§ 1 Calcolare la funzione primitiva

1. $x^n (n \neq -1)$.
2. $5x^3$.
3. $3\sqrt{x}$. [1])
4. $y\sqrt{2y}$.
5. $6t^{-\frac{2}{3}}$.
6. $x^3 + 4x^2 - 3$.
7. $2x\sqrt{x} - \sqrt[3]{x^2}$.
8. $(1 + x + x^2)^2$.
9. $(z^4 + 1)z^{-\frac{1}{2}}$.
10. $(1 + t^2)\sqrt{t}$.
11. $(x + 2)^7$
12. $(\frac{1}{2}x - 3)^{10}$.
13. $\sqrt[3]{x - 2}$.
14. $(4x + 3)^{\frac{1}{3}}$.
15. $(x + 3)^{-\frac{1}{2}}$.
16. $(2y + 5)^{-\frac{1}{3}}$.
17. $2(z^2 - 6z + 9)^{-1}$.
18. $x\sqrt{a^2 + x^2}$. [2])
19. $x^2\sqrt{2x^3 + 9}$.
20. $x(a^2 - x^2)^{-\frac{1}{2}}$.
21. $\dfrac{3y^2}{(2y^3 - 1)^2}$.
22. $\dfrac{4t}{(5 - 4t^2)\sqrt{5 - 4t^2}}$.

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23. $\dfrac{1}{x}$.
24. $\dfrac{1}{ax + b}$.
25. $\dfrac{\ln^2 x}{x}$.
26. $\dfrac{1}{x \ln x}$.
27. $\dfrac{1}{x \ln^2 x}$.
28. $\dfrac{1}{x \ln x \ln \ln x}$.
29. $\dfrac{2x + 1}{x^2 + x + 1}$.
30. $\dfrac{x}{x^2 - 1}$.
31. $\dfrac{3x}{5 - 4x^2}$.
32. $\dfrac{3x}{(5 - 4x^2)^2}$.

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33. e^x.
34. e^{3x+5}.
35. 2^x
36. 4^{3x+5}.

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37. $a^x e^x$; $(a > 0)$.
38. xe^{x^2}.
39. $4x^2 e^{-x^3}$.
40. $10^{\frac{1}{x^2}} x^{-3}$.
41. $\dfrac{e^x}{\sqrt{e^x + 4}}$.
42. $\dfrac{2^x}{2^x + 3}$.

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43. $\sin x$.
44. $\cos x$.
45. $\cos 4x$.
46. $\sin \pi x \cos \pi x$.
47. $(\sin 2x + \cos 2x)^2$.
48. $\sin 2x \sin x$.
49. $\cos x \cos 2x$.
50. $\cos 2x \sin x$.
51. $\sin 3x \cos x$.
52. $\sin^2 x$.
53. $\cos^2 \dfrac{x}{2}$.
54. $\sin^3 x \cos x$.
55. $\sin^3 x$.
56. $\cos^3 x$.
57. $\operatorname{tg} x$.
58. $\operatorname{cotg} x$.
59. $\dfrac{2 \cos x}{\sin^2 x}$.
60. $\dfrac{\sin \sqrt{x}}{\sqrt{x}}$.
61. $x \cos 2x^2$.
62. $\dfrac{\sin 2x}{4 - \cos^2 x}$.
63. $e^{\cos t} \sin z$.
64. $e^{\sin t \cos t} \cos 2t$.
65. $\dfrac{1}{\cos^2 x}$.
66. $\dfrac{1}{\sin^2 x}$.
67. $\dfrac{1}{3 \cos^2 2x}$.
68. $\dfrac{1}{1 + \cos z}$.

69. $\dfrac{\operatorname{tg} 2z}{\operatorname{cotg} 2z}$.

70. $\dfrac{1}{\sin x}$.

71. $\dfrac{1}{\cos x}$.

72. $\dfrac{1}{\sin 3x}$.

73. $\dfrac{\cos x}{\sin \frac{1}{2}x}$.

74. $\dfrac{\ln \operatorname{tg} \frac{1}{2}x}{\sin x}$.

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75. $\dfrac{1}{x^2+1}$.

76. $\dfrac{1}{a^2+x^2}$.

77. $\dfrac{1}{9x^2+4}$.

78. $\dfrac{1}{(x+1)^2+4}$.

79. $\dfrac{1}{x^2+x+1}$.

80. $\dfrac{x}{(x^2+4)^2+1}$.

81. $\dfrac{x}{x^4+4x^2+14}$.

82. $\dfrac{e^x}{e^{2x}+4}$.

83. $\dfrac{1}{x(9+\ln^2 x)}$.

84. $\dfrac{\cos x}{2+\sin^2 x}$.

85. $\dfrac{1}{x^2+4x+8}$.

86. $\dfrac{2x+4}{x^2+4x+8}$.

87. $\dfrac{2x+6}{x^2+4x+8}$.

88. $\dfrac{x+4}{x^2+4x+8}$.

89. $\dfrac{x+2}{(x^2+4x+8)^3}$.

90. $\dfrac{1}{4x^2-4x+7}$.

91. $\dfrac{8x}{4x^2-4x+7}$.

92. $\dfrac{4x^2}{4x^2-4x+7}$.

93. $\dfrac{z^3}{z^2-z+1}$.

94. $\dfrac{y^2-a^2}{y^2+a^2}$.

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95. $\dfrac{1}{\sqrt{1-x^2}}$.

96. $\dfrac{1}{\sqrt{a^2-x^2}}$, $(a>0)$.

97. $\dfrac{1}{\sqrt{a^2-b^2x^2}}$, $(a>0)$.

98. $(10-4z^2)^{-1/2}$.

99. $\dfrac{1}{\sqrt{10-(2x+1)^2}}$.

100. $\dfrac{1}{\sqrt{2x-x^2}}$.

101. $\dfrac{1}{\sqrt{15+4x-4x^2}}$.

102. $\dfrac{x}{\sqrt{15+4x^2-4x^4}}$.

103. $\dfrac{x}{\sqrt{15-4x-4x^2}}$.

104. $\dfrac{t}{\sqrt{23+12t^2-4t^4}}$.

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105. $\dfrac{1}{\sqrt{x^2+a^2}}$.

106. $\dfrac{1}{\sqrt{x^2-a^2}}$.

107. $\dfrac{1}{\sqrt{x^2+2x+7}}$.

108. $\dfrac{1}{\sqrt{16z^2-24z-7}}$.

109. $\dfrac{x+6}{\sqrt{x^2-5x+1}}$.

110. $\dfrac{3x-1}{\sqrt{x^2+x+1}}$.

§2. Calcolare la primitiva. Integrazione per parti.

1. xe^x.
2. xe^{-x}.
3. x^2e^{2x}.
4. xa^x; $(a > 0)$.
5. x^2a^x; $(a > 0)$.
6. $\ln x$.
7. $x^2 \ln x$.
8. $\ln^2 x$.
9. $x^{-2} \ln x$.
10. $x \cos 2x$.
11. $x^2 \sin 2x$.
12. $x \cos^2 \frac{1}{2}x$.
13. $e^x \sin 2x$.
14. $e^{\frac{x}{3}} \cos x$.
15. $e^{ax} \sin bx$.
16. $e^{ax} \cos bx$.
17. $\dfrac{x}{\cos^2 2x}$.
18. $\dfrac{1}{\cos^3 x}$.
19. $\dfrac{1}{\sin^3 x}$.
20. $\dfrac{\operatorname{tg}^2 x}{\cos x}$.
21. $\operatorname{arctg} x$.
22. $x \operatorname{arctg} x$.
23. $x^2 \operatorname{arctg} \dfrac{x}{2}$.
24. $x^2 \operatorname{arccotg} x$.
25. $\arcsin x$.
26. $\arccos x$.

§3. Calcolare la primitiva. Frazioni parziali.

1. $\dfrac{1}{x^2 - 1}$.
2. $\dfrac{1}{a^2 - x^2}$.
3. $\dfrac{x + 7}{x^2 + 2x - 8}$.
4. $\dfrac{3x + 4}{x^2 + 5x + 6}$.
5. $\dfrac{x - 6}{x^2 - x}$.
6. $\dfrac{x}{(x^2 + 4)^2 - 9}$.
7. $\dfrac{1}{-6 + 5x - x^2}$.
8. $\dfrac{1}{x^3 - 3x^2 + 2x}$.
9. $\dfrac{x^2 + x + 1}{x^2 - 7x + 10}$.
10. $\dfrac{x^3}{(x + 1)(x^2 - 4)}$.
11. $\dfrac{3x - 1}{(2x + 1)(x^2 - 5x + 4)}$.
12. $\dfrac{x^3 + x + 1}{x^4 - 6x^3 + 11x^2 - 6x}$.
13. $\dfrac{3x + 4}{(x + 2)^2(x - 6)}$.
14. $\dfrac{1}{x^2(x^2 - 4)}$.
15. $\dfrac{x^5 - 2}{x^4 - 2x^3}$.
16. $\dfrac{1}{x^4 + 5x^2 + 4}$.
17. $\dfrac{-3x^2 + 7x - 16}{x^3 - 5x^2 + 7x - 3}$.
18. $\dfrac{x^2 + 9x + 29}{(x - 4)(x^2 + 2x + 3)}$.
19. $\dfrac{2x^2 + 6x - 1}{x^3 + x^2 + x}$.
20. $\dfrac{5x^2 - 20x + 1}{(x^2 + 4)(2x^2 + x + 1)}$.
21. $\dfrac{1}{x^3 - 8}$.
22. $\dfrac{1}{x^4 - 16}$.
23. $\dfrac{\sin x}{4 - \cos^2 x}$.
24. $\dfrac{\cos x}{1 - \sin^2 x}$.

§4. Calcolare la primitiva. Funzioni trigonometriche.

1. $\sin^5 x$.
2. $\cos^5 2x$.
3. $\sin x \cos^5 x$.
4. $\sin^3 2z \cos^3 2z$.
5. $\cos^3 x \sqrt{\sin x}$.
6. $\operatorname{tg}^3 x$.
7. $\sin^3 x \cos^4 x$.
8. $\sin^4 x$.
9. $\cos^6 \frac{1}{2}x$.
10. $\sin^2 x \cos^4 x$.
11. $\sin^4 x \cos^2 x$.
12. $\operatorname{tg}^2 x$.
13. $\operatorname{cotg}^4 \frac{1}{2}x$.
14. $\dfrac{\sin^3 x}{\sqrt{\cos x}}$.
15. $\dfrac{\sin^2 x}{\cos^4 x}$.
16. $\dfrac{1}{\cos^4 x}$.
17. $\dfrac{1}{\sin^4 x}$.
18. $\dfrac{1}{\cos^5 x}$.

§5. Calcolare la primitiva. Funzioni irrazionali.

1. $\dfrac{\sqrt{x}}{4+\sqrt{x}}$.
2. $\dfrac{2+\sqrt{x}}{-1+\sqrt{x}}$.
3. $\dfrac{\sqrt{1+y}}{y-3}$.
4. $\dfrac{1}{8+\sqrt{4+t}}$.
5. $\dfrac{1}{\sqrt{x}+\sqrt[3]{x}}$.
6. $\dfrac{x^2}{\sqrt{3+4x}}$.
7. $x\sqrt[3]{3x+4}$.
8. $t^{-2}(t+1)^{1/2}$, $(t>0)$.
9. $\dfrac{1}{\sqrt{a^2+x^2}}$.
10. $\dfrac{x^3}{\sqrt{4+x^2}}$.
11. $\dfrac{1}{x^2\sqrt{a^2+x^2}}$.
12. $\dfrac{1}{z^4\sqrt{5+z^2}}$.
13. $\dfrac{1}{\sqrt{a^2-x^2}}$, $(a>0)$.
14. $\dfrac{t^3}{\sqrt{a^2-t^2}}$.
15. $x^2\sqrt{3-x^2}$.
16. $x^3\sqrt{9-x^2}$.
17. $\dfrac{\sqrt{4-9x^2}}{x^2}$.
18. $\dfrac{1}{x\sqrt{4-x^2}}$.
19. $\dfrac{1}{\sqrt{x^2-a^2}}$.
20. $\dfrac{z^3}{\sqrt{z^2-4}}$.
21. $\dfrac{\sqrt{x^2-4}}{x}$.
22. $x^3\sqrt{x^2-6}$.
23. $\dfrac{1}{x\sqrt{x^2-a^2}}$.
24. $\dfrac{1}{x^2\sqrt{x^2-16}}$.
25. $\dfrac{1}{x^3\sqrt{x^2-8}}$.
26. $z\sqrt{3-2z-z^2}$.
27. $x^5\sqrt{2x^3+4}$.
28. $x^3\sqrt{x^2+6}$.
29. $\dfrac{t^2}{\sqrt{(a^2-t^2)^3}}$, $(a>0)$.
30. $\dfrac{\sqrt{x-4}}{x^{3/2}}$.
31. $\sqrt{4-\sqrt{x}}$.
32. $\dfrac{e^x(e^x-4)}{e^x+2}$.
33. $\dfrac{1}{\sin x - 2\cos x - 2}$.
34. $\dfrac{1}{2-\sin x}$.

§ 6. Calcolare la primitiva. Esercizi vari.

1. $\dfrac{1}{x\sqrt{-4+\ln^2 x}}$.
2. $\dfrac{1}{x\sqrt{4+\ln^2 x}}$.
3. $\dfrac{1}{x\sqrt{4-\ln^2 x}}$.
4. $\dfrac{3x}{(4-2x^2)^2}$.
5. $\dfrac{\cos x}{e^{\sin x}}$.
6. $\dfrac{2x+4}{\sqrt{1-4x^2}}$.
7. $\dfrac{\sqrt{3+\ln x}}{x}$.
8. $\dfrac{x+4}{\sqrt{12x-4x^2}}$
9. $\dfrac{2}{x^{3/4}(x^{1/4}+3)}$.
10. $\dfrac{2x-7}{\sqrt{3-6x-9x^2}}$
11. $\dfrac{1}{\sin x \operatorname{tg} x}$.
12. $\dfrac{2^{\operatorname{arctg} x}}{x^2+1}$.
13. $\dfrac{\cos x-\sin x}{\cos x+\sin x}$.
14. $\dfrac{\sin \frac{1}{2}x}{3+4\cos \frac{1}{2}x}$.
15. $\dfrac{2^x}{\sqrt{9-4^x}}$.
16. $\dfrac{\sin x}{\sqrt{16-\cos^2 x}}$.
17. $\dfrac{2x^2+4x+3}{x+2}$.
18. $\dfrac{6x-1}{\sqrt{4x^2-4x+10}}$.
19. $(1+\operatorname{tg} x)^2$.
20. $(1+\sin^2 x)^2 \sin 2x$.
21. $(2a)^{5x}$, $(a>0)$.
22. $x^{-1/2}e^{\sqrt{x}}$.
23. $(2x+e^x)^2$.
24. $x^n \ln x$.
25. $\dfrac{e^{\frac{x}{2}}}{e^x-1}$.
26. $\dfrac{3x+4\sin 2x}{\cos^2 x}$.
27. $\dfrac{1}{(x^2-4)\sqrt{x^2-1}}$.
28. $\dfrac{1}{(x-2)\sqrt{x^2-1}}$.
29. $\dfrac{1}{(x+1)\sqrt{2x^2+x-1}}$.
30. $\dfrac{1}{(x-b)\sqrt{x^2+a^2}}$.
31. $\dfrac{1}{(4x-3)\sqrt{x^2+1}}$.
32. $\dfrac{\ln x}{(x+1)\sqrt{x+1}}$.
33. $\dfrac{1}{x}\sqrt{\dfrac{x+4}{x+1}}$.
34. $\dfrac{1}{x^3\sqrt{1-x^2}}$.
35. $\dfrac{a+\sqrt{a-x}}{x}$, $(a>0)$.
36. $\sin x\sqrt{\dfrac{1-\cos x}{2+\cos x}}$.
37. $\dfrac{\sqrt{x}}{(1+\sqrt[3]{x})^2}$.
38. $\dfrac{1}{(1+\sqrt[3]{x})^2\sqrt{x}}$.
39. $\dfrac{x^3}{\sqrt{2x-x^2}}$.
40. $\dfrac{a+\sqrt{a^2-x^2}}{x^2}$, $(a>0)$.
41. $\dfrac{1}{(1+x^2)^{5/2}}$.
42. $\dfrac{\operatorname{arctg} x}{(1+x^2)^{3/2}}$.
43. $\dfrac{\operatorname{arctg} x}{x^3}$.
44. $\dfrac{1}{\sqrt{x}}\operatorname{arctg} x$.
45. $\dfrac{x\ln x}{\sqrt{x^2+1}}$.
46. $\dfrac{72}{(x-1)(x+5)^2}$.
47. $\dfrac{5x^2+2x+2}{x^3-1}$.
48. $\dfrac{1}{(x-1)^2(x+1)}$.

49. $\dfrac{1}{(1+x)^2(1-x^3)}$.

50. $\dfrac{x^4}{x^3+1}$.

51. $\dfrac{1}{(x^2-x+1)^2}$.

52. $\dfrac{1}{x^4+x^3+x+1}$.

53. $\dfrac{x^4-1}{x^4+1}$.

54. $\dfrac{1}{x^4+a^4}$.

55. $\dfrac{\cos 4x + 2\cos 2x}{\cos^2 x}$.

56. $\dfrac{1}{2\sin^2 x - 3\cos x}$.

57. $\dfrac{\sin^n x}{\cos^{n+2} x}$.

58. $\dfrac{\sin x}{\sin^3 x + \cos^3 x}$.

59. $xe^x \sin x$.

60. $\sin 2x \ln \sin x$.

61. $\dfrac{\ln(1+x^2)}{x^3}$.

62. $\dfrac{\ln x}{\sqrt{x+1}}$.

63. $\dfrac{1}{(1-x^2)^{3/2}}$.

64. $\dfrac{\operatorname{arcsin} x}{(1-x^2)^{3/2}}$.

65. $\dfrac{e^{2\operatorname{arctg} x}}{\sqrt{(1+x^2)^3}}$.

66. $\dfrac{x}{\sqrt{1+x^2}} \operatorname{arctg} x$.

67. $\dfrac{e^{2x}}{\sqrt{4-e^{2x}}}$.

68. $\dfrac{(1+x)e^x}{1+x^2e^{2x}}$.

69. $\dfrac{x^3 \operatorname{arcsin} x}{\sqrt{1-x^4}}$.

70. $\sin x \sin 2x \sin 3x$.

71. $\dfrac{1}{\sqrt{(x-a)(x-b)}}$; $(a<b<x)$.

72. $\dfrac{1}{\sqrt{(a-x)(b-x)}}$; $(x<a<b)$.

73. $\dfrac{1}{\sqrt{(x-a)(b-x)}}$; $(a<x<b)$.

74. $\dfrac{1+\sqrt{x}}{\sqrt{1+x}}$.

§7. Calcolare i seguenti integrali

16. $\int_1^4 (x^2+1)dx$.

17. $\int_1^5 \frac{5}{x} dx$.

18. $\int_{-2}^{2} (4-x^2)dx$.

19. $\int_0^5 (5+4x-x^2)dx$.

20. $\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$.

21. $\int_0^{\frac{\pi}{2}} \sin^2 x\, dx$.

22. $\int_0^2 (4x-x^3)dx$.

23. $\int_0^9 2\sqrt{x}\, dx$.

24. $\int_1^e \ln x\, dx$.

25. $\int_0^2 \frac{8}{x^2+4} dx$.

26. $\int_{-1}^{3} (x^3-7x^2+7x+15)dx$.

27. $\int_1^2 \frac{dx}{x(1+x^2)}$.

28. $\int_{\frac{1}{e}}^{e} \frac{dx}{x(1+\ln^2 x)}$.

29. $\int_0^4 x\sqrt{16-x^2}\, dx$.

30. $\int_0^a \sqrt{a^2-x^2}\, dx$; $(a>0)$.

31. $\int_{-2}^{2} 3\sqrt{4-x^2}\, dx$.

32. $\int_{-3}^{1} |x|\, dx$.

33. $\int_0^2 \sqrt{(x-1)^2}\, dx$.

36. $\int_1^2 \left(\frac{\sqrt{x}}{x^2} - \frac{2}{x}\sqrt{2x}\right) dx.$

37. $\int_0^1 x^3 e^x\, dx.$

38. $\int_0^{\frac{\pi}{2}} \sin^2 7x\, dx.$

39. $\int_1^{64} \frac{dx}{(\sqrt{x} + \sqrt[3]{x})(1 + \sqrt[6]{x})}.$

40. $\int_0^1 \frac{x^3 + 3x - 1}{(x+1)(x^2 - 2x + 2)} dx.$

41. $\int_0^{\frac{\pi}{2}} e^{\sin x} \sin 2x\, dx.$

42. $\int_0^1 \frac{x}{1 + \sqrt{x}} dx.$

43. $\int_0^1 e^{x^2}(1 + 2x^2) dx.$

44. $\int_0^2 x \ln (1 + x^2) dx.$

45. $\int_0^1 \frac{\arcsin \frac{x}{2}}{\sqrt{4 - x^2}} dx.$

46. $\int_0^{13} \frac{1 + \sqrt[3]{2x+1}}{2x + 1} dx.$

47. $\int_{\frac{\pi}{2}}^{\frac{2}{3}\pi} \cos x \ln (\operatorname{tg} \tfrac{1}{2}x)\, dx.$

48. $\int_0^{\frac{2}{3}\pi} \frac{dx}{2 + \cos x}.$

49. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sin x \sin 2x\, dx.$

50. $\int_0^{\frac{\pi}{6}} \frac{\operatorname{tg} x}{\cos 2x} dx.$

51. $\int_0^{\sqrt{\pi}} x \cos (x^2) dx.$

52. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{\sin x(\sin x - \cos x)}.$

53. $\int_0^{\frac{\pi}{4}} \frac{1 + \sin 2x}{\sin^2 x + 3\cos^2 x} dx.$

54. $\int_1^2 \frac{dx}{x(1+x^n)}, \quad (n \neq 0).$

55. $\int_0^{\frac{1}{2}\sqrt{2}} \sqrt{\frac{1+x}{1-x}}\, dx.$

56. $\int_0^1 \sqrt{x - x^2}\, dx.$

57. $\int_{-1}^1 \sqrt{x^4 + x^2}\, dx.$

58. $\int_2^3 \frac{dx}{\sqrt[3]{x^2} - x}.$

59. $\int_0^1 x^n (1 - x)^n\, dx,$

60. $\int_0^{\frac{\pi}{4}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}.$

61. $\int_0^1 (1 - t^3)^{50} t^5\, dt.$

62. $\int_0^1 \frac{x}{1 + x^4} dx.$

63. $\int_0^{\frac{\pi}{2}} \sin^{79} x \cos^3 x\, dx.$

64. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{x \arcsin x}{\sqrt{1 - x^2}} dx.$

65. $\int_0^1 x^7 (1 - x)^{80} dx.$

66. $\int_0^{\frac{\pi}{6}} \sin^5 x \cos^3 x\, dx.$

67. $\int_{\operatorname{arctg} \frac{2}{4}}^{\operatorname{arctg} \frac{4}{3}} \frac{dx}{5 + 3\cos x + 4 \sin x}.$

9d. Integrali generalizzati

Da calcolare.

1. $\int_0^{\infty} \frac{dx}{(x+1)^2}$.
2. $\int_1^{\infty} \frac{dx}{x^2}$.
3. $\int_0^{\infty} xe^{-x^2}dx$.
4. $\int_{-\infty}^{0} e^x dx$.
5. $\int_0^{\infty} \frac{dx}{x^2+1}$.
6. $\int_2^{\infty} \frac{dx}{x(x-1)}$.
7. $\int_4^{\infty} \frac{dx}{x\sqrt{x^2-4}}$.
8. $\int_1^{\infty} \frac{\sqrt{x}}{e^{\sqrt{x}}} dx$.
9. $\int_{\sqrt{3}}^{\infty} \frac{x^2-3}{e^x} dx$.
10. $\int_{e\sqrt{2}}^{\infty} \frac{\ln^2 x - 2}{x^2} dx$.
11. $\int_0^{\infty} \frac{x}{(1+x^2)^{3/2}} dx$.
12. $\int_1^{\infty} \frac{\sqrt{x}}{(1+x)^2} dx$.
15. $\int_0^1 \frac{dx}{\sqrt{1-x}}$.
16. $\int_0^1 \ln x\, dx$.
17. $\int_0^1 \frac{dx}{\sqrt{x}}$.
18. $\int_0^3 \frac{x}{\sqrt{9-x^2}} dx$.
19. $\int_{-3}^{3} \frac{t}{\sqrt{9-t^2}} dt$.
20. $\int_0^a \frac{dx}{\sqrt{a^2-x^2}}$, $(a>0)$.
21. $\int_a^{2a} \frac{dz}{\sqrt{z^2-a^2}}$.
22. $\int_0^a \frac{x^2 dx}{\sqrt{a^2-x^2}}$, $(a>0)$.
23. $\int_0^4 \frac{dx}{\sqrt{8x-x^2}}$.
24. $\int_0^{\infty} \frac{dx}{x^2 e^{\frac{1}{x}}}$.
25. $\int_0^{\infty} \frac{\ln(1+x^2)}{x^2} dx$.
26. $\int_{-\infty}^{\infty} \frac{e^{-\frac{1}{|x|}}}{x^2} dx$.
27. $\int_{1/2}^1 \frac{dx}{x^3\sqrt{1-x^2}}$.
28. $\int_{-1}^{1} \frac{1}{\sqrt[3]{x}} dx$.

29. $\int_0^2 \frac{dx}{(x-1)^{2/3}}$.

30. $\int_{-1}^{3} \frac{dx}{\sqrt{|x^2-2x|}}$.

Determinare se sono convergenti o divergenti:

31. 1°. $\int_1^\infty \frac{dx}{x+\sqrt{x}+\cos x}$; 2°. $\int_1^\infty \frac{dx}{x+x\sqrt{x}+\cos x}$;

3°. $\int_0^1 \frac{dx}{x+\sqrt{x}+\operatorname{tg} x}$; 4°. $\int_0^1 \frac{dx}{x+x\sqrt{x}+\operatorname{tg} x}$.

Calcolare

36. $\int_0^\infty \frac{e^{-x}}{e^{-2x}+1}\,dx$.

37. $\int_0^\infty e^{-x}(x^2+x)\,dx$.

38. $\int_0^\infty e^{x-e^x}\,dx$.

39. $\int_a^\infty \frac{dx}{x^4\sqrt{a^2+x^2}}$, $(a>0)$.

40. $\int_1^\infty \frac{dt}{(1+t)\,t\sqrt{t}}$.

41. $\int_0^\infty \frac{\ln x}{(1+x)^2}\,dx$.

42. $\int_0^\infty \frac{dx}{x^3+1}$.

43. $\int_0^\infty e^{-x}\cos x\,dx$.

44. $\int_{-\infty}^\infty 2^{x-2^x}\,dx$.

45. $\int_1^\infty \frac{dx}{x\sqrt[3]{x^2-1}}$.

46. $\int_0^{\frac{1}{2}\pi} \left(\frac{1}{x}-\frac{1}{\sin x}\right)dx$.

47. $\int_2^\infty \frac{\sqrt{x^2-1}-x}{x^2-1}\,dx$.

48. $\int_0^1 \frac{1-\sqrt{1-t^2}}{t\sqrt{1-t^2}}\,dt$.

49. $\int_1^\infty \left(\frac{1}{x}-\frac{\sqrt{x^2-x}}{x^2}\right)dx$.

50. $\int_0^{\frac{1}{2}\pi} \left(\frac{1}{x}-\frac{1}{\sin x\sqrt{\cos x}}\right)dx$.

51. $\int_1^\infty \left(1-x\operatorname{arctg}\frac{1}{x}\right)dx$.

52. $\int_0^\infty \frac{7x-3}{x^3-3x^2+x+5}\,dx$.

53. $\int_0^1 \frac{x-\operatorname{arctg} x}{x^3}\,dx$.

54. $\int_2^\infty \frac{dx}{(x+3)\sqrt{x^2-4}}$.

55. $\int_{\frac{b^2}{a}}^\infty \frac{dx}{x\sqrt{ax-b^2}}$; $(a>0)$.

56. $\int_0^\infty \frac{dx}{(x+\sqrt{x^2+1})^n}$, $(n>1)$.

57. $\int_0^\infty (\sqrt{x^2+1}-x)^3\,dx$.

58. $\int_b^\infty \frac{dx}{(x+a)\sqrt{x-b}}$, $(a+b>0)$.

59. $\int_1^2 \frac{dx}{(x+1)\sqrt{x^2-1}}$.

60. $\int_0^\infty \frac{x}{(1+x)^3}\,dx$.

61. $\int_0^1 \frac{x^3\arcsin x}{\sqrt{1-x^2}}\,dx$.

10

62. $\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}$, $(a<b)$.

63. $\int_a^b \frac{x\,dx}{\sqrt{(x-a)(b-x)}}$, $(a<b)$.

64. $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$, $(a>0,\ b>0)$.

65. $\int_0^\infty \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$, $(a>0,\ b>0)$.

66. $\int_0^\infty e^{-x} x^n dx$. $n = 1, 2, 3, \ldots$

67. $\int_0^\infty \frac{\operatorname{arctg} x}{(1+x)^2}\,dx$.

68. $\int_0^1 x^2 \ln(1-x)\,dx$.

69. $\int_0^1 \frac{x \ln x}{\sqrt{1-x^2}}\,dx$.

70. $\int_0^{\frac{1}{4}\pi} \frac{\ln(1-\operatorname{tg}^2 x)}{\sin^2 x}\,dx$.

71. $\int_0^{2\pi} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x}$.

72. $\int_{-\infty}^{\infty} \frac{dt}{(1+e^{|t|})^2}$.

73. Calcolare per $a > 0$:

1°. $\int_0^\infty e^{-ax} \sin bx\,dx$;

2°. $\int_0^\infty e^{-ax} \cos bx\,dx$;

3°. $\int_0^\infty x^2 e^{-ax} \sin bx\,dx$.

RISULTATI

§ 1.

1. $\frac{x^{n+1}}{n+1} + C.$
2. $\frac{5}{4}x^4 + C.$
3. $2x\sqrt{x} + C.$
4. $\frac{2}{5}y^2\sqrt{2y} + C.$
5. $18\sqrt[3]{t} + C.$
6. $\frac{x^4}{4} + \frac{4}{3}x^3 - 3x + C.$
7. $\frac{4}{5}x^2\sqrt{x} - \frac{3}{5}x\sqrt[3]{x^2} + C.$
8. $\frac{x^5}{5} + \frac{x^4}{2} + x^3 + x^2 + x + C.$
9. $\frac{2}{9}z^{9/2} + 2z^{1/2} + C.$
10. $\frac{2}{3}t^{3/2} + \frac{2}{7}t^{7/2} + C.$
11. $\frac{1}{8}(x+2)^8 + C.$
12. $\frac{2}{11}(\frac{1}{2}x - 3)^{11} + C.$
13. $\frac{3}{4}(x-2)^{4/3} + C.$
14. $\frac{3}{16}(4x+3)^{4/3} + C.$
15. $2\sqrt{x+3} + C.$
16. $\frac{3}{4}(2y+5)^{2/3} + C.$
17. $-\frac{2}{z-3} + C.$
18. $\frac{1}{3}(a^2+x^2)^{3/2} + C.$
19. $\frac{1}{9}(2x^3+9)^{3/2} + C.$
20. $-\sqrt{a^2-x^2} + C.$
21. $-\frac{1}{2(2y^3-1)} + C.$
22. $\frac{1}{\sqrt{5-4t^2}} + C.$

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23. $\ln|x| + C.$
24. $\frac{1}{a}\ln|ax+b| + C.$
25. $\frac{1}{3}\ln^3 x + C.$
26. $\ln|\ln x| + C.$
27. $-\frac{1}{\ln x} + C.$
28. $\ln|\ln\ln x| + C.$
29. $\ln(x^2+x+1) + C.$
30. $\frac{1}{2}\ln|x^2-1| + C.$
31. $-\frac{3}{8}\ln|4x^2-5| + C.$
32. $\frac{3}{8(5-4x^2)} + C.$

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33. $e^x + C.$
34. $\frac{1}{3}e^{3x+5} + C.$
35. $\frac{2^x}{\ln 2} + C.$
36. $\frac{4^{3x+5}}{3\ln 4} + C.$
37. $\frac{(ae)^x}{1+\ln a} + C.$
38. $\frac{1}{2}e^{x^2} + C.$
39. $-\frac{4}{3}e^{-x^3} + C.$
40. $-\frac{10^{\frac{1}{x^3}}}{2\ln 10} + C.$
41. $2\sqrt{e^x+4} + C.$
42. $\frac{\ln(2^x+3)}{\ln 2} + C.$

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43. $-\cos x + C.$
44. $\sin x + C.$
45. $\frac{1}{4}\sin 4x + C.$
46. $\frac{1}{2\pi}\sin^2 \pi x + C.$
47. $x + \frac{1}{2}\sin^2 2x + C.$
48. $\frac{2}{3}\sin^3 x + C.$
49. $\sin x - \frac{2}{3}\sin^3 x + C.$
50. $\cos x - \frac{2}{3}\cos^3 x + C.$
51. $\frac{3}{2}\sin^2 x - \sin^4 x + C.$
52. $\frac{1}{2}x - \frac{1}{4}\sin 2x + C.$
53. $\frac{1}{2}(x + \sin x) + C.$
54. $\frac{1}{4}\sin^4 x + C.$
55. $\frac{1}{3}\cos^3 x - \cos x + C.$
56. $\sin x - \frac{1}{3}\sin^3 x + C.$
57. $-\ln|\cos x| + C.$
58. $\ln|\sin x| + C.$
59. $-\frac{2}{\sin x} + C.$
60. $-2\cos\sqrt{x} + C.$
61. $\frac{1}{4}\sin 2x^2 + C.$
62. $\ln(4 - \cos^2 x) + C.$
63. $-e^{\cos x} + C.$
64. $e^{\sin t \cos t} + C.$

65. $\operatorname{tg} x + C$.
66. $-\operatorname{cotg} x + C$.
67. $\frac{1}{6}\operatorname{tg} 2x + C$.
68. $\operatorname{tg} \frac{1}{2}z + C$.
69. $\frac{1}{2}\operatorname{tg} 2z - z + C$.
70. $\ln\left|\operatorname{tg} \frac{1}{2}x\right| + C$.
71. $\ln\left|\operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C$.
72. $\frac{1}{3}\ln\left|\operatorname{tg}\frac{3x}{2}\right| + C$.
73. $2\ln\left|\operatorname{tg}\frac{x}{4}\right| + 4\cos\frac{x}{2} + C$.
74. $\frac{1}{2}\ln^2 \operatorname{tg} \frac{1}{2}x + C$.

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75. $\operatorname{arctg} x + C$.
76. $\frac{1}{a}\operatorname{arctg}\frac{x}{a} + C$.
77. $\frac{1}{6}\operatorname{arctg}\frac{3x}{2} + C$.
78. $\frac{1}{2}\operatorname{arctg}\frac{x+1}{2} + C$.
79. $\frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}} + C$.
80. $\frac{1}{2}\operatorname{arctg}(x^2+4) + C$.
81. $\frac{1}{2\sqrt{10}}\operatorname{arctg}\frac{x^2+2}{\sqrt{10}} + C$.
82. $\frac{1}{2}\operatorname{arctg}\frac{e^x}{2} + C$.
83. $\frac{1}{3}\operatorname{arctg}\frac{\ln x}{3} + C$.
84. $\frac{1}{\sqrt{2}}\operatorname{arctg}\frac{\sin x}{\sqrt{2}} + C$.
85. $\frac{1}{2}\operatorname{arctg}\frac{x+2}{2} + C$.
86. $\ln(x^2+4x+8) + C$.
87. $\ln(x^2+4x+8) + \operatorname{arctg}\frac{x+2}{2} + C$.
88. $\frac{1}{2}\ln(x^2+4x+8) + \operatorname{arctg}\frac{x+2}{2} + C$.
89. $-\frac{1}{4}(x^2+4x+8)^{-2} + C$.
90. $\frac{1}{2\sqrt{6}}\operatorname{arctg}\frac{2x-1}{\sqrt{6}} + C$.
91. $\ln(4x^2-4x+7) + \frac{2}{\sqrt{6}}\operatorname{arctg}\frac{2x-1}{\sqrt{6}} + C$.
92. $x + \frac{1}{2}\ln(4x^2-4x+7) - \frac{5}{2\sqrt{6}}\operatorname{arctg}\frac{2x-1}{\sqrt{6}} + C$.
93. $\frac{1}{2}z^2 + z - \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2z-1}{\sqrt{3}} + C$.
94. $y - 2a\operatorname{arctg}\frac{y}{a} + C$.

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95. $\arcsin x + C$.
96. $\arcsin\frac{x}{a} + C$.
97. $\frac{1}{b}\arcsin\frac{bx}{a} + C$.
98. $\frac{1}{2}\arcsin\frac{2z}{\sqrt{10}} + C$.
99. $\frac{1}{2}\arcsin\frac{2x+1}{\sqrt{10}} + C$.
100. $\arcsin(x-1) + C$.
101. $\frac{1}{2}\arcsin\frac{2x-1}{4} + C$.
102. $\frac{1}{4}\arcsin\frac{2x^2-1}{4} + C$.
103. $-\frac{1}{4}\sqrt{15-4x-4x^2} - \frac{1}{4}\arcsin\frac{2x+1}{4} + C$.
104. $\frac{1}{4}\arcsin\frac{2t^2-3}{4\sqrt{2}} + C$.

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105. $\ln(x + \sqrt{x^2+a^2}) + C$.
106. $\ln\left|x + \sqrt{x^2-a^2}\right| + C$.
107. $\ln(x + 1 + \sqrt{x^2+2x+7}) + C$.
108. $\frac{1}{4}\ln\left|4z - 3 + \sqrt{16z^2-24z-7}\right| + C$.
109. $\sqrt{x^2-5x+1} + \frac{17}{2}\ln\left|x - \frac{5}{2} + \sqrt{x^2-5x+1}\right| + C$.
110. $3\sqrt{x^2+x+1} - \frac{5}{2}\ln(x + \frac{1}{2} + \sqrt{x^2+x+1}) + C$.

§ 2.

1. $xe^x - e^x + C.$
2. $-e^{-x}(x+1) + C.$
3. $\frac{e^{2x}}{4}(2x^2 - 2x + 1) + C.$
4. $\frac{x a^x}{\ln a} - \frac{a^x}{\ln^2 a} + C.$
5. $\frac{x^2 a^x}{\ln a} - \frac{2x a^x}{\ln^2 a} + \frac{2a^x}{\ln^3 a} + C.$
6. $x \ln x - x + C.$
7. $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C.$
8. $x \ln^2 x - 2x \ln x + 2x + C.$
9. $-\frac{\ln x}{x} - \frac{1}{x} + C.$
10. $\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C.$
11. $-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C.$
12. $\frac{1}{4}x^2 + \frac{1}{2}(x \sin x + \cos x) + C.$
13. $\frac{1}{5}e^x(\sin 2x - 2\cos 2x) + C.$
14. $\frac{9}{10}e^{\frac{x}{3}}(\frac{1}{3}\cos x + \sin x) + C.$
15. $\frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2} + C.$
16. $\frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2} + C.$
17. $\frac{1}{2}x \operatorname{tg} 2x + \frac{1}{4}\ln|\cos 2x| + C.$
18. $\frac{\operatorname{tg} x}{2\cos x} + \frac{1}{2}\ln\left|\operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C.$
19. $-\frac{\operatorname{cotg} x}{2\sin x} + \frac{1}{2}\ln\left|\operatorname{tg}\frac{x}{2}\right| + C.$
20. $\frac{\operatorname{tg} x}{2\cos x} - \frac{1}{2}\ln\left|\operatorname{tg}\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C.$
21. $x \operatorname{arctg} x - \frac{1}{2}\ln(1 + x^2) + C.$
22. $\frac{1}{2}(x^2 + 1)\operatorname{arctg} x - \frac{1}{2}x + C.$
23. $\frac{x^3}{3}\operatorname{arctg}\frac{x}{2} - \frac{1}{3}x^2 + \frac{4}{3}\ln(x^2 + 4) + C.$
24. $\frac{1}{3}x^3 \operatorname{arccotg} x + \frac{1}{6}x^2 - \frac{1}{6}\ln(1 + x^2) + C.$
25. $x \arcsin x + \sqrt{1 - x^2} + C.$
26. $x \arccos x - \sqrt{1 - x^2} + C.$

§ 3.

1. $\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C.$
2. $\frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C.$
3. $\frac{3}{2}\ln|x-2| - \frac{1}{2}\ln|x+4| + C.$
4. $5\ln|x+3| - 2\ln|x+2| + C.$
5. $6\ln|x| - 5\ln|x-1| + C.$
6. $\frac{1}{12}\ln\frac{x^2+1}{x^2+7} + C.$
7. $\ln\left|\frac{x-2}{x-3}\right| + C.$
8. $\frac{1}{2}\ln|x| - \ln|x-1| + \frac{1}{2}\ln|x-2| + C.$
9. $x + \frac{31}{3}\ln|x-5| - \frac{7}{3}\ln|x-2| + C.$
10. $x + \frac{1}{3}\ln|x+1| + \frac{2}{3}\ln|x-2| - 2\ln|x+2| + C.$
11. $\frac{11}{27}\ln|x-4| - \frac{5}{27}\ln|2x+1| - \frac{2}{9}\ln|x-1| + C.$
12. $-\frac{1}{6}\ln|x| + \frac{3}{2}\ln|x-1| - \frac{11}{2}\ln|x-2| + \frac{31}{6}\ln|x-3| + C.$

13. $-\frac{1}{4(x+2)}+\frac{11}{32}\ln\left|\frac{x-6}{x+2}\right|+C.$

14. $\frac{1}{16}\ln\left|\frac{x-2}{x+2}\right|+\frac{1}{4x}+C.$

15. $\frac{1}{2}x^2+2x-\frac{1}{2}x^{-1}-\frac{1}{2}x^{-2}+\frac{1}{4}\ln|x|+\frac{15}{4}\ln|x-2|+C.$

16. $\frac{1}{3}\operatorname{arctg} x-\frac{1}{6}\operatorname{arctg}\frac{x}{2}+C.$

17. $\frac{5}{2}\ln|x-1|-\frac{11}{2}\ln|x-3|-\frac{6}{x-1}+C.$

18. $3\ln|x-4|-\ln(x^2+2x+3)-\frac{3}{\sqrt{2}}\operatorname{arctg}\frac{x+1}{\sqrt{2}}+C.$

19. $\frac{3}{2}\ln(x^2+x+1)-\ln|x|+\frac{11}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}+C.$

20. $\frac{3}{2}\ln\frac{x^2+4}{2x^2+x+1}+\frac{1}{2}\operatorname{arctg}\frac{x}{2}+\frac{3}{\sqrt{7}}\operatorname{arctg}\frac{4x+1}{\sqrt{7}}+C.$

21. $\frac{1}{12}\ln|x-2|-\frac{1}{24}\ln(x^2+2x+4)-\frac{\sqrt{3}}{12}\operatorname{arctg}\frac{x+1}{\sqrt{3}}+C.$

22. $\frac{1}{32}\ln\left|\frac{x-2}{x+2}\right|-\frac{1}{16}\operatorname{arctg}\frac{x}{2}+C.$

23. $\frac{1}{4}\ln\frac{2-\cos x}{2+\cos x}+C.$

24. $\frac{1}{2}\ln\frac{1+\sin x}{1-\sin x}+C=\ln\left|\operatorname{tg}\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+C.$

§ 4.

1. $-\cos x+\frac{2}{3}\cos^3 x-\frac{1}{5}\cos^5 x+C.$
2. $\frac{1}{2}\sin 2x-\frac{1}{3}\sin^3 2x+\frac{1}{10}\sin^5 2x+C.$
3. $-\frac{1}{6}\cos^6 x+C.$
4. $\frac{1}{8}\sin^4 2z-\frac{1}{12}\sin^6 2z+C.$
5. $\frac{2}{5}(\sin x)^{5/2}-\frac{2}{7}(\sin x)^{7/2}+C.$
6. $\frac{1}{2\cos^2 x}+\ln|\cos x|+C.$
7. $-\frac{1}{5}\cos^5 x+\frac{1}{7}\cos^7 x+C.$
8. $\frac{3}{8}x-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C.$
9. $\frac{5}{16}x+\frac{1}{2}\sin x+\frac{3}{32}\sin 2x-\frac{1}{24}\sin^3 x+C.$
10. $\frac{1}{16}x-\frac{1}{64}\sin 4x+\frac{1}{48}\sin^3 2x+C.$
11. $\frac{1}{16}x-\frac{1}{64}\sin 4x-\frac{1}{48}\sin^3 2x+C.$
12. $-x+\operatorname{tg} x+C.$
13. $x+2\operatorname{cotg}\frac{1}{2}x-\frac{2}{3}\operatorname{cotg}^3\frac{1}{2}x+C.$
14. $-2\sqrt{\cos x}+\frac{2}{5}(\cos x)^{5/2}+C.$
15. $\frac{1}{3}\operatorname{tg}^3 x+C.$
16. $\operatorname{tg} x+\frac{1}{3}\operatorname{tg}^3 x+C.$
17. $-\operatorname{cotg} x-\frac{1}{3}\operatorname{cotg}^3 x+C.$
18. $\frac{\sin x}{4\cos^4 x}+\frac{3\sin x}{8\cos^2 x}+\frac{3}{8}\ln\left|\operatorname{tg}\left(\frac{x}{2}+\frac{\pi}{4}\right)\right|+C.$

§ 5.

1. $x-8\sqrt{x}+32\ln(4+\sqrt{x})+C.$
2. $x+6\sqrt{x}+6\ln|-1+\sqrt{x}|+C.$
3. $2\sqrt{1+y}+2\ln\left|\frac{-2+\sqrt{1+y}}{2+\sqrt{1+y}}\right|+C.$
4. $2\sqrt{t+4}-16\ln(8+\sqrt{t+4})+C.$
5. $2x^{1/2}-3x^{1/3}+6x^{1/6}-6\ln(1+x^{1/6})+C.$
6. $\frac{1}{32}\{\frac{1}{5}(3+4x)^{5/2}-2(3+4x)^{3/2}+9(3+4x)^{1/2}\}+C.$
7. $\frac{1}{7}(x-1)(3x+4)^{4/3}+C.$

8. $\ln \dfrac{\sqrt{t+1}-1}{\sqrt{t}} - \dfrac{\sqrt{t+1}}{t} + C.$

9. $\ln (x + \sqrt{x^2+a^2}) + C.$ 10. $\frac{1}{3}(x^2-8)\sqrt{x^2+4} + C.$

11. $-\dfrac{\sqrt{a^2+x^2}}{a^2x} + C.$ 12. $\dfrac{(2z^2-5)\sqrt{z^2+5}}{75z^3} + C.$

13. $\arcsin \dfrac{x}{a} + C.$ 14. $-\frac{1}{3}(t^2+2a^2)\sqrt{a^2-t^2} + C.$

15. $\frac{9}{8}\arcsin \dfrac{x}{\sqrt{3}} + \frac{1}{8}x(2x^2-3)\sqrt{3-x^2} + C.$

16. $-\frac{1}{5}(x^2+6)(9-x^2)^{3/2} + C.$

17. $-\dfrac{\sqrt{4-9x^2}}{x} - 3\arcsin \dfrac{3x}{2} + C.$

18. $\frac{1}{2}\ln \dfrac{2-\sqrt{4-x^2}}{|x|} + C.$ 19. $\ln |x + \sqrt{x^2-a^2}| + C.$

20. $\frac{1}{3}(z^2+8)\sqrt{z^2-4} + C.$ 21. $\sqrt{x^2-4} - 2\arccos \dfrac{2}{|x|} + C.$

22. $\frac{1}{5}(x^2+4)(x^2-6)^{3/2} + C.$ 23. $\dfrac{1}{a}\arccos \dfrac{a}{|x|} + C.$

24. $\dfrac{\sqrt{x^2-16}}{16x} + C.$

25. $\dfrac{\sqrt{x^2-8}}{16x^2} + \dfrac{1}{32\sqrt{2}}\arccos \dfrac{2\sqrt{2}}{|x|} + C.$

26. $\frac{1}{6}(2z^2+z-9)\sqrt{3-2z-z^2} - 2\arcsin \dfrac{z+1}{2} + C.$

27. $\frac{1}{45}(2x^3+4)^{3/2}(3x^3-4) + C.$ 28. $\frac{1}{5}(x^2-4)(x^2+6)^{3/2} + C.$

29. $\dfrac{t}{\sqrt{a^2-t^2}} - \arcsin \dfrac{t}{a} + C.$

30. $2\ln (\sqrt{x} + \sqrt{x-4}) - \dfrac{2\sqrt{x-4}}{\sqrt{x}} + C.$

31. $-\frac{4}{15}(8+3\sqrt{x})(4-\sqrt{x})^{3/2} + C.$

32. $e^x - 6\ln (e^x+2) + C.$ 33. $\ln |-2 + \operatorname{tg} \frac{1}{2}x| + C.$

34. $\dfrac{2}{\sqrt{3}}\operatorname{arctg} \dfrac{-1+2\operatorname{tg} \frac{1}{2}x}{\sqrt{3}} + C.$

§ 6.

1. $\ln |\ln x + \sqrt{\ln^2 x - 4}| + C.$
2. $\ln (\ln x + \sqrt{\ln^2 x + 4}) + C.$
3. $\arcsin \dfrac{\ln x}{2} + C.$ 4. $\dfrac{3}{4(4-2x^2)} + C.$
5. $-e^{-\sin x} + C.$ 6. $-\frac{1}{2}\sqrt{1-4x^2} + 2\arcsin 2x + C.$
7. $\frac{2}{3}(3+\ln x)^{3/2} + C.$
8. $-\frac{1}{4}\sqrt{12x-4x^2} + \frac{11}{4}\arcsin \dfrac{2x-3}{3} + C.$
9. $8\ln (x^{1/4}+3) + C.$
10. $-\frac{2}{9}\sqrt{3-6x-9x^2} - \frac{23}{9}\arcsin \dfrac{3x+1}{2} + C.$
11. $-\operatorname{cosec} x + C.$ 12. $\dfrac{2^{\operatorname{arctg} x}}{\ln 2} + C.$
13. $\ln |\sin x + \cos x| + C.$ 14. $-\frac{1}{2}\ln |3 + 4\cos \frac{1}{2}x| + C.$

15. $\frac{1}{\ln 2}\arcsin\frac{2^x}{3}+C.$ **16.** $-\arcsin\frac{\cos x}{4}+C.$

17. $x^2+3\ln|x+2|+C.$

18. $\frac{3}{2}\sqrt{4x^2-4x+10}+\ln(2x-1+\sqrt{4x^2-4x+10})+C.$

19. $\operatorname{tg} x-2\ln|\cos x|+C.$ **20.** $\frac{1}{3}(1+\sin^2 x)^3+C.$

21. $\frac{(2a)^{5x}}{5\ln 2a}+C.$ **22.** $2e^{\sqrt{x}}+C.$

23. $\frac{4}{3}x^3+\frac{1}{2}e^{2x}+4e^x(x-1)+C.$

24. Voor $n\neq -1$: $x^{n+1}\left\{\frac{\ln x}{n+1}-\frac{1}{(n+1)^2}\right\}+C;$

voor $n=-1$: $\frac{1}{2}\ln^2 x+C.$

25. $\ln\frac{\left|e^{\frac{x}{2}}-1\right|}{e^{\frac{x}{2}}+1}+C.$ **26.** $3x\operatorname{tg} x-5\ln|\cos x|+C.$

27. $\frac{1}{4\sqrt{3}}\ln\left|\frac{2\sqrt{x^2-1}-x\sqrt{3}}{2\sqrt{x^2-1}+x\sqrt{3}}\right|+C.$

28. $\frac{1}{\sqrt{3}}\ln\left|\frac{3\sqrt{x^2-1}-(x+1)\sqrt{3}}{3\sqrt{x^2-1}+(x+1)\sqrt{3}}\right|+C.$

29. $\frac{2\sqrt{2x^2+x-1}}{3(x+1)}+C.$

30. $\frac{1}{\sqrt{a^2+b^2}}\ln\left|\frac{b-x+\sqrt{x^2+a^2}-\sqrt{a^2+b^2}}{b-x+\sqrt{x^2+a^2}+\sqrt{a^2+b^2}}\right|+C.$

31. $\frac{1}{5}\ln\left|\frac{2x+1-2\sqrt{x^2+1}}{x-2-\sqrt{x^2+1}}\right|+C.$

32. $-\frac{2\ln x}{\sqrt{x+1}}+2\ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}+C.$

33. $\ln x^2+\ln|2x+5+2\sqrt{x^2+5x+4}|+$

$-2\ln|8+5x+4\sqrt{x^2+5x+4}|+C.$

34. $-\frac{\sqrt{1-x^2}}{2x^2}+\frac{1}{2}\ln\left|\frac{1-\sqrt{1-x^2}}{x}\right|+C.$

35. $2\sqrt{a-x}+a\ln|x|+\sqrt{a}\ln\left|\frac{\sqrt{a-x}-\sqrt{a}}{\sqrt{a-x}+\sqrt{a}}\right|+C.$

36. $3\operatorname{arctg}\sqrt{\frac{1-\cos x}{2+\cos x}}-\sqrt{(1-\cos x)(2+\cos x)}+C.$

37. $\frac{6}{5}x^{5/6}-4x^{1/2}+18x^{1/6}-21\operatorname{arctg}\sqrt[6]{x}+\frac{3\sqrt[6]{x}}{1+\sqrt[3]{x}}+C.$

38. $3\operatorname{arctg}\sqrt[6]{x}-\frac{3\sqrt[6]{x}}{1+\sqrt[3]{x}}+C.$

39. $\frac{5}{2}\arcsin(x-1)-\frac{1}{6}(2x^2+5x+15)\sqrt{2x-x^2}+C.$

40. $-\frac{a}{x}-\frac{\sqrt{a^2-x^2}}{x}-\arcsin\frac{x}{a}+C.$

41. $\frac{x}{\sqrt{1+x^2}}+C.$ **42.** $\frac{x\operatorname{arctg} x}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}+C.$

42. $-\frac{1}{2}\left(\frac{1}{x}+\operatorname{arctg} x+\frac{\operatorname{arctg} x}{x^2}\right)+C.$

43. $-\frac{1}{2}\left(\frac{1}{x}+\operatorname{arctg} x+\frac{\operatorname{arctg} x}{x^2}\right)+C.$

44. $2\sqrt{x}\operatorname{arctg} x+\frac{1}{\sqrt{2}}\ln(x+1+\sqrt{2x})-\sqrt{2}\operatorname{arctg}(1+\sqrt{2x})+$

$-\frac{1}{\sqrt{2}}\ln(1+x-\sqrt{2x})-\sqrt{2}\operatorname{arctg}(-1+\sqrt{2x})+C.$

45. $\sqrt{1+x^2}\,(\ln x - 1) - \ln\dfrac{\sqrt{1+x^2}-1}{x} + C.$

46. $\dfrac{12}{x+5} + \ln\dfrac{(x-1)^2}{(x+5)^2} + C.$ 47. $\ln(x^2+x+1)\,|x-1|^3 + C.$

48. $\dfrac{1}{2(1-x)} + \frac{1}{4}\ln\left|\dfrac{x+1}{x-1}\right| + C.$

49. $-\dfrac{1}{2(1+x)} + \frac{3}{4}\ln|1+x| - \frac{1}{3}\ln(1+x+x^2) - \frac{1}{12}\ln|1-x| + C.$

50. $\frac{1}{2}x^2 - \frac{1}{6}\ln(x^2-x+1) + \frac{1}{3}\ln|x+1| - \dfrac{1}{\sqrt{3}}\operatorname{arctg}\dfrac{2x-1}{\sqrt{3}} + C.$

51. $\dfrac{2x-1}{3(x^2-x+1)} + \dfrac{4\sqrt{3}}{9}\operatorname{arctg}\dfrac{2x-1}{\sqrt{3}} + C.$

52. $-\dfrac{1}{3(x+1)} + \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln(x^2-x+1) +$
$\dfrac{1}{3\sqrt{3}}\operatorname{arctg}\dfrac{2x-1}{\sqrt{3}} + C.$

53. $x + \dfrac{\sqrt{2}}{4}\ln\dfrac{x^2-x\sqrt{2}+1}{x^2+x\sqrt{2}+1} - \dfrac{1}{\sqrt{2}}\operatorname{arctg}(1+x\sqrt{2}) +$
$-\dfrac{1}{\sqrt{2}}\operatorname{arctg}(-1+x\sqrt{2}) + C.$

54. $\dfrac{1}{4a^3\sqrt{2}}\ln\dfrac{x^2+ax\sqrt{2}+a^2}{x^2-ax\sqrt{2}+a^2} + \dfrac{1}{2a^3\sqrt{2}}\operatorname{arctg}\dfrac{ax\sqrt{2}}{a^2-x^2} + C.$

55. $2\sin 2x - \operatorname{tg} x + C.$

56. $\dfrac{2}{5\sqrt{3}}\ln\left|\dfrac{\sqrt{3}\operatorname{tg}\dfrac{x}{2}-1}{\sqrt{3}\operatorname{tg}\dfrac{x}{2}+1}\right| + \dfrac{2}{5\sqrt{3}}\operatorname{arctg}\dfrac{\operatorname{tg}\frac{1}{2}x}{\sqrt{3}} + C.$

57. $\dfrac{1}{n+1}\operatorname{tg}^{n+1}x + C.$

58. $-\frac{1}{3}\ln|1+\operatorname{tg}x| + \frac{1}{6}\ln(\operatorname{tg}^2 x - \operatorname{tg}x + 1) +$
$\dfrac{1}{\sqrt{3}}\operatorname{arctg}\dfrac{-1+2\operatorname{tg}x}{\sqrt{3}} + C.$

59. $\frac{1}{2}e^x(x\sin x - x\cos x + \cos x) + C.$

60. $\sin^2 x\,(-\frac{1}{2} + \ln\sin x) + C.$

61. $\ln\dfrac{|x|}{\sqrt{1+x^2}} - \dfrac{1}{2x^2}\ln(1+x^2) + C.$

62. $2\sqrt{x+1}\ln x - 4\sqrt{x+1} - 2\ln\left|\dfrac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right| + C.$

63. $\dfrac{x}{\sqrt{1-x^2}} + C.$ 64. $\dfrac{x\arcsin x}{\sqrt{1-x^2}} + \ln\sqrt{1-x^2} + C.$

65. $\dfrac{e^{2\operatorname{arctg}x}}{5\sqrt{1+x^2}}(x+2) + C.$

66. $\sqrt{1+x^2}\operatorname{arctg}x - \ln(x+\sqrt{1+x^2}) + C.$

67. $-\sqrt{4-e^{2x}} + C.$ 68. $\operatorname{arctg}(xe^x) + C.$

69. $-\frac{1}{2}\arcsin x\sqrt{1-x^4} + \frac{1}{4}x\sqrt{1+x^2} + \frac{1}{4}\ln(x+\sqrt{1+x^2}) + C.$

70. $\frac{1}{24}\cos 6x - \frac{1}{16}\cos 4x - \frac{1}{8}\cos 2x + C.$

71. $2\ln(\sqrt{x-a} + \sqrt{x-b}) + C.$

72. $-2\ln(\sqrt{a-x} + \sqrt{b-x}) + C.$

73. $2\operatorname{arctg}\sqrt{\dfrac{x-a}{b-x}} + C.$

74. $2\sqrt{1+x} + \sqrt{x(1+x)} - \ln(\sqrt{x} + \sqrt{1+x}) + C.$

16. 24. **17.** $5 \ln 5$. **18.** $\frac{32}{3}$. **19.** $\frac{100}{3}$. **20.** $\frac{\pi}{4}$. **21.** $\frac{\pi}{4}$. **22.** 4.

23. 36. **24.** 1. **25.** π. **26.** $\frac{128}{3}$. **27.** $\frac{3}{2}\ln 2 - \frac{1}{2}\ln 5$. **28.** $\frac{\pi}{2}$ **29.** $\frac{64}{63}$.

30. $\frac{1}{4}\pi a^2$. **31.** 6π. **32.** 5. **33.** 1.

36. $3\sqrt{2} - 6$. **37.** $6 - 2e$. **38.** $\frac{\pi}{4}$. **39.** $18 \ln \frac{3}{2} - 4$. **40.** $1 - \ln 4 + \frac{\pi}{4}$.

41. 2. **42.** $\frac{5}{3} - \ln 4$. **43.** e. **44.** $2\frac{1}{2}\ln 5 - 2$. **45.** $\frac{\pi^2}{72}$. **46.** $3 + \frac{3}{2}\ln 3$.

47. $\frac{\sqrt{3}}{4}\ln 3 - \frac{\pi}{6}$. **48.** $\frac{\pi}{2\sqrt{3}}$. **49.** $-\frac{4}{3}$. **50.** $\ln \frac{1}{2}\sqrt{6}$.

51. 0. **52.** $\ln \frac{3+\sqrt{3}}{2}$. **53.** $\frac{1}{2}\ln \frac{3}{2} + \frac{\pi}{6\sqrt{3}}$.

54. $\frac{n+1}{n}\ln 2 - \frac{\ln(1+2^n)}{n}$. **55.** $1 - \frac{1}{2}\sqrt{2} + \frac{\pi}{4}$. **56.** $\frac{\pi}{8}$.

57. $\frac{4}{3}\sqrt{2} - \frac{2}{3}$. **58.** $3 \ln \frac{-1+\sqrt[3]{2}}{-1+\sqrt[3]{3}}$. **59.** $\frac{(n!)^2}{(2n+1)!}$

60. $\frac{1}{ab}\operatorname{arctg}\frac{b}{a}$. **61.** $\frac{1}{3 \cdot 51 \cdot 52}$. **62.** $\frac{\pi}{8}$. **63.** $\frac{1}{40 \cdot 82}$.

64. $1 - \frac{\pi}{6}\sqrt{3}$. **65.** $\frac{7!80!}{88!}$. **66.** $\frac{13}{6144}$. **67.** $\frac{1}{35}$.

§ 8.

1. 1. **2.** 1. **3.** $\frac{1}{2}$. **4.** 1. **5.** $\frac{\pi}{2}$. **6.** $\ln 2$. **7.** $\frac{\pi}{12}$. **8.** $\frac{10}{e}$.

9. $2(1+\sqrt{3})e^{-\sqrt{3}}$. **10.** $2(1+\sqrt{2})e^{-\sqrt{2}}$. **11.** 1. **12.** $\frac{1}{2} + \frac{\pi}{4}$

15. 2. **16.** -1. **17.** 2 **18.** 3. **19.** 0. **20.** $\frac{\pi}{2}$. **21.** $\ln(2\frac{a}{|a|} + \sqrt{3})$.

22. $\frac{a^2\pi}{4}$. **23.** $\frac{\pi}{2}$. **24.** 1. **25.** π. **26.** 2. **27.** $\sqrt{3} + \frac{1}{2}\ln(2+\sqrt{3})$.

28. 0. **29.** 6. **30.** $\pi + 2\ln(2+\sqrt{3})$.

35. 1°. div.; 2°. conv.; 3°. conv.; 4°. div.

36. $\frac{\pi}{4}$. **37.** 3. **38.** $\frac{1}{e}$. **39.** $\frac{2-\sqrt{2}}{3a^4}$. **40.** $2 - \frac{\pi}{2}$. **41.** 0.

42. $\frac{2}{9}\pi\sqrt{3}$. **43.** $\frac{1}{2}$. **44.** $\frac{1}{\ln^2 2}$. **45.** $\frac{1}{3}\pi\sqrt{3}$ **46.** $\ln \frac{\pi}{4}$.

47. $\ln(4\sqrt{3} - 6)$. **48.** $\ln 2$. **49.** $2 - \ln 4$.

50. $-\frac{\pi}{4} + \ln \frac{\pi}{4\sqrt{2}}$. **51.** $\frac{\pi}{4} - \frac{1}{2}$. **52.** $2\pi - \frac{1}{2}\ln 5 + 4 \operatorname{arctg} 2$.

53. $\frac{\pi}{4} - \frac{1}{2}$. **54.** $\frac{1}{\sqrt{5}}\ln \frac{3+\sqrt{5}}{2}$. **55.** $\frac{\pi}{|b|}$. **56.** $\frac{n}{n^2-1}$. **57.** $\frac{3}{8}$.

58. $\frac{\pi}{\sqrt{a+b}}$. **59.** $\frac{1}{\sqrt{3}}$. **60.** $\frac{1}{2}$. **61.** $\frac{7}{9}$. **62.** π. **63.** $\frac{1}{2}\pi(a+b)$.

64. $\frac{\pi}{2ab(a+b)}$. **65.** $\frac{\pi}{2(a+b)}$. **66.** $n!$ **67.** $\frac{\pi}{4}$. **68.** $-\frac{11}{18}$.

69. $-1 + \ln 2$. **70.** $-\ln 4$. **71.** $\frac{2\pi}{ab}$. **72.** $-1 + \ln 4$.

73. 1°: $\frac{b}{a^2+b^2}$; 2°: $\frac{a}{a^2+b^2}$; 3°: $\frac{2b(3a^2-b^2)}{(a^2+b^2)^3}$.