Calcolare la derivata prima delle seguenti funzioni:

1. $y = 3x^5 - 2x^3 + 3x - 5$.

2. $y = \dfrac{3}{x^2} - \dfrac{7}{x^3}$.

3. $y = 5x^2 + \dfrac{2}{x^3}$.

4. $y = \dfrac{2x^6 - 3x^5}{x^3}$.

5. $y = \dfrac{x^2 - 7}{x + 1}$.

6. $y = \dfrac{2x^2 - 3x + 5}{2x^3 - 1}$.

7. $y = \dfrac{1}{x^2 + 1}$.

8. $y = \dfrac{1}{x^5(x + 1)^3}$.

9. $y = (3x + 1)^3$.

10. $y = \dfrac{(x + 2)^3}{(2x - 1)^2}$.

11. $y = (3x+6)(3x^2+1)(2x^3+6)$.

12. $y = (3x^4 - 2x^2 + 1)^4$.

13. $y = \sqrt{5x}$.

14. $y = 2\sqrt[3]{x^2}$.

15. $y = x\sqrt{x^2 + 5}$.

16. $y = \sqrt{3 + \dfrac{2}{x}}$.

17. $y = (2x + 3)\sqrt{x + 6}$.

18. $y = 2x^n + \sqrt[3]{x^m}$.

19. $y = \sqrt{\dfrac{1 + x^2}{1 - x^2}}$.

20. $y = \dfrac{3x}{\sqrt{6 - 2x^2}}$.

21. $y = \dfrac{\sqrt{7 - x^n}}{n}$.

22. $y = \dfrac{2a^3}{x^2 + 2a^2}$.

23. $y = \left(\dfrac{x^2 - 4}{x^2 + 4}\right)^2$.

24. $y = \dfrac{1}{(x^p + 1)(x^q + 1)}$.

----

25. $y = 4 \sin 3x$.

26. $y = \sin x \cos^2 x$.

27. $y = 3 \cotg^2 x$.

28. $y = \sin^2 \tfrac{1}{2}x$.

29. $y = \sqrt{\sin 2x}$.

30. $y = \sin^2 x \sec x$.

31. $y = x^2 \tg^2 x$.

32. $y = 2a \cosec^2 \tfrac{1}{2}x$.

33. $y = \sqrt{x} \cdot \sec x$.

34. $y = \dfrac{\cos x}{x}$.

35. $y = \dfrac{\sin x}{1 - \cos x}$.

36. $y = \dfrac{\sec x + \tg x}{\sec x - \tg x}$.

37. $y = \dfrac{1 - \sin x}{1 + \sin x}$.

38. $y = \dfrac{1 - \tg^2 x}{\tg x}$.

39. $y = \dfrac{\sin^2 (x^2 + 6)}{4}$.

40. $y = \sqrt{x} \cdot \cos(1 - x^2)$.

----

41. $y = \arcsin \tfrac{1}{2}x$.

42. $y = \arccos 4x$.

43. $y = \arcsin \sqrt{x}$.

44. $y = \arccos \dfrac{1}{x}$.

45. $y = 4 \arcsin x^2$.

46. $y = \arccos \dfrac{1}{\sqrt{1 + x^2}}$.

47. $y = \arctg 3x$.

48. $y = \arccotg \tfrac{1}{2}x$.

49. $y = \arctg (x^2 + 2)$.

50. $y = \arccotg \dfrac{1}{x}$.

51. $y = \arcsin \dfrac{1 - x}{1 + x}$.

52. $y = x^2 \arcsin x$.

----

53. $y = x \ln x$.

54. $y = 3x^2 \ln x$.

55. $y = \dfrac{\ln 5x}{3}$.

56. $y = \ln^2 x$.

57. $y = \ln \sin x$.

58. $y = \ln \cos x$.

59. $y = \ln \sqrt{3x + 4}$.

60. $y = \ln \sqrt{\operatorname{tg} x}$.

61. $y = \ln \sqrt{r^2 - x^2}$.

62. $y = \ln \sec^2 x$.

63. $y = \ln(x + \sqrt{x^2 - a^2})$.

64. $y = \ln(\sec x + \operatorname{tg} x)$.

65. $y = \ln \dfrac{x^2 - 4}{x^2 + 4}$.

66. $y = \ln \sqrt{\dfrac{1 + x^2}{1 - x^2}}$.

67. $y = \ln \ln x$.

68. $y = \ln(x^2 \sin x)$.

69. $y = \ln \{(x^2 - 1)\sqrt{x + 1}\}$.

70. $y = \operatorname{tg} \ln (2x + 3)^2$.

---

71. $y = 4e^{1/2x}$.

72. $y = e^{-x^3}$.

73. $y = 2^{3x}$.

74. $y = 10^{x^3 - 1}$.

75. $y = \dfrac{3^{x^2}}{9}$.

76. $y = \dfrac{1}{(2e)^x}$.

77. $y = 4x^2 e^{x^2}$.

78. $y = e^x \ln x$.

79. $y = x^x$.

80. $y = x^{\frac{1}{x}}$.

81. $y = x^{\sin x}$.

82. $y = (\ln x)^x$.

---

83. $y = \ln(x + \sqrt{x^2 - 1})$.

84. $y = \dfrac{1}{\operatorname{arctg} x}$.

85. $y = \ln \cos \sqrt{\dfrac{2a}{x}}$.

86. $y = \operatorname{arctg} (\arcsin \sqrt{x})$.

87. $y = \ln x \cdot \ln x^{-1}$.

88. $y = x^{\sqrt{x}}$.

89. $y = (e^x + 2)^x$.

90. $y = {}^x\log (1 + x)$.

91. $y = e^{x(\ln x)^{-1}}$.

92. $y = (1 + x)^{x-1}$.

93. $y = \operatorname{arctg} \dfrac{x}{\sqrt{1 - x^2}}$.

94. $y = bx\sqrt{4 - b^2 x^2} + 4\arcsin \dfrac{b}{2}x$.

95. $y = \arcsin (2x^2 - 1)$.

96. $y = \arcsin nx^n$.

97. $y = \dfrac{\sin 3x \cos x^2}{2}$.

98. $y = \operatorname{arctg} \dfrac{1}{\sqrt{1 + x^2}}$.

99. $y = \dfrac{\operatorname{tg} 3x^2}{1 + x^2}$.

100. $y = \dfrac{\sqrt[3]{1 + 2x}}{\sqrt{1 + x^2}}$.

101. $y = \sin (n \arccos x)$.

102. $y = 3^{\cos x}$.

103. $y = \ln \sqrt{1 + x^2}$.

104. $y = \arcsin \dfrac{x}{\sqrt{1 + x^2}}$.

105. $y = (x - \sqrt{1 - x^2})^2$.

106. $y = \dfrac{e^{\sqrt{1 + 2x}}}{\sqrt{1 + 2x}}$.

107. $y = \ln | \cos x - \sec x |$.

108. $y = \dfrac{\sin(x^3) \cos^3 x}{3}$.

109. $y = x^{\ln x}$.

110. $y = \sqrt{2x + 1} \operatorname{tg} (2x + 1)$.

111. $y = \ln \sqrt{\dfrac{1 - \cos mx}{1 + \cos mx}}$.

112. $y = \operatorname{arctg} \dfrac{x\sqrt{3}}{x + 2}$.

113. $y = e^{\arcsin x}$.

114. $y = \dfrac{x \arcsin x}{\sqrt{1 - x^2}} + \ln \sqrt{1 - x^2}$.

115. $y = x \arcsin x + \sqrt{1 - x^2}$.

116. $y = (1 + x) \operatorname{arctg} \sqrt{x} - \sqrt{x}$.

117. $y = \arcsin \sqrt{1 - x^2}$.

118. $y = \operatorname{tg} (\arcsin x)$.

119. $y = \operatorname{arctg} \dfrac{\cos x}{1 + \sin x}$.

120. $y = \operatorname{arctg} \dfrac{a + b \cos x}{b + a \cos x}$.

121. $y = \operatorname{arctg} \dfrac{a+x}{1-ax}$.

122. $y = 2 \arcsin \sqrt{\dfrac{x-b}{a-b}}$.

123. $y = 2 \operatorname{arctg} \sqrt{\dfrac{x-b}{a-x}}$.

124. $y = \operatorname{arccotg} \dfrac{x^2-a^2}{2ax}$.

125. $y = a \arccos \dfrac{a-x}{a} - \sqrt{2ax-x^2}$.

126. $y = x^{\operatorname{arcsec} x}$.

127. $y = \operatorname{arctg} \left(\arcsin \dfrac{x}{a} \right)$.

128. $y = x^{x^x}$.

129. $y = e^{|x|}$.

130. $y = \ln |\cos x|$.

Tracciare i grafici delle seguenti funsioni:
CIOÈ, calcolare i massimi, i mekihmi, ed i flessi

1. 1°. $y = x^3 + x^2 - 8x + 6$;   2°. $y = x^3 + 3x^2 + 3x - 7$;
   3°. $y = x^3 - x^2 + 12$.

2. 1°. $y = \dfrac{x-2}{x^2}$;   2°. $y = \dfrac{x^2}{x-2}$.

3. 1°. $y = \dfrac{x^2 - 16x + 60}{x^2 - 10x + 16}$;   2°. $y = \dfrac{x^2 + 8x + 37}{x^2 - 4x - 12}$.

4. 1°. $y = \dfrac{x^2 - 10x - 3}{x^2 + 2x + 3}$;   2°. $y = \dfrac{2x^2 + 4x + 7}{x^2 + 2x - 2}$.

5. 1°. $y = \dfrac{x^3}{x^2 - 12}$;   2°. $y = \dfrac{x^2 - 3}{x^3}$.

6. 1°. $y = xe^x$;   2°. $y = xe^{-x}$.

7. 1°. $y = x \ln x$, $(x > 0)$;   2°. $y = x^{-1} \ln x$, $(x > 0)$.

8. 1°. $y = \dfrac{\sqrt{x^2 + 1}}{x}$;   2°. $y = \dfrac{\sqrt{x^2 - 1}}{x}$, $(|x| \geqq 1)$.

9. 1°. $y = \dfrac{\sqrt[3]{x}}{x^2 + 1}$;   2°. $y = \dfrac{\sqrt[3]{x^2}}{x^2 + 1}$.

10. 1°. $y = \dfrac{x^3 + 1}{|x|}$;   2°. $y = \dfrac{2|x|^3 + 1}{x}$.

11. $y = x^5 - 5x^4 + 5x^3$.

12. $y = \sqrt[3]{x^4 - 2x^2}$.

13. $y = 1 + x - x\sqrt{1+x}$; $x \geqq -1$.

14. $y = x^2 \operatorname{arctg} x$.

15. $y = 2\sin x + p \sin^2 x$, voor $p = 2$, $p = 1$, $p = \frac{1}{2}$ en $0 \leqq x < 2\pi$.

16. $y = 1 + \dfrac{2x \operatorname{arctg} x}{1 + x^2} + (\operatorname{arctg} x)^2$.

17. $y = 2 \operatorname{cosec} x - \operatorname{cotg} x$.

18. $y = x^{2/5}(x-5)^{3/5}$; $-1 \leqq x \leqq 6$.

_____

19. $y = \dfrac{x\sqrt[3]{x}}{1 + x^2}$.

20. $y = \left(x^2 + \dfrac{1}{2e} \right)^{x^2 + \frac{1}{2e}}$.

44)^{x} $\quad f(x) = \dfrac{\ln x}{\ln^2 x + 1}$, $\quad x > 0$

45)^{x} $\quad f(x) = e^{-4x} - e^{-2x} + 1$

46)^{x} $\quad f(x) = x^{-x}$, $\quad x > 0$

$\quad$ [calcolare $\lim\limits_{x \to 0^+} f(x)$, $\lim\limits_{x \to 0^+} f'(x)$]

1. $15x^4 - 6x^2 + 3.$ 2. $\dfrac{-6x + 21}{x^4}.$ 3. $\dfrac{10x^5 - 6}{x^4}.$

4. $6x^2 - 6x.$ 5. $\dfrac{x^2 + 2x + 7}{(x+1)^2}.$

6. $\dfrac{-4x^4 + 12x^3 - 30x^2 - 4x + 3}{(2x^3 - 1)^2}.$ 7. $\dfrac{-2x}{(x^2+1)^2}.$

8. $\dfrac{-(8x+5)}{x^6(x+1)^4}.$ 9. $9(3x+1)^2.$ 10. $\dfrac{(x+2)^2(2x-11)}{(2x-1)^3}.$

11. $3(3x^2+1)(2x^3+6) + 6x(3x+6)(2x^3+6) + 6x^2(3x+6)(3x^2+1).$

12. $16x(3x^4-2x^2+1)^3(3x^2-1).$ 13. $\dfrac{\sqrt{5}}{2\sqrt{x}}$ 14. $\dfrac{4}{3\sqrt[3]{x}}$ 15. $\dfrac{2x^2+5}{\sqrt{x^2+5}}.$

16. $\dfrac{-1}{|x|\sqrt{3x^2+2x}}.$ 17. $\dfrac{6x+27}{2\sqrt{x+6}}.$ 18. $2nx^{n-1} + \dfrac{m}{3x}\sqrt[3]{x^m}.$

19. $\dfrac{2x}{(1-x^2)\sqrt{1-x^4}}.$ 20. $\dfrac{18}{(6-2x^2)^{3/2}}.$ 21. $\dfrac{-x^{n-1}}{2\sqrt{7-x^n}}.$

22. $\dfrac{-4a^3x}{(x^2+2a^2)^2}.$ 23. $\dfrac{32x(x^2-4)}{(x^2+4)^3}.$ 24. $\dfrac{-(p+q)x^{p+q-1}-px^{p-1}-qx^{q-1}}{(x^p+1)^2(x^q+1)^2}.$

---

25. $12\cos 3x.$ 26. $\cos^3 x - 2\sin^2 x \cos x.$ 27. $\dfrac{-6\cos x}{\sin^3 x}.$ 28. $\frac{1}{2}\sin x.$

29. $\dfrac{\cos 2x}{\sqrt{\sin 2x}}.$ 30. $\sin x(\operatorname{tg}^2 x + 2).$ 31. $2x\operatorname{tg} x(x\sec^2 x + \operatorname{tg} x).$

32. $\dfrac{-4a\sin x}{(1-\cos x)^2}.$ 33. $\dfrac{2x\sec x\operatorname{tg} x + \sec x}{2\sqrt{x}}.$ 34. $\dfrac{-x\sin x - \cos x}{x^2}.$

35. $\dfrac{1}{\cos x - 1}.$ 36. $\dfrac{2\cos x}{(1-\sin x)^2}.$ 37. $\dfrac{-2\cos x}{(1+\sin x)^2}.$ 38. $\dfrac{-1}{\sin^2 x\cos^2 x}.$

39. $\frac{1}{2}x\sin(2x^2+12).$ 40. $\dfrac{4x^2\sin(1-x^2) + \cos(1-x^2)}{2\sqrt{x}}.$

---

41. $\dfrac{1}{\sqrt{4-x^2}}.$ 42. $\dfrac{-4}{\sqrt{1-16x^2}}.$ 43. $\dfrac{1}{2\sqrt{x-x^2}}.$ 44. $\dfrac{1}{|x|\sqrt{x^2-1}}.$

45. $\dfrac{8x}{\sqrt{1-x^4}}.$ 46. $\dfrac{x}{|x|(1+x^2)}.$ 47. $\dfrac{3}{1+9x^2}.$ 48. $\dfrac{-2}{4+x^2}.$

49. $\dfrac{2x}{x^4+4x^2+5}.$ 50. $\dfrac{1}{1+x^2}.$ 51. $\dfrac{-1}{(1+x)\sqrt{x}}.$

52. $\dfrac{x^2}{\sqrt{1-x^2}} + 2x\arcsin x.$

---

53. $1 + \ln x.$ 54. $3x(\ln x^2 + 1).$ 55. $\dfrac{1}{3x}.$ 56. $\dfrac{2}{x}\ln x.$ 57. $\operatorname{cotg} x.$

58. $-\operatorname{tg} x.$ 59. $\dfrac{3}{6x+8}.$ 60. $\dfrac{1}{\sin 2x}.$ 61. $\dfrac{-x}{r^2-x^2}.$ 62. $2\operatorname{tg} x.$

63. $\dfrac{1}{\sqrt{x^2-a^2}}.$ 64. $\sec x.$ 65. $\dfrac{16x}{(x^2+4)(x^2-4)}.$ 66. $\dfrac{2x}{1-x^4}.$ 67. $\dfrac{1}{x\ln x}.$

68. $\dfrac{2}{x} + \operatorname{cotg} x.$ 69. $\dfrac{5x-1}{2(x^2-1)}.$ 70. $\dfrac{4}{(2x+3)\cos^2\{\ln(2x+3)^2\}}.$

---

71. $2e^{\frac{1}{2}x}.$ 72. $-2xe^{-x^2}.$ 73. $3\ln 2 \cdot 2^{3x}.$ 74. $10^{x^2-1}(\ln 10)(2x).$

75. $\dfrac{2x}{9}\cdot 3^{x^2}\ln 3.$ 76. $-(2e)^{-x}\ln(2e).$ 77. $8x(x^2+1)e^{x^2}.$

78. $e^x \left(\dfrac{1}{x} + \ln x \right).$

79. $x^x (1 + \ln x).$ **80.** $\dfrac{x^{\frac{1}{x}}(1 - \ln x)}{x^2}.$ **81.** $x^{\sin x} \left(\dfrac{\sin x}{x} + \cos x \cdot \ln x \right).$

82. $(\ln x)^x \left(\dfrac{1}{\ln x} + \ln \ln x \right).$

83. $\dfrac{1}{\sqrt{x^2 - 1}}.$ **84.** $\dfrac{-1}{(1 + x^2)(\operatorname{arctg} x)^2}.$ **85.** $\dfrac{a}{|x| \sqrt{2ax}} \operatorname{tg} \sqrt{\dfrac{2a}{x}}.$

86. $\dfrac{1}{2\sqrt{x - x^2}\{1 + (\arcsin \sqrt{x})^2\}}.$ **87.** $\dfrac{-2 \ln x}{x}.$ **88.** $\dfrac{2 + \ln x}{2\sqrt{x}} \cdot x^{\sqrt{x}}.$

89. $\left\{ \ln(e^x + 2) + \dfrac{xe^x}{e^x + 2} \right\} (e^x + 2)^x.$ **90.** $\dfrac{x \ln x - (1 + x) \ln(1 + x)}{(x^2 + x) \ln^2 x}.$

91. $y \cdot \dfrac{\ln x - 1}{\ln^2 x}.$ **92.** $y \cdot \dfrac{x - (1 + x) \ln(1 + x)}{x^2(1 + x)}.$ **93.** $\dfrac{1}{\sqrt{1 - x^2}}.$

94. $2b\sqrt{4 - b^2 x^2}.$ **95.** $\dfrac{2|x|}{x\sqrt{1 - x^2}}.$ **96.** $\dfrac{n^2 x^{n-1}}{\sqrt{1 - n^2 x^{2n}}}.$

97. $\dfrac{3 \cos 3x \cos x^2 - 2x \sin 3x \sin x^2}{2}.$ **98.** $\dfrac{-x}{(2 + x^2)\sqrt{1 + x^2}}.$

99. $\dfrac{6x(1 + x^2) - x \sin 6x^2}{(1 + x^2)^2 \cos^2 3x^2}.$ **100.** $\dfrac{-4x^2 - 3x + 2}{3(1 + 2x)^{2/3}(1 + x^2)^{3/2}}.$

101. $\dfrac{-n \cos(n \arccos x)}{\sqrt{1 - x^2}}.$ **102.** $-\sin x \cdot \ln 3 \cdot 3^{\cos x}.$ **103.** $\dfrac{x}{1 + x^2}.$

104. $\dfrac{1}{1 + x^2}.$ **105.** $\dfrac{4x^2 - 2}{\sqrt{1 - x^2}}.$ **106.** $\dfrac{\sqrt{1 + 2x} - 1}{(1 + 2x)\sqrt{1 + 2x}} e^{\sqrt{1 + 2x}}.$

107. $\dfrac{2 + 2\cos^2 x}{\sin 2x}.$ **108.** $x^2 \cos(x^3) \cos^3 x - \sin(x^3) \sin x \cos^2 x.$

109. $\dfrac{2 \ln x}{x} x^{\ln x}.$ **110.** $\dfrac{\operatorname{tg}(2x + 1)}{\sqrt{2x + 1}} + \dfrac{2\sqrt{2x + 1}}{\cos^2(2x + 1)}.$

111. $\dfrac{m}{\sin mx}.$ **112.** $\dfrac{\sqrt{3}}{2(x^2 + x + 1)}.$ **113.** $\dfrac{1}{\sqrt{1 - x^2}} e^{\arcsin x}.$ **114.** $\dfrac{\arcsin x}{(1 - x^2)^{3/2}}.$

115. $\arcsin x.$ **116.** $\operatorname{arctg} \sqrt{x}.$ **117.** $\dfrac{-|x|}{x\sqrt{1 - x^2}}.$ **118.** $\dfrac{1}{(1 - x^2)\sqrt{1 - x^2}}.$

119. $-\dfrac{1}{2}.$ **120.** $\dfrac{(a^2 - b^2) \sin x}{(a^2 + b^2)(1 + \cos^2 x) + 4ab \cos x}.$

GRAFICI

1) 1°) minimo per $x = \frac{4}{3}$, massimo per $x = -2$

2°) nessun valore estremo.

3°) minimo per $x = 2/3$, massimo per $x = 0$

2) 1°) massimo per $x = 4$ 2°) massimo per $x = 0$, minimo per $x = 4$

3) 1°) nessun valore estremo   2°) minimo per $x = -\frac{26}{3}$, massimo per $x = -1$

4) 1°) minimo per $x = 1$, massimo per $x = -2$

2°) massimo per $x = -1$

5) 1°) minimo per $x = 6$, massimo per $x = -6$

2°) minimo per $x = -3$, massimo per $x = 3$

6) 1°) minimo per $x = -1$ 2°) massimo per $x = 1$

7) 1°) minimo per $x = 1/e$ 2°) massimo per $x = e$

8) 1°) nessun valore estremo

2°) minimo per $x = 1$, massimo per $x = -1$

9) 1°) minimo per $x = -\frac{1}{5}\sqrt{5}$, massimo per $x = \frac{1}{5}\sqrt{5}$

2°) minimo per $x = 0$, massimo per $x = \pm\frac{1}{2}\sqrt{2}$

10) 1°) minimo per $x = \frac{1}{2}\sqrt[3]{4}$

2°) minimo per $x = \frac{1}{2}\sqrt[3]{2}$, massimo per $x = -\frac{1}{2}\sqrt[3]{2}$

11) minimo per $x = 3$, massimo per $x = 1$

12) minimo per $x = \pm 1$, massimo per $x = 0$

13) massimo per $x = 0$, minimo per $x = -1$

14) nessun valore estremo

15) $p=2$: massimo per $x=\frac{\pi}{2}$ e $x=\frac{3\pi}{2}$, minimo per $x=\frac{7\pi}{6}$

minimo per $x=\frac{11\pi}{6}$, minimo al bordo per $x=0$

$p=1$ e $p=\frac{1}{2}$: massimo per $x=\frac{\pi}{2}$, minimo per $x=\frac{3\pi}{2}$,

minimo al bordo per $x=0$

16) minimo per $x=0$

17) massimo per $x=\frac{5\pi}{3}+2k\pi$, minimo per $x=\frac{\pi}{3}+2k\pi$

18) massimo per $x=0$, minimo per $x=2$, minimo al bordo per $x=1$, massimo al bordo per $x=6$

19) massimo per $x=\pm\sqrt{2}$, minimo per $x=0$

20) massimo per $x=0$, minimo per $x=\pm\frac{1}{\sqrt{2e}}$

44) minimo per $x=\frac{1}{e}$, massimo per $x=e$, $\lim\limits_{x\to 0^+} f(x)=0$, $\lim\limits_{x\to 0^+} f'(x)=-\infty$, $\lim\limits_{x\to+\infty} f(x)=0$

45) 1) minimo per $x=\frac{1}{2}\ln 2$

2) flesso per $x=\ln 2$

46) $\lim\limits_{x\to 0^+} f(x)=1$, $\lim\limits_{x\to+\infty} f(x)=0$, $\lim\limits_{x\to 0^+} f'(x)=+\infty$, massimo per $x=\frac{1}{e}$