17.07.08 VERSIONE A

$0<\frac{8}{5n^2}<\varepsilon \Leftrightarrow n>\sqrt{\frac{8}{5\varepsilon}}$

1a) $\lim_{n\to+\infty} a_n = \lim_{n\to\infty}\left(\frac{8}{5n^2}-\frac{3}{5}\right) = -\frac{3}{5}$ se $n>\sqrt{\frac{8}{5\varepsilon}}$, allora $\left|a_n+\frac{3}{5}\right|<\varepsilon$

b) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{5+\frac{7}{n^2}+\frac{1}{n^4}}{\frac{3}{n^3}+4+\frac{6}{n^4}} = \frac{7}{4}$

2a) $\left[-x^2+\sqrt{x^4+1}\right](x^2+1) = \frac{-x^4+(x^4+1)}{x^2+\sqrt{x^4+1}}(x^2+1) = \frac{x^2+1}{x^2+x^2\sqrt{1+\frac{1}{x^4}}} \to \frac{1}{2},\ x\to+\infty$

2b) $\lim_{x\to-3}\frac{(x+3)\,tg(x+3)}{\ln^2(x+4)} = \lim_{x\to-3}\frac{tg(x+3)}{x+3}\left[\frac{x+3}{\ln(x+4)}\right]^2$

$= \lim_{z\to 0}\frac{tg\, z}{z}\left[\frac{z}{\ln(1+z)}\right]^2 = 1$

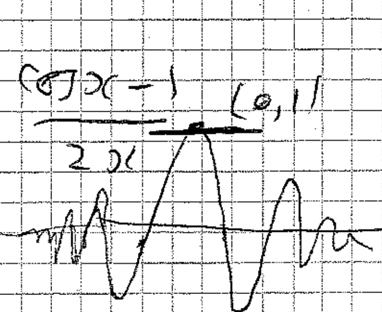
3a) $f'(x) = \frac{-1}{\sqrt{1-(\sqrt{1-x^2})^2}}\ \frac{-2x}{2\sqrt{1-x^2}} = -\frac{x}{|x|}\ \frac{1}{\sqrt{1-x^2}}\ \overset{\forall x>0}{\downarrow}\ \frac{-1}{\sqrt{1-x^2}}$

$\hookrightarrow f'\left(\frac{1}{2}\sqrt{2}\right) = -\sqrt{2}$ $\quad f\left(\frac{1}{2}\sqrt{2}\right) = \frac{\pi}{4}$

$y-\frac{\pi}{4} = -\sqrt{2}\left(x-\frac{1}{2}\sqrt{2}\right)$

3b) $f'(0) = \lim_{x\to 0}\frac{\frac{\sin(x)}{x}-1}{x} = \lim_{x\to 0}\frac{\sin x - x}{x^2} = \lim_{x\to 0}\frac{\cos x - 1}{2x}$

$= \lim_{x\to 0}\frac{-\sin x}{2} = 0$ $\qquad y=1$



4) $f(x) = 16-[x(x+4)]^2 = [4+x(x+4)][4-x(x+4)]$

$= -(x+2)^2(x+2-2\sqrt{2})(x+2+2\sqrt{2})$

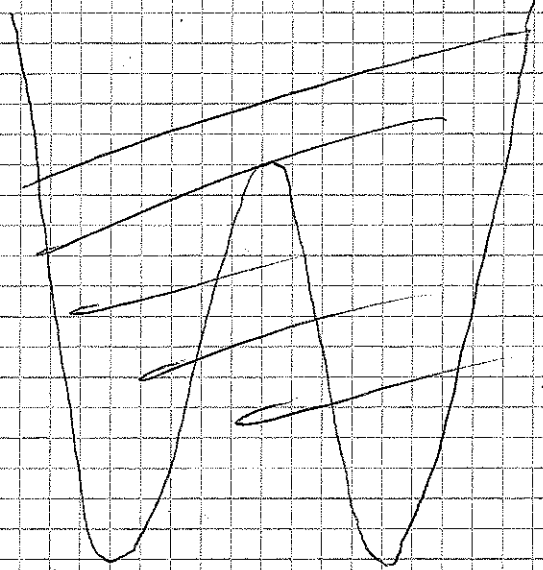
$f(x)$	$-$	$-2-2\sqrt{2}$	$+$	-2	$+$	$-2+2\sqrt{2}$	$-$

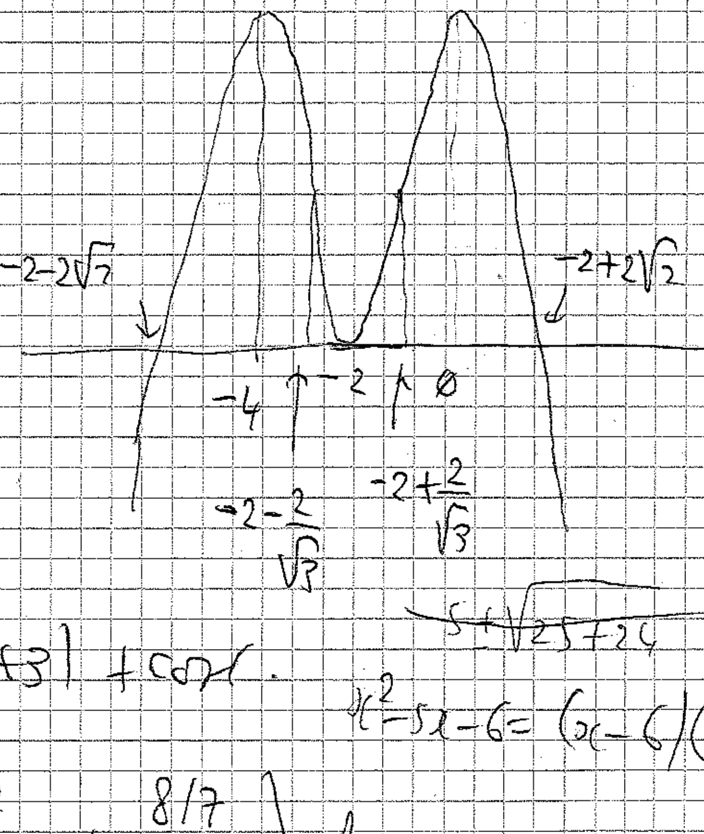
$$f'(x) = -2x(x+4)\frac{d}{dx}\{x(x+4)\} = -2x(x+4)(2x+4)$$
$$= -4x(x+2)(x+4) = -4\{x^3+6x^2+8x\}$$

$$\begin{array}{cccccccc} + & & - & & + & & - & f'(x) \\ & -4 & & -2 & & 0 & & \\ \text{cresc} & \text{MAX} & \text{decr} & \text{MIN} & \text{cresc} & \text{MAX} & \text{decr} & f(x) \end{array}$$

$$f''(x) = -4\{3x^2+12x+8\} = -12(x+2)^2+16$$
$$= -12\left[x+2+\frac{2}{\sqrt{3}}\right]\left[x+2-\frac{2}{\sqrt{3}}\right]$$

$$\begin{array}{cccccc} - & -2-\frac{2}{\sqrt{3}} & + & -2+\frac{2}{\sqrt{3}} & - & f''(x) \\ \text{concava} & & \text{convessa} & & \text{concava} & f(x) \end{array}$$





5) $$\int \frac{dx}{\cos^2(2x+3)} = \frac{1}{2}\,tg(2x+3) + cost.$$

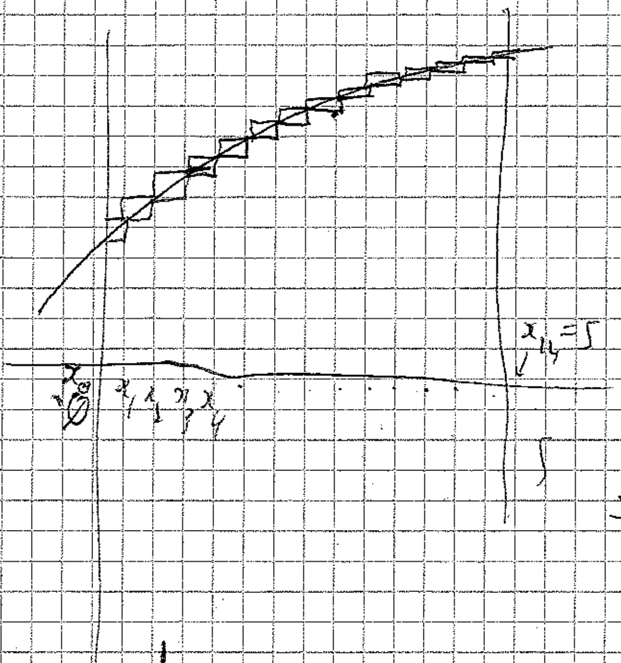
$$5+\sqrt{25+24}$$

$$x^2-5x-6 = (x-6)(x+1)$$

$$\int \frac{3x-5}{x^2-5x-6}\,dx = \int\left(\frac{13/7}{x-6} + \frac{8/7}{x+1}\right)dx$$
$$= \frac{13}{7}\ln|x-6| + \frac{8}{7}\ln|x+1| + cost.$$

$$\int \frac{dx}{\sqrt[5]{3x+1}} = \frac{5}{12}(3x+1)^{\frac{4}{5}} + cost.$$

6) Sia $f:[a,b]\to\mathbb{R}$ continua e sia $F(x)=\int_a^x f(t)\,dt$.
allora $F:[a,b]\to\mathbb{R}$ è derivabile e $F'(x)=f(x)$



$0=x_0<x_1<\dots<x_n=5$

$M_j=\max\{f(x):x_{j-1}\le x\le x_j\}$

$m_j=\min\{f(x):x_{j-1}\le x\le x_j\}$

$$\underbrace{\sum_{j=1}^{n} m_j(x_j-x_{j-1})}_{S_j}\le\int_0^5 f(t)\,dt\le\underbrace{\sum_{j=1}^{N} M_j(x_j-x_{j-1})}_{S_j^+}$$

7) $\int_\varepsilon^1 \ln(1/x)\,dx=\left[x\ln(1/x)\right]_\varepsilon^1-\int_\varepsilon^1 x\cdot\frac{-1}{x}\,dx=$

$=-\varepsilon\ln(1/\varepsilon)+(1-\varepsilon)\longrightarrow 1,\ \varepsilon\to 0^+$ convergente

$-\int_0^a xe^{-5x}\,dx=\left[-\frac{1}{5}xe^{-5x}\right]_0^a+\int_0^a\frac{1}{5}e^{-5x}\,dx$

$=\left[-\frac{1}{5}xe^{-5x}-\frac{1}{25}e^{-5x}\right]_0^a=-\left(\frac{1}{5}a+\frac{1}{25}\right)e^{-5a}+\frac{1}{25}\to\frac{1}{25}$ se $a\to+\infty$ convergente

VERSIONE B

1a) $\lim_{n\to\infty}a_n=\lim_{n\to\infty}\left(\frac{6}{4n^2}-\frac{5}{4}\right)$

$\left|\frac{5}{4}-a_n\right|=\frac{6}{4n^2}<\varepsilon\Leftrightarrow n>\sqrt{\frac{6}{4\varepsilon}}$

se $n>\sqrt{\frac{6}{4\varepsilon}}$, allora $\left|\frac{5}{4}-a_n\right|<\varepsilon$

1b) $\lim_{n\to\infty}a_n=\lim_{n\to\infty}\dfrac{5+\frac{1}{n}+\frac{10}{n^4}}{\frac{2}{n^4}+\frac{3}{n^3}+8}=\frac{5}{8}$

2a) $\left[-x^2+\sqrt{x^4+4}\right](x^2+1) = \dfrac{-x^4+(x^4+4)}{x^2+\sqrt{x^4+4}}(x^2+1)$

$$= \frac{4(x^2+1)}{x^2+x^2\sqrt{1+\frac{4}{x^4}}} \to 2, \quad x\to+\infty$$

2b) $$\lim_{x\to3}\frac{(x-3)\operatorname{arctg}(x-3)}{\ln^2(x-2)} = \lim_{x\to3}\frac{\operatorname{arctg}(x-3)}{x-3}\left[\frac{x-3}{\ln(x-2)}\right]^2$$

$$= \lim_{z\to0}\frac{\operatorname{arctg} z}{z}\left[\frac{z}{\ln(1+z)}\right]^2 = 1$$

3a) $f\left(\frac{1}{2}\sqrt{2}\right) = \frac{\pi}{4}$

$$f'(x) = \frac{1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}\,\frac{-2x}{2\sqrt{1-x^2}} = -\frac{x}{|x|}\,\frac{1}{\sqrt{1-x^2}} = -\frac{1}{\sqrt{1-x^2}} \quad \text{se } x>0$$

$f'\left(\frac{1}{2}\sqrt{2}\right) = -\sqrt{2}$ $\qquad y-\frac{\pi}{4} = -\sqrt{2}\left(x-\frac{1}{2}\sqrt{2}\right)$

3b) Come la versione A

4) $f(x) = \left[4+x(x-4)\right]\left[4-x(x-4)\right]$

$= (x-2)^2\left(x-2-2\sqrt{2}\right)\left(x-2+2\sqrt{2}\right)$

		$2-2\sqrt{2}$		2		$2+2\sqrt{2}$	
$f(x)$	$+$		$-$		$+$		$-$

$$f'(x) = -2x(x-4)\frac{d}{dx}\{x(x-4)\} = -4x(x-4)(x-2)$$

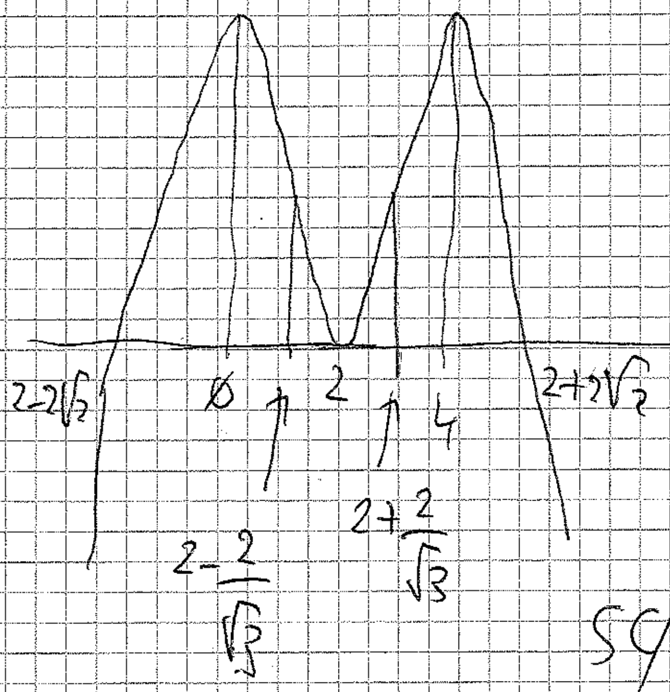
		0		2		4	
$f'(x)$	$+$		$-$		$+$		$-$
$f(x)$	cresc.	MAX	decr.	MIN	cresc.	MAX	decr.

$$f''(x) = -4\frac{d}{dx}\left(x^3-6x^2+8x\right) = -4\left(3x^2-12x+8\right)$$

$$= -12(x-2)^2+16 = -12\left(x-2-\frac{2}{\sqrt{3}}\right)\left(x-2+\frac{2}{\sqrt{3}}\right)$$

$-$ $\quad 2+\frac{2}{\sqrt{3}}$ $\quad +$ $\quad 2-\frac{2}{\sqrt{3}}$ $\quad -$ $\quad f''(x)$

concava | convessa | concava



5a) $\displaystyle\int \frac{dx}{\cos^2(3x+1)} = \frac{1}{3}\,tg(3x+1) + cost$

5b) $\displaystyle\int \frac{4-3x}{x^2+5x-6} = \int\left(\frac{-22/7}{x+6} + \frac{1/7}{x-1}\right)dx$

$\displaystyle = -\frac{22}{7}\ln|x+6| + \frac{1}{7}\ln|x-1| + cost$

$A+B=-3$

$-A+6B=4$

5c) $\displaystyle\int \frac{dx}{\sqrt[6]{3x+1}} = \frac{2}{5}(3x+1)^{5/6} + cost.$

6) → Versione A

7a) Versione A

7b) $\displaystyle\int_0^a x e^{-4x}\,dx = \left[-\frac{1}{4}xe^{-4x}\right]_0^a + \int_0^a \frac{1}{4}e^{-4x}\,dx$

$\displaystyle = \left[-\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x}\right]_0^a = -\left(\frac{1}{4}a + \frac{1}{16}\right)e^{-4a} + \frac{1}{16} \to \frac{1}{16},\ a\to+\infty$

Convergente