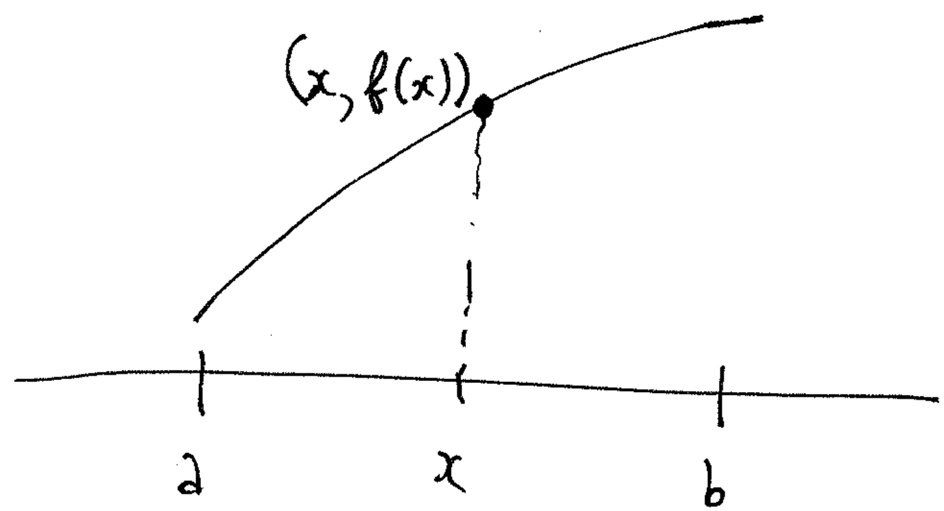
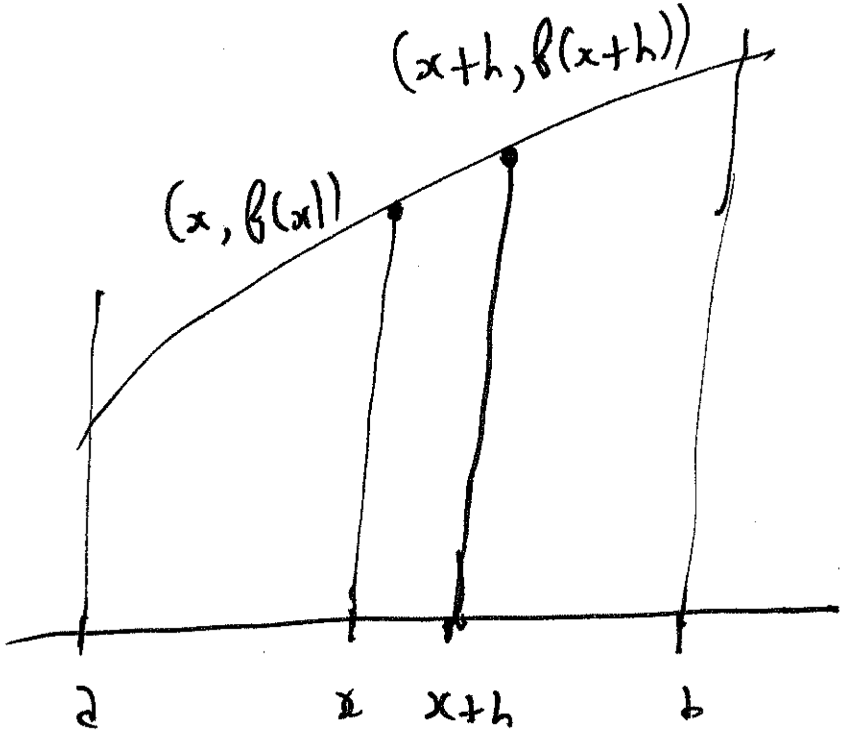
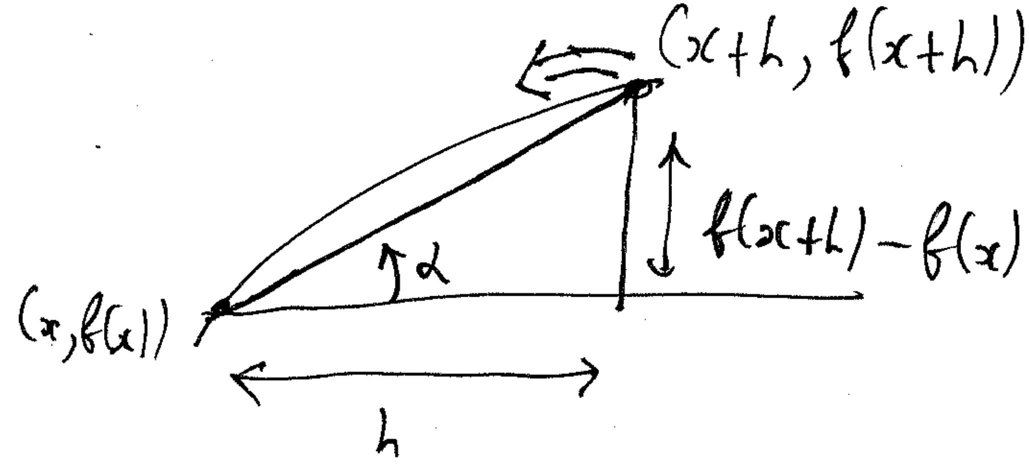
$f : (a,b) \longrightarrow \mathbb{R}$

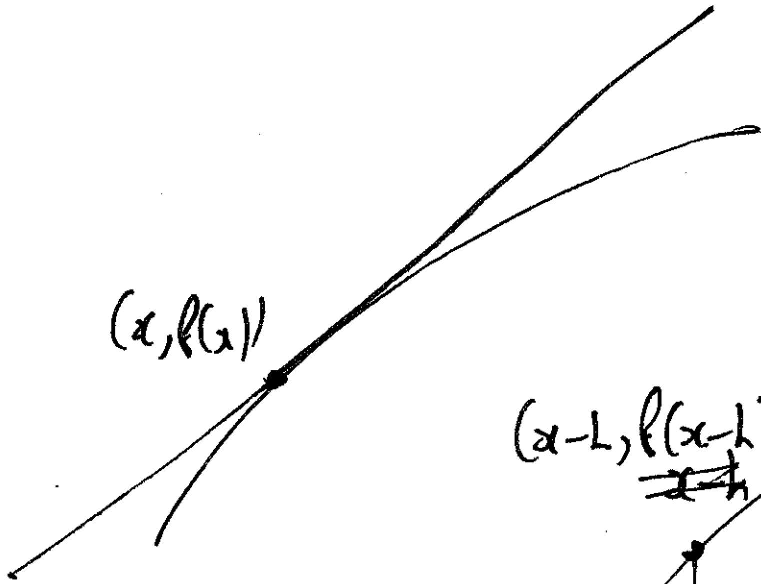


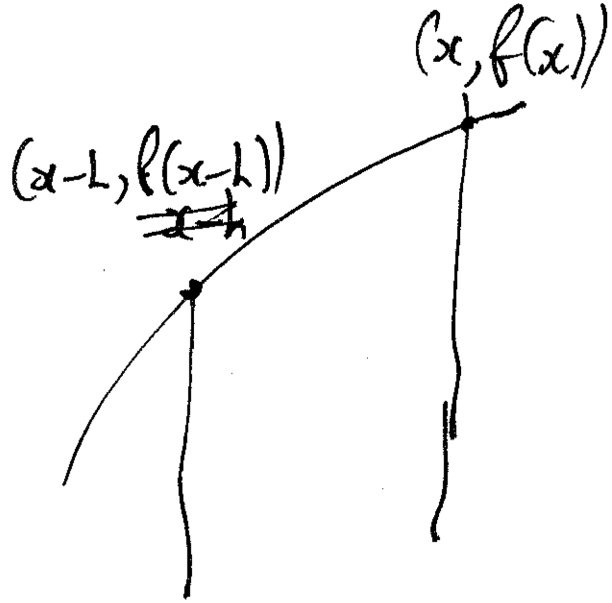
$\text{Grafico} = \{(x, f(x)) : a < x < b\}$





$$\operatorname{tg}\alpha = \frac{f(x+h)-f(x)}{h}$$



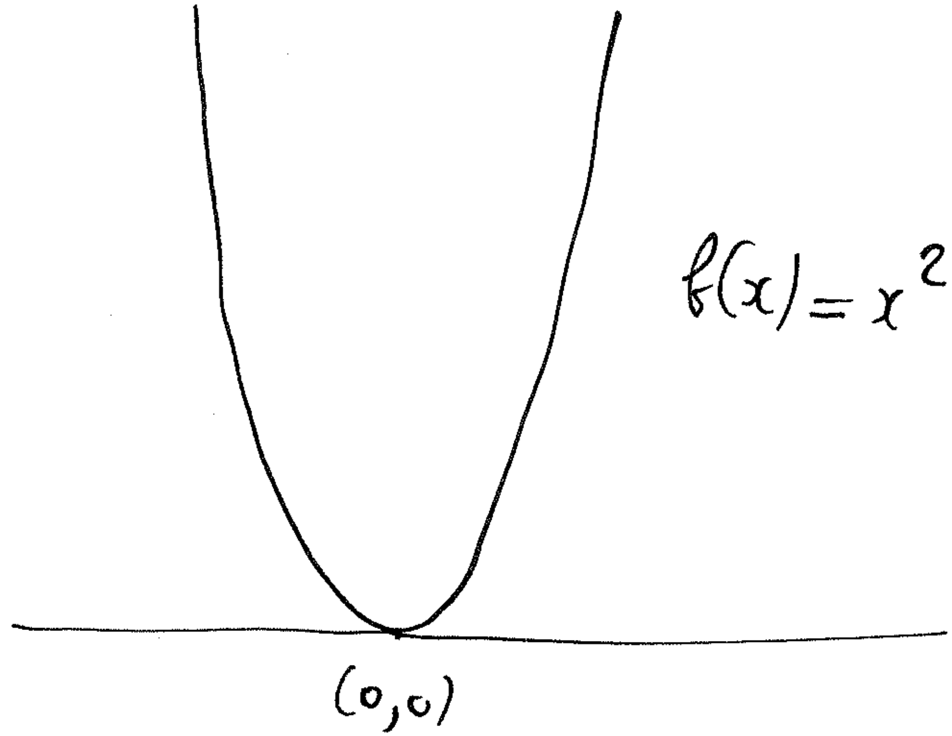


$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0} \frac{f(x)-f(x-h)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{df}{dx} \qquad Df(x)$$



$f(x) = x^2$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} = 2x + h$$

$$\downarrow \text{ se } h \to 0$$

$$2x$$

$$f'(x) = 2x$$

$f(x) = \text{costante} \rightarrow f'(x) = 0$

$f(x) = x^2 \rightarrow f'(x) = 2x$

$$f(x) = x^3 \qquad \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - x^3}{h} = \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= 3x^2 + 3xh + h^2 \longrightarrow 3x^2 \text{ se } h \to 0$$

$f'(x) = 3x^2$

$$f(x) = x^n \qquad n = 1, 2, 3, \ldots$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^n - x^n}{h} = \frac{\sum_{k=0}^{n} \binom{n}{k} x^{n-k} h^k - x^n}{h}$$

$$= \frac{\cancel{x^n} + \sum_{k=1}^{n} \binom{n}{k} x^{n-k} h^k - \cancel{x^n}}{h} = \sum_{k=1}^{n} \binom{n}{k} x^{n-k} h^{k-1}$$

$$\xrightarrow[\text{se } h \to 0]{} \binom{n}{1} x^{n-1} h^0 = n x^{n-1} \qquad f'(x) = n x^{n-1}$$

$$f(x) = \frac{1}{x} = x^{-1}, \quad x \neq 0$$

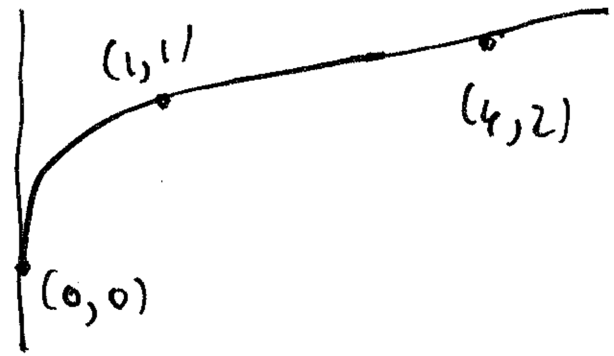
$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{\overset{-h}{\cancel{x - (x+h)}}}{(x+h)\,x\,h} = \frac{-1}{(x+h)x}$$

$$\xrightarrow{h \to 0} \frac{-1}{x^2} = (-1)\,x^{(-1)-1}$$

$f(x) = \sqrt{x}\,, x > 0$ $\qquad\qquad f(x) = x^{\frac{1}{2}}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{\overset{h}{\cancel{(x+h) - x}}}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \frac{1}{\sqrt{x+h} + \sqrt{x}} \xrightarrow{h \to 0} \frac{1}{2\sqrt{x}}$$



$$\| $$

$$f'(x) = \frac{1}{2} x^{\frac{1}{2} - 1}$$

$$(f+g)'(x) = f'(x) + g'(x)$$

$$\frac{(f+g)(x+h) - (f+g)(x)}{h} = \frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h}$$

$$\to f'(x) + g'(x)$$

$$(cf)'(x) = c\, f'(x)$$

$$\frac{(cf)(x+h) - (cf)(x)}{h} = c\, \frac{f(x+h) - f(x)}{h} \to c\, f'(x)$$

$$(fg)'(x) = f(x)g'(x) + f'(x)g(x)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{esiste finito}$$

ALLORA: $\lim_{h \to 0} f(x+h) = f(x) \longrightarrow$ CONTINUITÀ

$$\frac{f(x+h)g(x+h) - f(x)g(x)}{h} = f(x+h)\frac{g(x+h) - g(x)}{h} + \frac{f(x+h) - f(x)}{h} g(x) \longrightarrow f(x)g'(x) + f'(x)g(x)$$

$$\frac{\dfrac{f(x+h)}{g(x+h)} - \dfrac{f(x)}{g(x)}}{h} = \frac{g(x)\,\dfrac{f(x+h)-f(x)}{h} - f(x)\,\dfrac{g(x+h)-g(x)}{h}}{g(x+h)\,g(x)}$$

$$\xrightarrow[h\to 0]{} \frac{g(x)\,f'(x) - f(x)\,g'(x)}{g(x)^2}$$

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

IPOTESI: $\underline{\underline{g(x) \neq 0}}$

$$x^2 - 3x + 2 \xrightarrow{d/dx} 2x - 3 + 0 = 2x - 3$$

$$5x^3 + 4x^2 - 3x + 7 \xrightarrow{\frac{d}{dx}} 15x^2 + 8x - 3$$

$$\frac{x^2 + 4x}{x + 2} \xrightarrow{\frac{d}{dx}} \frac{(x+2)(2x+4) - (x^2+4x)\cdot 1}{(x+2)^2} = \frac{x^2 + 4x + 8}{(x+2)^2}$$

$$\frac{x+1}{x^2+4} \xrightarrow{d/dx} \frac{(x^2+4)\cdot 1 - (x+1)\cdot 2x}{(x^2+4)^2} = \frac{-x^2 - 2x + 4}{(x^2+4)^2}$$

$$(g \circ f)(x) = g(f(x))$$

$$\frac{g(f(x+h)) - g(f(x))}{h} = \frac{g(f(x+h)) - g(f(x))}{f(x+h) - f(x)} \times \underbrace{\frac{f(x+h) - f(x)}{h}}_{\to f'(x)}$$

$$\lim_{k \to 0} \frac{g(y+k) - g(y)}{k} = g'(y)$$

dove $y = f(x)$

$$\boxed{\begin{gathered} g'(f(x))\, f'(x) \\ \parallel \\ (g \circ f)'(x) \end{gathered}}$$

$$\frac{d}{dx}\left(x^2+5x\right)^{1000}$$

$$\parallel$$

$$1000\left(x^2+5x\right)^{999}(2x+5)$$

$$f(x) = x^2+5x$$

$$g(y) = y^{1000}$$

$$\frac{d}{dx}x^4 = \frac{d}{dx}(x^2)^2$$

$$= 2x^2 \cdot 2x = 4x^3$$

$$\begin{tikzcd} f : (a,b) \arrow[r] & (c,d) \arrow[l, bend left, "f^{-1}"] \end{tikzcd}$$ derivabile, ~~bigett~~ ~~invertibile~~ biunivoca

$\cancel{f^{-1}(f(x)) = x}$

$$1 = \frac{d}{dx} x = f'(f^{-1}(x)) \left(\frac{d}{dx} f^{-1}\right)(x)$$

$$f(f^{-1}(x)) = x$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^3$, $f^{-1}(y) = \sqrt[3]{y} = y^{1/3}$

$$(f^{-1})'(x) = \frac{1}{3(\sqrt[3]{x})^2} = \frac{1}{3} x^{-2/3}$$

$f: (0,\infty) \to (0,\infty)$, $f(x) = x^2$, $f^{-1}(y) = \sqrt{y}$

$$(f^{-1})'(x) = \frac{1}{2(f^{-1}(x))} = \frac{1}{2\sqrt{x}}$$

Logaritmo Naturale

$$\lim_{x \to 0} (1+x)^{1/x} = e$$

$$e^w = z \leftrightarrow \ln(z) = \log_e z = w$$

$$\ln(1) = \log_e 1 = \emptyset$$

$$1 = \lim_{x \to 0} \ln\{(1+x)^{1/x}\} = \lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$$

$$\| $$

$$\ln e$$

$$\cancel{f(x) = \ln(1+x)}$$

$$e^{p-q} = \frac{e^p}{e^q} \leftrightarrow p-q = \ln\left(\frac{e^p}{e^q}\right)$$

$$(e^w)^{\frac{1}{h}} = z^{\frac{1}{h}} \leftrightarrow \frac{w}{h} = \ln\left(z^{\frac{1}{h}}\right)$$

$$f(x) = \ln(x), \quad x > 0$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\ln(x+h) - \ln(x)}{h} = \frac{\ln\left(\frac{x+h}{x}\right)}{h} = \cancel{\ln\left\{\left(\frac{x+h}{x}\right)^{h}\right\}}$$

$$= \frac{1}{x}\,\frac{x}{h}\ln\left(\frac{x+h}{x}\right) = \frac{1}{x}\ln\left\{\left(1+\frac{h}{x}\right)^{\frac{x}{h}}\right\} \xrightarrow{h\to 0} \frac{1}{x}\ln e = \frac{1}{x}$$

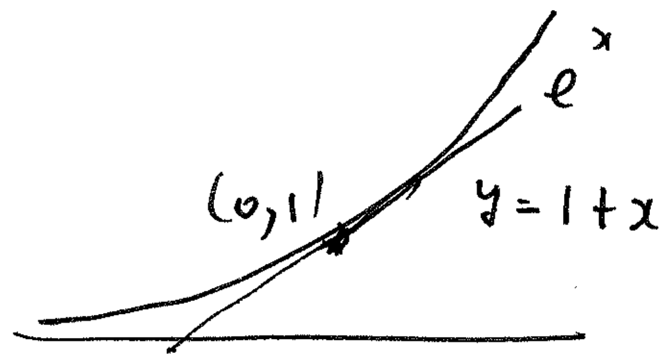
$$e^{\ln(x)} = x$$

$$e^{x} = y \iff x = \ln(y)$$

$$e^{x} = f^{-1}(x) \qquad \ln(y) = f(y)$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{\frac{1}{f^{-1}(x)}} = f^{-1}(x) = e^{x}$$

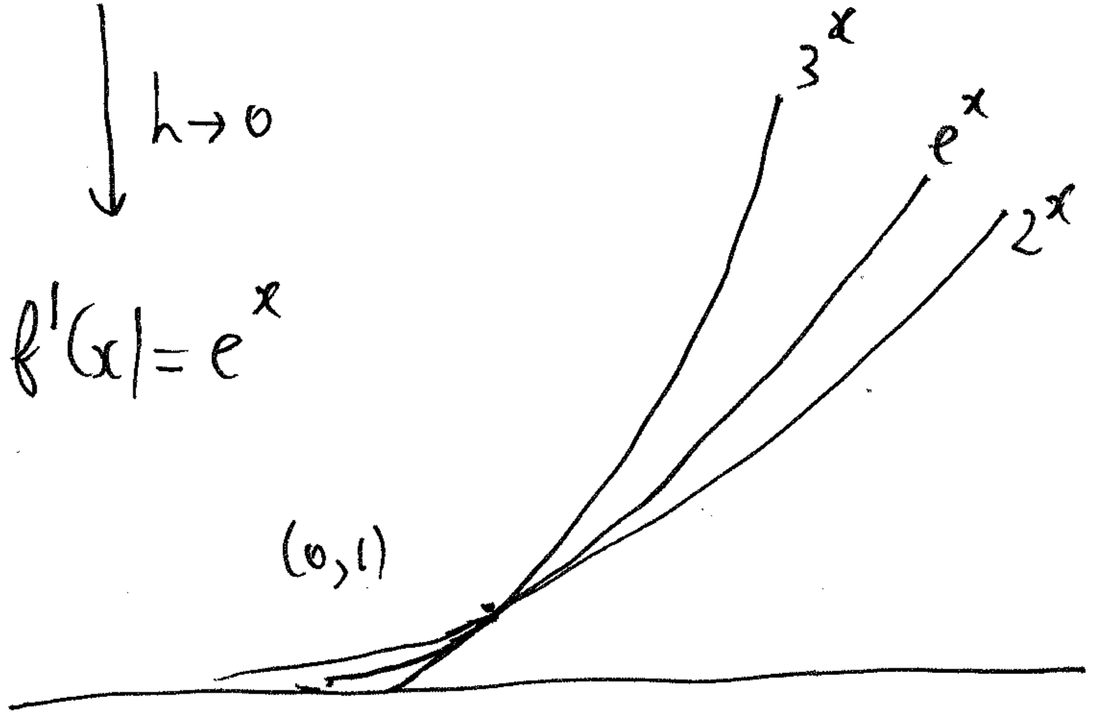
$$f(x) = e^x \longrightarrow f'(x) = e^x$$



$$\frac{f(x+h) - f(x)}{h} = \frac{e^{x+h} - e^x}{h} = \frac{e^x e^h - e^x}{h} = e^x \, \frac{e^h - 1}{h}$$

$$\Big\downarrow h \to 0 \qquad\qquad \Big\downarrow$$

$$f'(x) = e^x \qquad\qquad e^x \underbrace{\lim_{h \to 0} \frac{e^h - 1}{h}}_{=1}$$



$$f(x) = e^{2x} \longrightarrow f'(x) = e^{2x} \frac{d}{dx}(2x) = 2e^{2x}$$

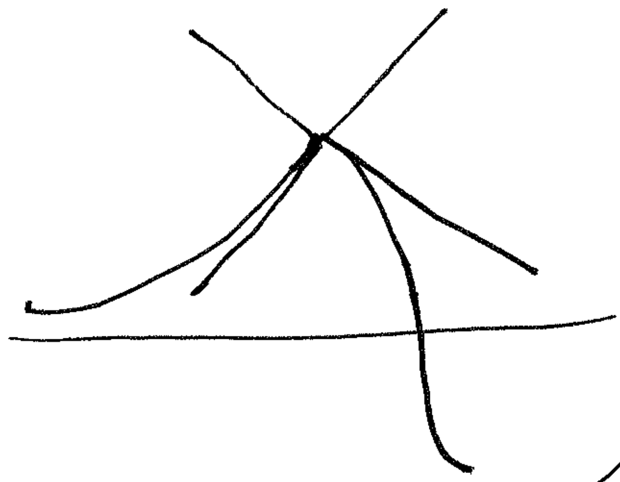
$$f(x) = e^{x^2} \longrightarrow f'(x) = e^{x^2} \frac{d}{dx}(x^2) = 2x\, e^{x^2}$$

$$f(x) = \ln(1+x^2) \longrightarrow f'(x) = \frac{1}{1+x^2} \frac{d}{dx}(1+x^2) = \frac{2x}{1+x^2}$$

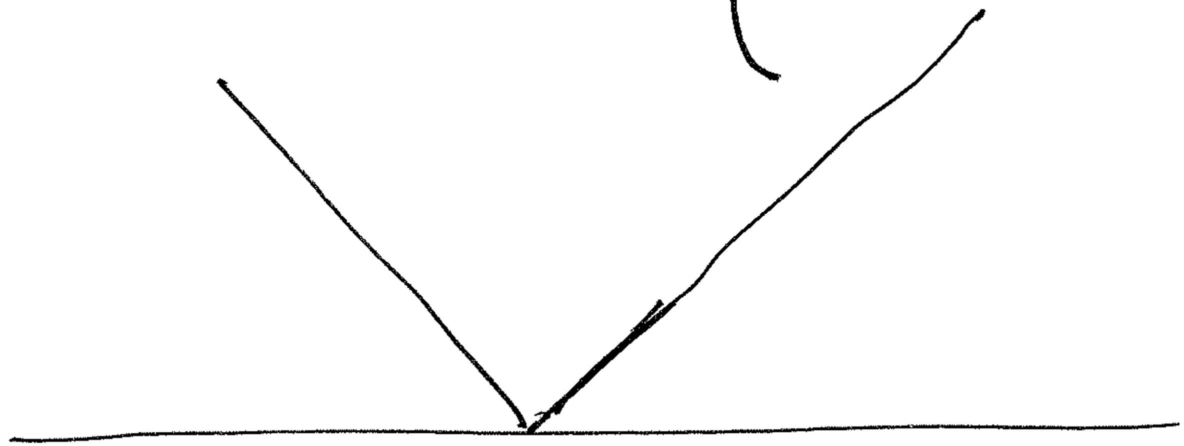
$$f(x) = \ln^2(x) \longrightarrow f'(x) = 2\ln(x) \frac{d}{dx}\ln(x) = \frac{2\ln(x)}{x}$$

$$f(x) = \ln(x^2) \longrightarrow f'(x) = \frac{1}{x^2} \frac{d}{dx}(2x) = \frac{2}{x}$$

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



$$f'(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$$



$$\lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h} = 1$$

$$\lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h} = -1$$