x}{2} + \text{cost} = \frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x + \text{cost}.$$

$$2\cos^2 x - 1 = 1 - 2\operatorname{sen}^2 x = \cos 2x \rightarrow \cos^2 x = \frac{1 + \cos 2x}{2}, \ \operatorname{sen}^2 x = \frac{1 - \cos 2x}{2}$$

$$\int tg^2 x \, dx = \int \frac{sen^2 x}{cos^2 x} dx = \int \frac{1-cos^2 x}{cos^2 x} dx$$

$$= \int \left(\frac{1}{cos^2 x} - 1 \right) dx = tg\, x - x + cost.$$

$$\int \frac{2x+5}{x^2+1} dx = \int \left(\frac{2x}{x^2+1} + \frac{5}{x^2+1} \right) dx = ln(x^2+1) + 5 \, arctg\, x$$

$$+ cost.$$

$$\int \frac{2x+5}{x^2-1} dx = \int \left(\frac{7/2}{x-1} - \frac{3/2}{x+1} \right) dx = \frac{7}{2} ln|x-1| - \frac{3}{2} ln|x+1|$$

$$+ cost.$$

$$\frac{7}{2}(x+1) - \frac{3}{2}(x-1) = 2x + 5$$

$$\frac{2x+5}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} \longrightarrow \frac{2x+5}{x-1} = A\,\frac{x+1}{x-1} + B \xrightarrow{x=-1} -\frac{3}{2} = B$$

$$\frac{2x+5}{x+1} = A + B\,\frac{x-1}{x+1} \xrightarrow{x=1} \frac{7}{2} = A$$

$$= \frac{A(x+1) + B(x-1)}{x^2-1} = \frac{(A+B)x + (A-B)}{x^2-1}$$

$$\left.\begin{aligned} A + B &= 2 \\ A - B &= 5 \end{aligned}\right\} \begin{aligned} 2A &= 7 \rightarrow A = 7/2 \\ 2B &= -3 \rightarrow B = -3/2 \end{aligned}$$

$$\int \frac{x^5-3x^4+x+3}{x^2-1}\,dx$$

$$(x^2-1)(x^2+1)=x^4-1$$

$$\int \frac{x^4+2}{x^2-1}\,dx=\int\left(\frac{x^4-1}{x^2-1}+\frac{3}{x^2-1}\right)dx=\int(x^2+1)\,dx$$

$$+\int\left(\frac{A}{x-1}+\frac{B}{x+1}\right)dx$$

$$\frac{3}{x^2-1}=\frac{A}{x-1}+\frac{B}{x+1}\xrightarrow{x=1}A=\frac{3}{2}$$

$$\frac{3}{x+1}=A+B\frac{x-1}{x+1}$$

$$\frac{3}{x-1}=A\frac{x+1}{x-1}+B\xrightarrow{x=-1}B=-\frac{3}{2}$$

$$=\frac{1}{3}x^3+x+\frac{3}{2}\ln|x-1|-\frac{3}{2}\ln|x+1|+\text{const.}$$

$$
\begin{array}{r|l}
x^2-1 & x^5-3x^4+x+3 \quad \big| \quad x^3+3x^2+x-3 \\
 & \underline{x^5-x^3} \\
 & 3x^4-x^3+x+3 \\
 & \underline{3x^4-3x^2} \\
 & x^3-3x^2+x+3 \\
 & \underline{x^3-x} \\
 & -3x^2+2x+3 \\
 & \underline{-3x^2+3} \\
 & 2x
\end{array}
$$

$$\frac{x^5-3x^4+x+3}{x^2-1} = x^3+3x^2+x-3 + \frac{2x}{x^2-1}$$

$$\Big\downarrow \int \dots dx$$

$$\frac{1}{4}x^4 + x^3 + \frac{1}{2}x^2 - 3x + \ln|x^2-1| + \text{cost.}$$

$$\int \frac{x}{x^2+x+1}\,dx = \int \left(\frac{1}{2}\,\frac{2x+1}{x^2+x+1} - \frac{1}{2}\,\frac{1}{x^2+x+1} \right) dx$$

$$= \int \left(\frac{1}{2}\,\frac{2x+1}{x^2+x+1} - \frac{1}{2}\,\frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}} \right) dx$$

$$= \int \left(\frac{1}{2}\,\frac{2x+1}{x^2+x+1} - \frac{2}{3}\,\frac{1}{\left(\frac{2}{3}x\sqrt{3}+\frac{1}{3}\sqrt{3}\right)^2+1} \right) dx$$

$$= \frac{1}{2}\ln(x^2+x+1) - \frac{2}{3}\,\frac{\sqrt{3}}{2}\,\underbrace{\arctan\left(\frac{2}{3}x\sqrt{3}+\frac{1}{3}\sqrt{3}\right)} + \text{cost}.$$

$$\xrightarrow{d/dx} \frac{1}{(\dots)^2+1}\,\frac{d}{dx}(\dots) = \frac{2/\sqrt{3}}{(\dots)^2+1}$$

$$\int \frac{dx}{x^2+4x+13} = \int \frac{dx}{(x+2)^2+9} = \int \frac{1}{9} \frac{1}{\left(\frac{x+2}{3}\right)^2+1} dx$$

$$= \frac{1}{3} \operatorname{arctg}\left(\frac{x+2}{3}\right) + \text{cost.}$$

$$\llcorner_{d/dx} \quad \frac{1}{(\ldots)^2+1} \frac{d}{dx} \frac{x+2}{3} = \frac{1}{3} \frac{1}{(\ldots)^2+1}$$

$$\int \frac{dx}{x^2+4x-5} = \int \frac{1}{(x+5)(x-1)} dx = \int \left(\frac{A}{x+5} + \frac{B}{x-1} \right) dx$$

$$A(x-1) + B(x+5) = 1 \qquad \begin{array}{l} A+B=0 \to A=-B \qquad A=-1/6 \\ -A+5B=1 \to 6B=1 \to B=1/6 \end{array}$$

$$\int \frac{dx}{x^2+4x-5} = -\frac{1}{6}\ln|x+5| + \frac{1}{6}\ln|x-1| + \text{cost.}$$

$$\int \frac{dx}{x^2+4x+4} = \int \frac{1}{(x+2)^2}\,dx = \frac{-1}{x+2} + \text{cost.}$$

$$\int \frac{3x+7}{x^2+4x+4}\,dx = \int \frac{3(x+2)+1}{(x+2)^2}\,dx = \int \left(\frac{3}{x+2} + \frac{1}{(x+2)^2}\right)dx$$

$$= 3\ln|x+2| - \frac{1}{x+2} + \text{cost.}$$