Prossime Lezioni:	Tutor
26-10, 11-13	27-10, 9-11
6-11, 11-13	10-11, 9-11

f è continua in x_0 $\iff$ $\lim_{x \to x_0} f(x) = f(x_0)$

$\iff$ Se $\lim_{n\to\infty} x_n = x_0$ $(x_n \neq x_0)$ allora $\lim_{n\to\infty} f(x_n) = f(x_0)$

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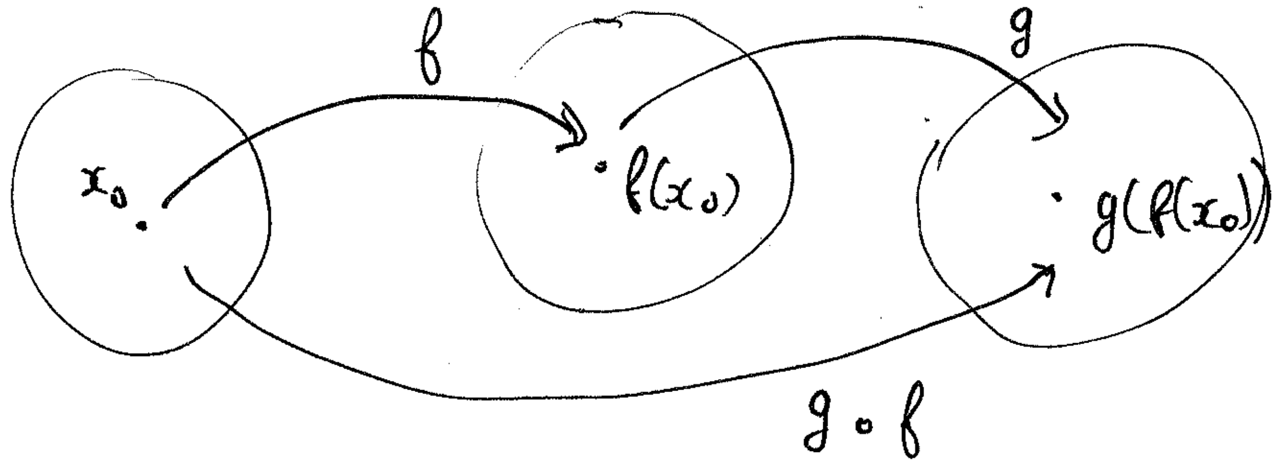
f è continua $\iff$ f è continua in tutti i punti del suo dominio

f, g continue in x_0, c costante $\Rightarrow$ $f+g$, $f-g$, cf, fg, $\frac{f}{g}$ continue in x_0

$\Uparrow$ SE $g(x_0) \neq 0$

f continua in x_0, g continua in $f(x_0)$ $\Rightarrow$ $g \circ f$ continua in x_0

$(g \circ f)(x) = g(f(x))$



FUNZIONE COMPOSTA

Sia I un intervallo, $I = (a,b) = \{x \in \mathbb{R} : a < x < b\}$,

~~$x_0 \in I$~~ $f : I \to \mathbb{R}$ continua

$\underline{x_0 \in I,\ f(x_0) > 0}$: In tal caso esiste $\delta > 0$

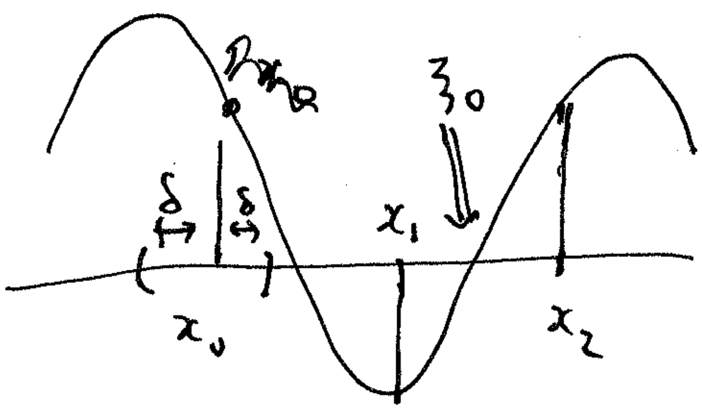
tale che $f(x) > 0$ per $x_0 - \delta < x < x_0 + \delta$

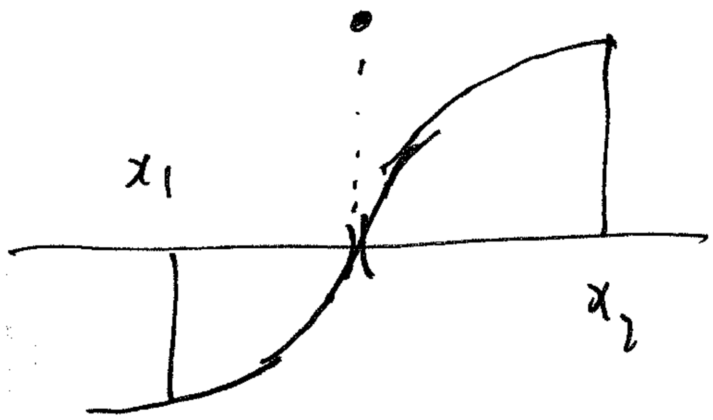
Teorema della PERMANENZA DEL SEGNO

$\underline{x_1, x_2 \in I,\ x_1 < x_2,\ f(x_1) < 0,\ f(x_2) > 0}$

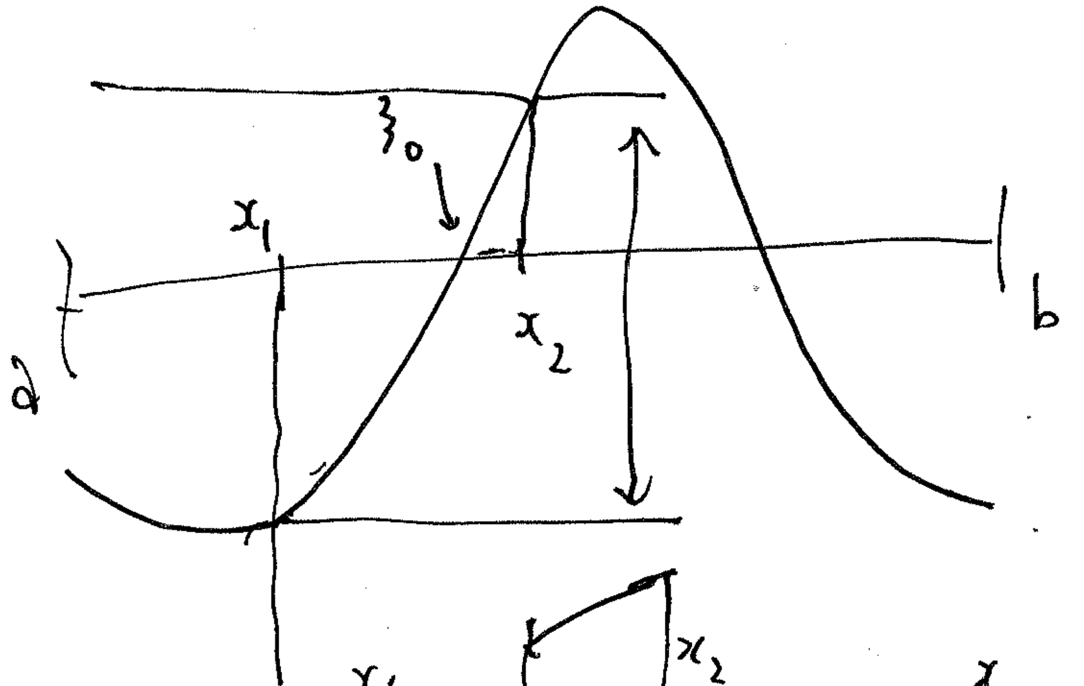
In tal caso esiste $\xi_0 \in I$ tale che

$x_1 < \xi_0 < x_2$, $f(\xi_0) = 0$ ESISTENZA DEGLI ZERO



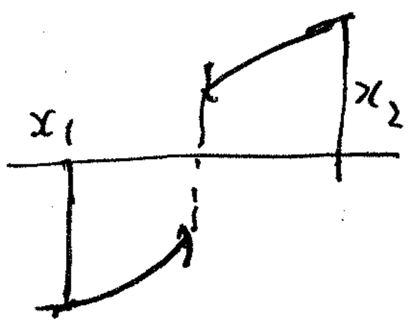


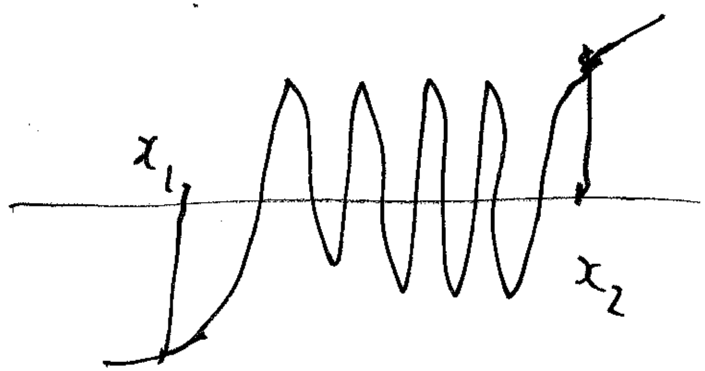
$$f(x) = x^3 - 2$$

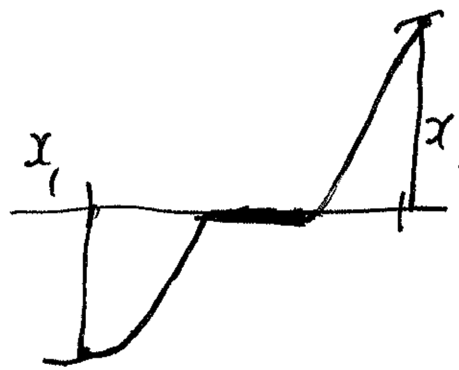


$$f(0) = -2, \quad f(2) = 6$$

$$\exists \xi \in (0, 2) : f(\xi) = 0$$







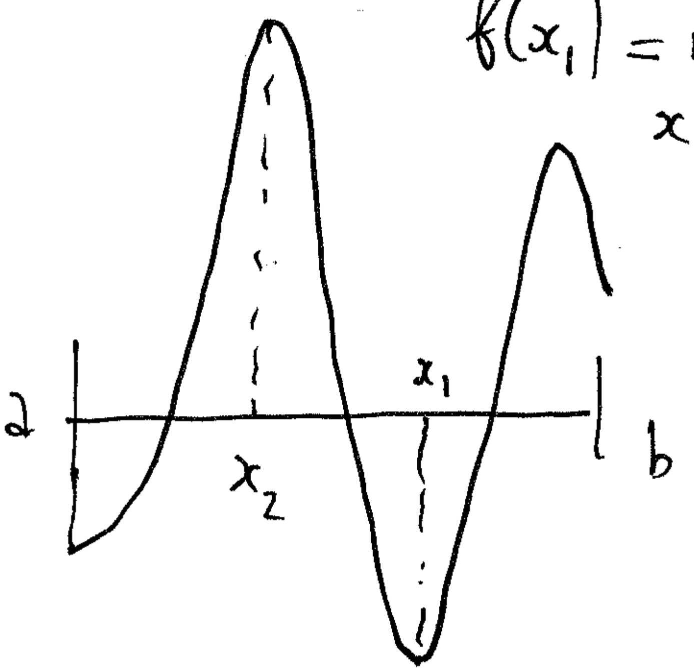
TEOREMA DI WEIERSTRASS

$[a,b] = \{x \in \mathbb{R} : a \le x \le b\}$

Sia $f : [a,b] \to \mathbb{R}$ continua

Allora esistono $x_1, x_2 \in [a,b]$ tali che

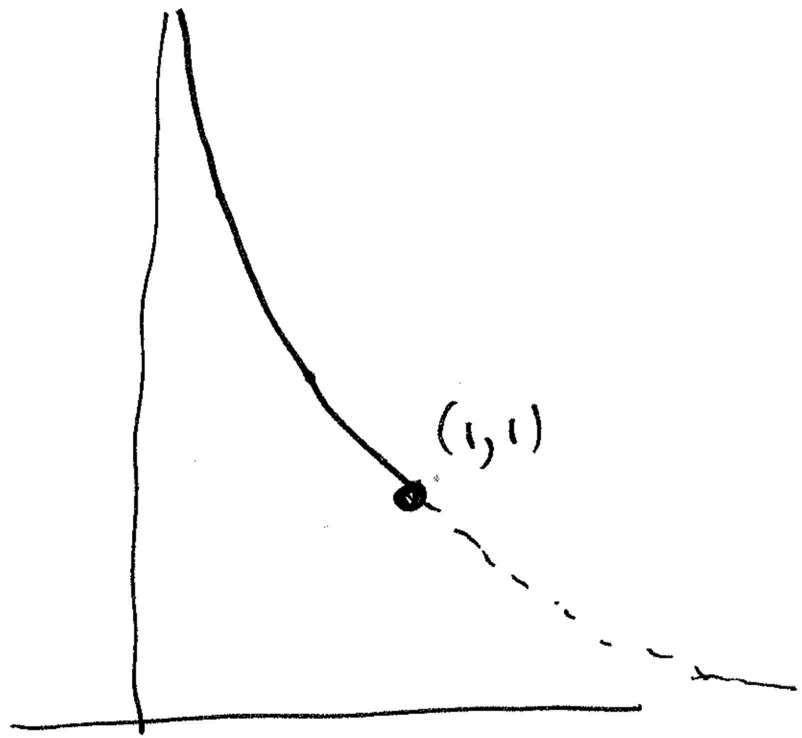
$$f(x_1) = \min_{x \in [a,b]} f(x) \quad , \quad f(x_2) = \max_{x \in [a,b]} f(x)$$



IMMAGINE

$$\left[\min f \,,\, \max f\right]$$

$$f : (0, 1] \longrightarrow \mathbb{R}, \quad f(x) = \frac{1}{x}$$

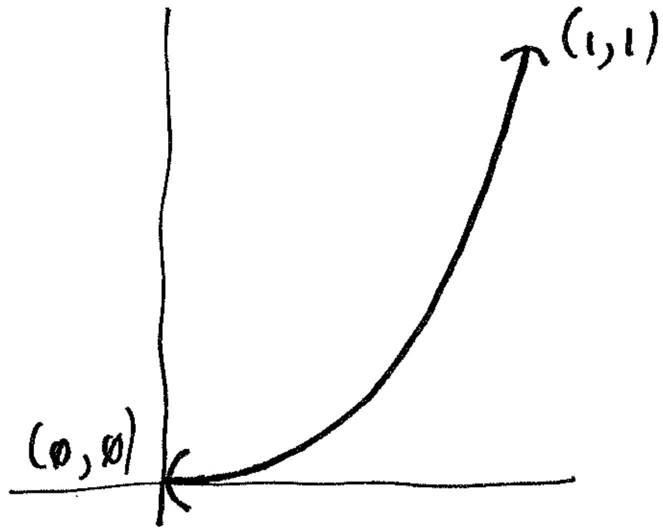


$$\lim_{x \to 0^+} f(x) = +\infty$$

$$\min f = 1 = f(1)$$

$$\sup f = +\infty$$

$$f : (0, 1) \longrightarrow \mathbb{R}, \quad f(x) = x^2$$



$\inf f = 0 \quad \nexists \qquad \sup f = 1$

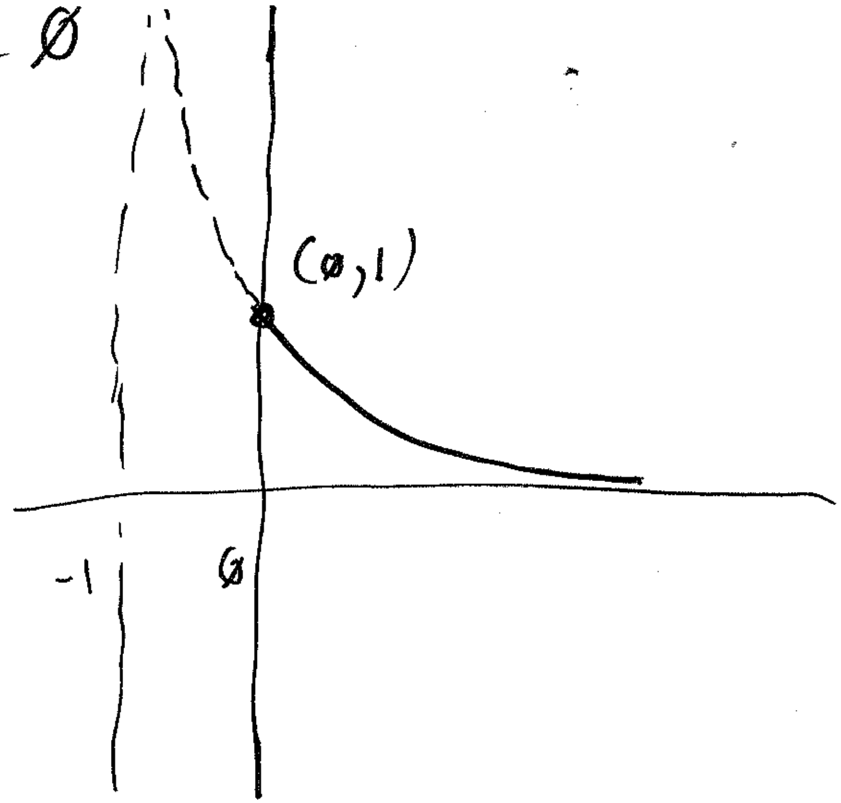
$f : [0, +\infty) = \{x \in \mathbb{R} : x \geq 0\} \to \mathbb{R}$, $f(x) = \dfrac{1}{x+1}$

$f(\emptyset) = 1$ $\qquad \lim_{x \to +\infty} f(x) = \emptyset$

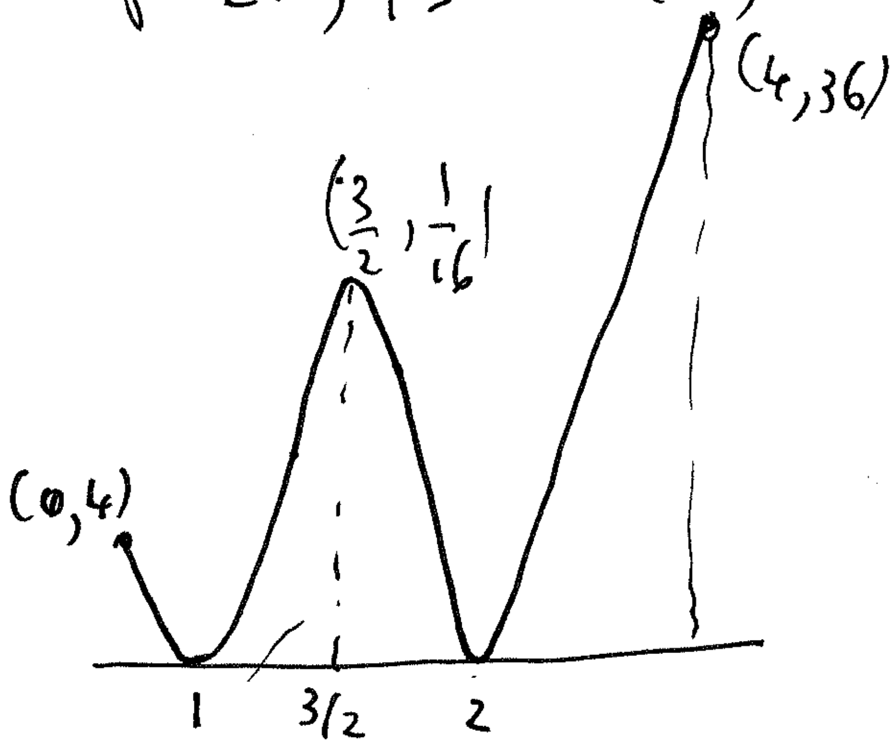
$\max f = 1 = f(\emptyset)$

$\inf f = \emptyset$

↳ NON C'È MINIMO



$f : [0, 4] \to \mathbb{R}, \quad f(x) = (x-1)^2 (x-2)^2$



$f(1) = f(2) = 0$

$f(0) = 4 \quad f(4) = 36$

$f(\frac{3}{2}) = \frac{1}{16} \qquad \max f = 36 = f(4)$

$\min f = 0 = f(1) = f(2)$

$f : [a,b] \to \mathbb{R}$ continua e strettamente crescente

$\min f = f(a) \qquad \max f = f(b)$

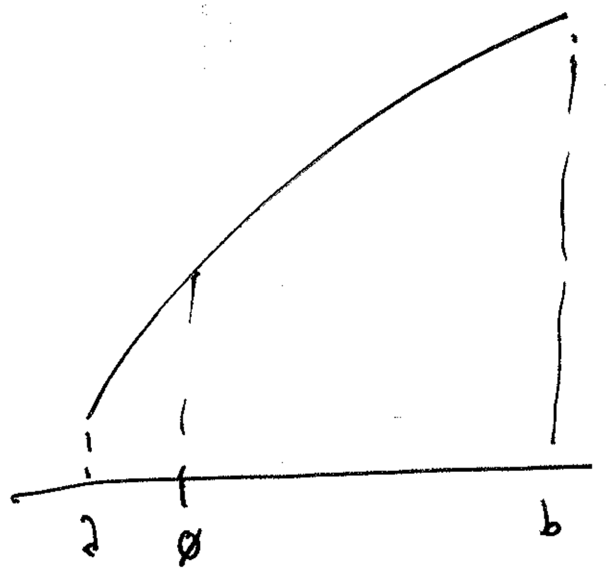
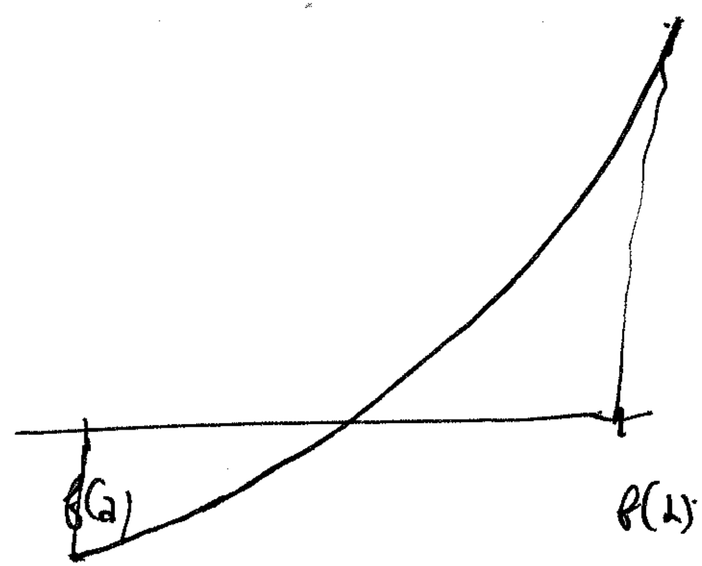


IMMAGINE $\to [f(a), f(b)]$

Vengono assunti tutti i valori intermedi una singola volta.

$f^{-1} : [f(a), f(b)] \to [a,b] \subseteq \mathbb{R}$

$\hookrightarrow$ CONTINUA



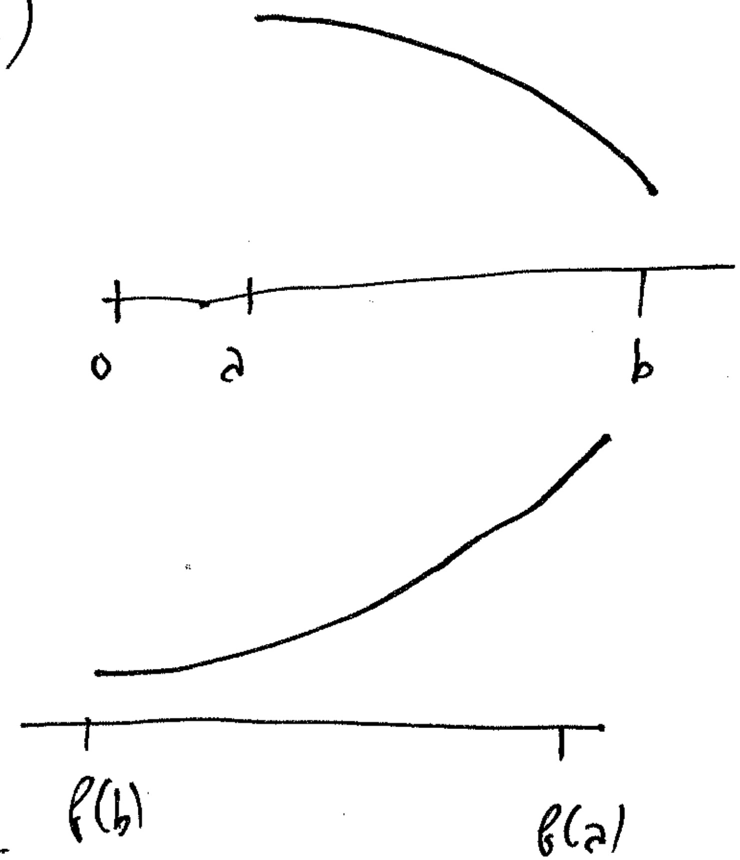
$f : [a,b] \to \mathbb{R}$ continua e strettamente decrescente

$\min f = f(b)$ $\quad$ $\max. f = f(a)$

IMMAGINE $\to [f(b), f(a)]$

$f^{-1} : [f(b), f(a)] \longrightarrow [a,b] \subseteq \mathbb{R}$

$\hookrightarrow$ CONTINUA



$$[x] = \max\{y \in \mathbb{R} : y \le x\}$$

$[2,3] = 2$

$[-1,7] = -2$

$$f(x) = x - [x]$$

$-2 \le -1,7 < -1$

$$[x] \le x < [x]+1$$

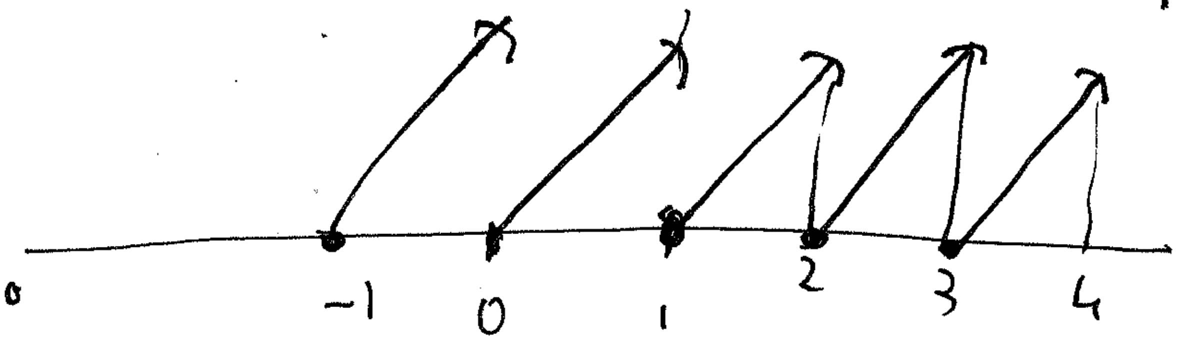
$2 \le 2,3 < 3$

$$0 \le f(x) < 1 \qquad \lim_{x \to n^-} f(x) = 1 \qquad \lim_{x \to n^+} f(x) = 0 = f(n)$$

$n \in \mathbb{Z}$

DISCONTINUITÀ DI

PRIMA SPECIE: x intero



$$\lim_{x \to +\infty} x\left(\sqrt{1+x} - x\right) = -\infty$$

$$x\left(\sqrt{1+x} - x\right)\frac{\sqrt{1+x}+x}{\sqrt{1+x}+x} = \frac{x\left(1+x-x^2\right)}{\sqrt{1+x}+x}$$

$$= \frac{1+x-x^2}{\sqrt{\frac{1}{x^2}+\frac{1}{x}}+1} \cancel{\sim} = \underbrace{-x^2}_{\to -\infty \text{ se } x \to +\infty} \underbrace{\frac{-\frac{1}{x^2}-\frac{1}{x}+1}{\sqrt{\frac{1}{x^2}+\frac{1}{x}}+1}}_{\to 1,\ x \to +\infty}$$

NON ESISTE

$$\lim_{x\to 0} \frac{\sqrt{1+4x+x^2}-x}{x} \times \frac{\sqrt{1+4x+x^2}+x}{\sqrt{1+4x+x^2}+x}$$

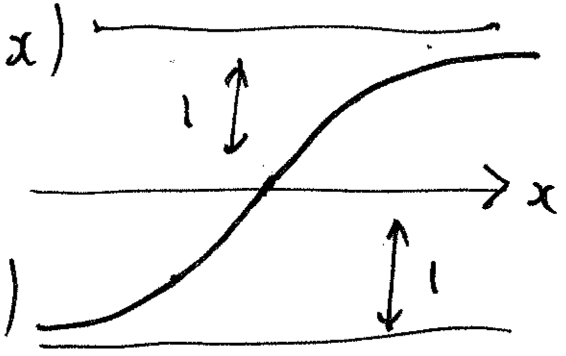
$$= \lim_{x\to 0} \frac{1+4x+\cancel{x^2}-\cancel{x^2}}{x\left(\sqrt{1+4x+x^2}+x\right)} = \lim_{x\to 0} \frac{1}{x} \underbrace{\frac{1+4x}{\sqrt{1+4x+x^2}+x}}_{\to 1,\ x\to 0}$$

$$\lim_{x\to 0^+} \frac{1}{x} = +\infty \qquad \lim_{x\to 0^+} \dots = +\infty$$

$$\lim_{x\to 0^-} \frac{1}{x} = -\infty \qquad \lim_{x\to 0^-} \dots = -\infty$$

$$\lim_{x\to+\infty} \overbrace{\frac{e^x - e^{-x}}{e^x + e^{-x}}}^{f(x)} = \lim_{x\to+\infty} \frac{1-e^{-2x}}{1+e^{-2x}} = \frac{1-0}{1+0} = 1$$

$$\lim_{x\to-\infty} \frac{e^x - e^{-x}}{\underbrace{e^x + e^{-x}}_{-f(-x)}} = \lim_{x\to-\infty} \frac{e^{2x}-1}{e^{2x}+1} = \frac{0-1}{0+1} = -1$$



$f(-x) = -f(x)$

$\lim_{x\to+\infty} e^x = +\infty \quad e>1$

$\lim_{x\to+\infty} e^{-2x} = 0 \quad 0<\frac{1}{e^2}<1$

$\left(\frac{1}{e^2}\right)^x$

$$\lim_{x \to 0} \left(1 + \operatorname{sen} x\right)^{1/x} = \lim_{x \to 0} \left[f(g(x))\right]^{h(x)} = f(g(\emptyset))^{h(\emptyset)}$$

$$= f(0)^{1} = e$$

$$g(x) = \operatorname{sen}(x) \to 0, \ x \to 0$$

$$h(x) = \begin{cases} \dfrac{\operatorname{sen}(x)}{x}, & x \neq 0 \\ 1, & x = \emptyset \end{cases}$$

$$\lim_{x \to 0} (1+x)^{1/x} = e$$

$$f : (-1, +\infty) \to \mathbb{R}$$

$$f(x) = \begin{cases} (1+x)^{1/x}, & x \neq 0 \\ e, & x = 0 \end{cases}$$

f continua

$$\lim_{x \to 0} (1 + \operatorname{sen} x)^{1/x} = \lim_{x \to 0} \left[(1 + \operatorname{sen} x)^{\frac{1}{\operatorname{sen} x}} \right]^{\frac{\operatorname{sen} x}{x}}$$

$$\lim_{x \to 0} \operatorname{sen} x = 0 \qquad = e^{\lim_{x \to 0} \frac{\operatorname{sen} x}{x}} = e^{1} = e$$

$$\lim_{z \to 0} (1 + z)^{1/z} = e$$

$$\lim_{x\to 0}\left(1+\operatorname{sen} x\right)^{\frac{1}{1-\cos(x)}} = \lim_{x\to 0}\left[\left(1+\operatorname{sen} x\right)^{\frac{1}{\operatorname{sen} x}}\right]^{\frac{\operatorname{sen} x}{1-\cos(x)}}$$

$$= e^{\lim_{x\to 0}\frac{\operatorname{sen} x}{1-\cos x}}.$$

NON ESISTE

$$\left.\begin{aligned}\operatorname{sen} x &\to \operatorname{sen} 0 = 0\\ \cos x &\to \cos 0 = 1\end{aligned}\right\} \text{se } x\to 0$$

$$\frac{\operatorname{sen} x}{1-\cos x} = \frac{2\operatorname{sen}(x/2)\cos(x/2)}{1-\left(1-2\operatorname{sen}^2(x/2)\right)} = \frac{\cos(x/2)}{\operatorname{sen}(x/2)} \to \begin{cases}+\infty, & x\to 0^+\\ -\infty, & x\to 0^-\end{cases}$$

$$e^{z} \longrightarrow \begin{cases} +\infty, & z \to +\infty \\ \emptyset, & z \to -\infty \end{cases}$$

$$e^{\frac{\operatorname{sen} x}{1-\cos(x)}} \longrightarrow \begin{cases} +\infty, & x \to 0^{+} \\ \emptyset, & x \to 0^{-} \end{cases}$$

$$\lim_{x \to +\infty} \frac{3x^2+2x+5}{5x^2+4x+2} = \lim_{x \to +\infty} \frac{3+\frac{2}{x}+\frac{5}{x^2}}{5+\frac{4}{x}+\frac{2}{x^2}} = \frac{3}{5}$$

$$\lim_{x \to +\infty} \frac{2x+5}{5x^2+4x+2} = \lim_{x \to +\infty} \frac{\frac{2}{x}+\frac{5}{x^2}}{5+\frac{4}{x}+\frac{2}{x^2}} = 0$$

$$\lim_{x \to +\infty} \frac{-3x^2+2x+5}{4x+2} = \lim_{x \to +\infty} \frac{x^2\left(-3+\frac{2}{x}+\frac{5}{x^2}\right)}{x\left(4+\frac{2}{x}\right)}$$

$$= \lim_{x \to +\infty} x \frac{-3 + \frac{2}{x} + \frac{5}{x^2}}{4 + \frac{2}{x}} = \cancel{+\infty} \; -\infty$$

$\longrightarrow \frac{-3}{4}, \; x \to +\infty$