$\int x e^{-2x}\,dx$

$\int x^2 \operatorname{sen} x\,dx$

$\int 1 \cdot \ln|x|\,dx$

$\int e^{2x} \cos 3x\,dx$

$(FG)' = FG' + F'G$

integrazione per parti

$FG' = (FG)' - F'G$

$\int FG'\,dx = FG - \int F'G\,dx$

$$\int \underset{G'}{\underset{\uparrow}{1}} \cdot \underset{F}{\underset{\uparrow}{\ln|x|}}\,dx = \underset{G}{\underset{\uparrow}{x}}\,\underset{F}{\underset{\uparrow}{\ln|x|}} - \int \underset{F'}{\underset{\uparrow}{\frac{1}{x}}} \cdot \underset{G}{\underset{\uparrow}{x}}\,dx$$

$$= x\ln|x| - x + \text{cost.}$$

$$\int \underset{F}{\underset{\uparrow}{x}}\,\underset{G'}{\underset{\uparrow}{e^{-2x}}}\,dx = \underset{F}{\underset{\uparrow}{x}} \cdot \underset{G}{\underset{\uparrow}{-\frac{1}{2}e^{-2x}}} - \int \underset{F'}{\underset{\uparrow}{1}} \cdot -\frac{1}{2}e^{-2x}\,dx = -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$$

$$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + \text{cost.}$$

$$\int \underset{\substack{\uparrow\\ F}}{x^2}\, \underset{\substack{\uparrow\\ G'}}{\operatorname{sen} x}\, dx = \underset{\substack{\uparrow\\ F}}{x^2} \cdot \underset{\substack{\uparrow\\ G}}{-\cos x} - \int \underset{\substack{\uparrow\\ F'}}{2x} \cdot -\cos x \, dx = -x^2 \cos x$$

$$+2\int \underset{\substack{\Uparrow\\ F}}{x}\, \underset{\substack{\Uparrow\\ G'}}{\cos x}\, dx = -x^2 \cos x + 2 . \underset{\substack{\Uparrow\\ F}}{x} . \underset{\substack{\Uparrow\\ G}}{\operatorname{sen} x} - 2\int \underset{\substack{\Uparrow\\ F'}}{1} . \underset{\substack{\Uparrow\\ G}}{\operatorname{sen} x}\, dx$$

$$= -x^2 \cos x + 2x \cos x + 2 \cos x + \text{cost.}$$

$$\int \underset{\overset{\uparrow}{F}}{e^{-2x}} \underset{\overset{\uparrow}{G'}}{\cos 3x}\, dx = \underset{\overset{\uparrow}{F}}{e^{-2x}} \cdot \underset{\overset{\uparrow}{G}}{\frac{1}{3} \operatorname{sen} 3x} - \int \underset{\overset{\uparrow}{F'}}{-2e^{-2x}} \cdot \underset{\overset{\uparrow}{G}}{\frac{1}{3} \operatorname{sen} 3x}\, dx$$

$$= \frac{1}{3} e^{-2x} \operatorname{sen} 3x + \frac{2}{3} \int \underset{\overset{\Uparrow}{F}}{e^{2x}} \underset{\overset{\Uparrow}{G'}}{\operatorname{sen} 3x}\, dx = \frac{1}{3} e^{-2x} \operatorname{sen} 3x + \frac{2}{3} \underset{\overset{\Uparrow}{F}}{e^{-2x}} \cdot \underset{\overset{\Uparrow}{G}}{-\frac{1}{3} \cos 3x}$$

$$- \frac{2}{3} \int \underset{\overset{\Uparrow}{F'}}{-2e^{-2x}} \cdot \underset{\overset{\Uparrow}{G}}{-\frac{1}{3} \cos 3x}\, dx = \frac{1}{3} e^{-2x} \operatorname{sen} 3x - \frac{2}{9} e^{-2x} \cos 3x - \frac{4}{9} \int e^{-2x} \cos 3x\, dx$$

$$\Rightarrow \frac{13}{9} \int e^{-2x} \cos 3x\, dx = \cdots \Rightarrow \int e^{-2x} \cos 3x\, dx = \frac{3}{13} e^{-2x} \operatorname{sen} 3x - \frac{2}{13} e^{-2x} \cos 3x + \text{cost},$$

$$A e^{-2x} \cos 3x + B e^{-2x} \operatorname{sen} 3x$$

$$\xrightarrow{d/dx} (-2A + 3B) e^{-2x} \cos 3x + (-3A - 2B) e^{-2x} \operatorname{sen} 3x$$

$$\overset{?}{=} e^{-2x} \cos 3x$$

$$\left.\begin{aligned} -2A + 3B &= 1 \\ -3A - 2B &= 0 \end{aligned}\right\} \quad A = -\frac{2}{13}, \; B = \frac{3}{13}$$

$$\int e^{-2x} \cos 3x \, dx = \frac{3}{13} e^{-2x} \operatorname{sen} 3x - \frac{2}{13} e^{-2x} \cos 3x + \text{cost.}$$

$$\int_{g(a)}^{g(b)} f(x)\,dx = \int_a^b f(g(t))\,g'(t)\,dt$$

$$x = g(t) \longrightarrow \frac{dx}{dt} = g'(t)$$

g è biunivoca su $[a,b]$, C^1

$$\int_0^1 \frac{2x+3}{1+(x^2+3x+2)^2}\,dx = \int_2^6 \frac{du}{1+u^2} = \Big[\operatorname{arctg} u\Big]_{u=2}^{6}$$

$$= \operatorname{arctg} 6 - \operatorname{arctg} 2$$

$$u = x^2 + 3x + 2$$

$$\frac{du}{dx} = 2x+3, \quad (2x+3)\,dx = du$$

$$\int_{\emptyset}^{\ln(5)} \frac{3e^x}{1+e^{2x}}\,dx = \int_1^5 \frac{3\,du}{1+u^2} = \Big[3 \operatorname{arctg} u\Big]_{u=1}^{5}$$

$$= 3\left(\operatorname{arctg} 5 - \frac{\pi}{4}\right)$$

$$u = e^x \to du = e^x\,dx$$

$$\int_{\emptyset}^{\pi/4} \operatorname{tg} x\,dx = \int_1^{\frac{1}{2}\sqrt{2}} \frac{-du}{u} = \int_{\frac{1}{2}\sqrt{2}}^{1} \frac{du}{u} = \Big[\ln|u|\Big]_{\frac{1}{2}\sqrt{2}}^{1} = \ln 1 - \ln\left(\tfrac{1}{2}\sqrt{2}\right)$$

$$2^{-1/2}$$

$$= \frac{1}{2}\ln 2$$

$$\frac{\operatorname{sen} x}{\cos x} \qquad u = \cos x \qquad du = -\operatorname{sen} x\,dx$$

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int_{\emptyset}^{\pi/3} \frac{1}{\cos x}\,dx = \int_{\emptyset}^{\frac{1}{3}\sqrt{3}} \frac{1+\underbrace{\operatorname{tg}^2\tfrac{1}{2}x}_{u^2}}{1-\underbrace{\operatorname{tg}^2\tfrac{1}{2}x}_{u^2}}\,\frac{2}{1+u^2}\,du = \int_{0}^{\frac{1}{3}\sqrt{3}} \frac{2}{1-u^2}\,du$$

$$1+\operatorname{tg}^2\alpha = 1+\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha} = \frac{\cos^2\alpha+\operatorname{sen}^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = \frac{1}{1+\operatorname{tg}^2\alpha}$$

$$\cos x = 2\cos^2\tfrac{1}{2}x - 1 = 2\,\frac{1}{1+\operatorname{tg}^2\tfrac{1}{2}x} - 1 = \frac{1-\operatorname{tg}^2\tfrac{1}{2}x}{1+\operatorname{tg}^2\tfrac{1}{2}x}$$

$$\operatorname{sen} x = \operatorname{tg} x \cos x = \frac{2\operatorname{tg}\tfrac{1}{2}x}{\cancel{1-\operatorname{tg}^2\tfrac{1}{2}x}}\;\frac{\cancel{1-\operatorname{tg}^2\tfrac{1}{2}x}}{1+\operatorname{tg}^2\tfrac{1}{2}x} = \frac{2\operatorname{tg}\tfrac{1}{2}x}{1+\operatorname{tg}^2\tfrac{1}{2}x}$$

$$u = \operatorname{tg}\tfrac{1}{2}x \rightarrow \frac{du}{dx} = \frac{1}{\cos^2\tfrac{1}{2}x}\,\frac{1}{2} = \frac{1}{2}\left(1+\operatorname{tg}^2\tfrac{1}{2}x\right) = \frac{1}{2}\left(1+u^2\right)$$

$u = \operatorname{tg} \frac{1}{2} x \rightarrow \frac{1}{2} x = \operatorname{arctg} u \rightarrow x = 2 \operatorname{arctg} u$

$dx = \frac{2}{1+u^2} du$

$\frac{A}{1-u} + \frac{B}{1+u} = \frac{2}{1-u^2}$

$A(1+u) + B(1-u) = 2$

$A + B = 2, \; A - B = 0 \rightarrow A = B = 1$

$* = \int_0^{\frac{1}{3}\sqrt{3}} \left(\frac{1}{1-u} + \frac{1}{1+u} \right) du$

$= \left[-\ln|1-u| + \ln|1+u| \right]_0^{\frac{1}{3}\sqrt{3}}$

$= \left[\ln \left| \frac{1+u}{1-u} \right| \right]_0^{\frac{1}{3}\sqrt{3}}$

$= \ln \frac{1+\frac{1}{3}\sqrt{3}}{1-\frac{1}{3}\sqrt{3}} - \cancel{\ln 1} = \ln \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$$\int_0^{\pi/3} \frac{1}{\cos x}\,dx = \left[\ln\left|\frac{1+u}{1-u}\right|\right]_0^{\frac{1}{3}\sqrt{3}} \qquad u = \mathrm{tg}\,\tfrac{1}{2}x$$

$$\int \frac{1}{\cos x}\,dx = \ln\left|\frac{1+\mathrm{tg}\,\frac{1}{2}x}{1-\mathrm{tg}\,\frac{1}{2}x}\right| + \mathrm{const.}$$

$$\int_4^{25} \frac{1}{\sqrt{x}-3}\,dx = \int_2^5 \frac{1}{t-3}\,2t\,dt = \int_2^5 \frac{2t}{t-3}\,dt$$

$t = \sqrt{x}$ $\quad x = t^2$

$dx = 2t\,dt$

$$= \int_2^5 \frac{2(t-3)+6}{t-3}\,dt$$

$$= \int_2^5 \left(2 + \frac{6}{t-3}\right) dt = \Big[2t + 6\ln|t-3|\Big]_2^5$$

$$= 6 + 6\{\ln 2 - \cancel{\ln 1}\} = 6 + 6\ln 2$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \text{arcsen}\, x + \text{cost.} \qquad \sqrt{1-x^2} \xrightarrow{\frac{d}{dx}} \frac{-2x}{2\sqrt{1-x^2}}$$

$$\int \sqrt{1-x^2}\, dx = \int \underset{G'}{1} \cdot \underset{F}{\sqrt{1-x^2}}\, dx = \underset{G}{x}\underset{F}{\sqrt{1-x^2}} - \int \underset{G}{x} \cdot \underset{F'}{\frac{-x}{\sqrt{1-x^2}}}\, dx$$

$$= x\sqrt{1-x^2} - \int \frac{1-x^2-1}{\sqrt{1-x^2}}\, dx = x\sqrt{1-x^2} - \int \left(\sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}}\right) dx$$

$$= x\sqrt{1-x^2} - \int \sqrt{1-x^2}\, dx + \underbrace{\int \frac{dx}{\sqrt{1-x^2}}}_{\text{arcsen}\, x} \rightarrow \int \sqrt{1-x^2}\, dx = \frac{1}{2} x \sqrt{1-x^2} + \frac{1}{2} \text{arcsen}\, x + \text{cost.}$$

$$\int \sqrt{1-x^2}\,dx = \int \sqrt{1-\operatorname{sen}^2 t}\,\cos t\,dt = \int \cos^2 t\,dt$$

$$x = \operatorname{sen} t \rightarrow dx = \cos t\,dt$$

$$\uparrow \qquad \uparrow$$

$$(-1,1) \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$= \int \left(\frac{1}{2} + \frac{1}{2}\cos 2t\right) dt = \frac{1}{2}t + \frac{1}{4}\operatorname{sen} 2t + \text{cost.}$$

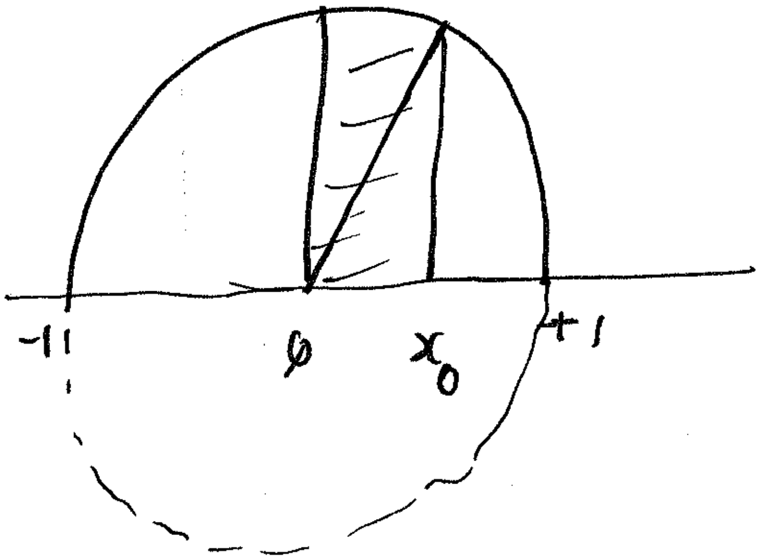
$$= \frac{1}{2}t + \frac{1}{2}\operatorname{sen} t \cos t + \text{cost.} = \frac{1}{2}\operatorname{arcsen} x + \frac{1}{2}x\sqrt{1-x^2} + \text{cost.}$$

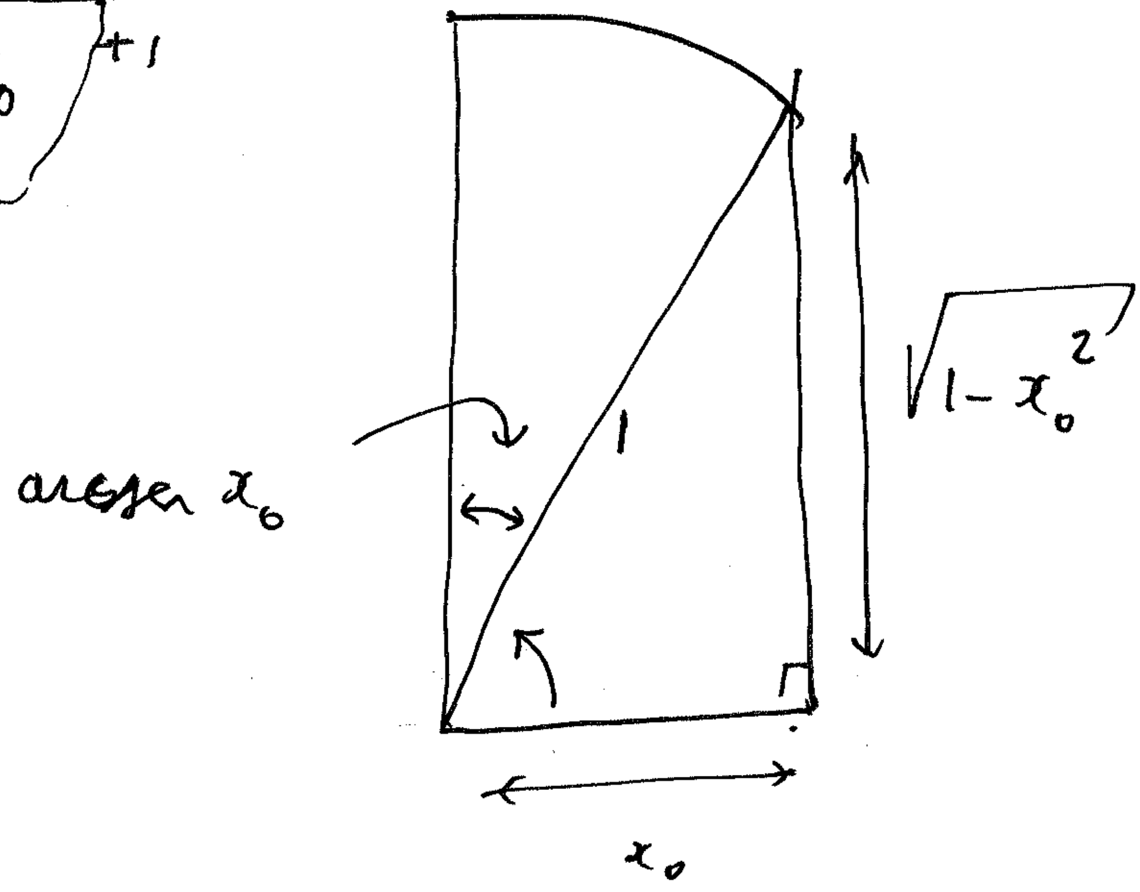
$$2\cos^2 t - 1 = \cos 2t \rightarrow \cos^2 t = \frac{1+\cos 2t}{2}$$

$$\operatorname{sen} 2t = 2\operatorname{sen} t \cos t$$

$$f(x) = \sqrt{1-x^2}$$

$$\int_0^{x_0} \sqrt{1-x^2}\, dx = \frac{1}{2} \operatorname{arcsen} x_0 + \frac{1}{2} x_0 \sqrt{1-x_0^2}$$





$$\frac{d}{dx}\ln\left(x+\sqrt{x^2+1}\right)=\frac{\frac{d}{dx}\left\{x+\sqrt{x^2+1}\right\}}{x+\sqrt{x^2+1}}$$

$$=\frac{1+\frac{1}{\cancel{2}\sqrt{x^2+1}}\cdot\cancel{2}x}{x+\sqrt{x^2+1}}=\frac{1+\frac{x}{\sqrt{x^2+1}}}{\sqrt{x^2+1}+x}=\frac{1}{\sqrt{x^2+1}}$$

$$\int\sqrt{x^2+1}\,dx=\int \underset{\underset{G'}{\uparrow}}{1}\cdot\underset{\underset{F}{\uparrow}}{\sqrt{x^2+1}}\,dx=\underset{\underset{G}{\uparrow}}{x}\underset{\underset{F}{\uparrow}}{\sqrt{x^2+1}}-\int \underset{\underset{G}{\uparrow}}{x}\cdot\underset{\underset{F'}{\uparrow}}{\frac{x}{\sqrt{x^2+1}}}\,dx$$

$$=x\sqrt{x^2+1}-\int\frac{x^2+1-1}{\sqrt{x^2+1}}\,dx=x\sqrt{x^2+1}-\int\sqrt{x^2+1}\,dx+\int\frac{dx}{\sqrt{x^2+1}}$$

$$\int \sqrt{x^2+1}\,dx = \frac{1}{2} x\sqrt{x^2+1} + \frac{1}{2}\ln\left(x+\sqrt{x^2+1}\right) + \text{cost.}$$

$$\int \operatorname{arctg} x \, dx = \int \underset{G'}{\underset{\uparrow}{1}} \cdot \underset{F}{\underset{\uparrow}{\operatorname{arctg} x}} \, dx = \underset{G}{\underset{\uparrow}{x}} \, \underset{F}{\underset{\uparrow}{\operatorname{arctg} x}} - \int \underset{G}{\underset{\uparrow}{x}} \, \frac{1}{1+x^2}\,dx$$

$$= x \operatorname{arctg} x - \frac{1}{2}\ln(1+x^2) + \text{cost.}$$

$$\int \frac{x}{1+x^2}\,dx \ \underset{dt=2x\,dx}{\overset{t=1+x^2}{=\!=\!=}} \ \frac{1}{2}\int \frac{dt}{t} = \frac{1}{2}\ln|t| + \text{cost.} = \frac{1}{2}\ln(1+x^2) + \text{cost.}$$

$$\int_0^{1/2} \underset{G'}{x}\ \underset{F}{\operatorname{arcsen} x}\ dx = \left[\underset{G}{\tfrac{1}{2}x^2}\ \underset{F}{\operatorname{arcsen} x}\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} \frac{1}{2}x^2 \frac{1}{\sqrt{1-x^2}}\, dx$$

$$= \left[\frac{1}{2}x^2 \operatorname{arcsen} x\right]_0^{\frac{1}{2}} - \frac{1}{2}\int_0^{\frac{1}{2}} \left(\frac{1}{\sqrt{1-x^2}} - \sqrt{1-x^2}\right) dx$$

$$= \left[\frac{1}{2}x^2 \operatorname{arcsen} x - \frac{1}{2}\operatorname{arcsen} x + \frac{1}{2}\left\{\frac{1}{2}\operatorname{arcsen} x + \frac{1}{2}x\sqrt{1-x^2}\right\}\right]_0^{1/2}$$

$$= \left[\left(\frac{1}{2}x^2 - \frac{1}{4}\right)\operatorname{arcsen} x + \frac{1}{4}x\sqrt{1-x^2}\right]_0^{\frac{1}{2}} = \frac{1}{8}\frac{\pi}{6} + \frac{1}{8}\sqrt{\frac{3}{4}} = \frac{\pi}{48} + \frac{1}{16}\sqrt{3}$$