

Esercizi sulle equazioni differenziali

Equazioni Separabili del tipo $u' = g(x)u$

1. $u' + u = 0$;
2. $u' - 4u = 0$;
3. $u' + x^2u = 0$;
4. $u' - (u/x) = 0$ per $x > 0$;
5. $u' = (1 - x^2)^{-1/2}u$;
6. $u' = \frac{3x - 2}{x^2 - 2x - 8}u$ per $-2 < x < 4$;
7. $u' = \frac{2x + 1}{x^2 - 2x - 1}u$ per $x < 1$;
8. $u' = (\ln x)u$ per $x > 0$;
9. $u' + 2u = 0, u(0) = 4$;
10. $u' = (u/2), u(2) = -1$;
11. $u' = \frac{x}{x^2 + 4x - 5}u, u(0) = 1$;
12. $u' + x(\ln(2x))u = 0, u(1/2) = 0$;
13. $u' + \sqrt{1 + 2x}u = 0, u(4) = 10$;
14. $u' = \frac{1}{x(1 - x)}u, u(1/4) = 1$.

Equazioni Lineari del Primo Ordine

15. $u' + 2u = x$;
16. $u' - u = \text{sen}(2x)$;
17. $u' + 2xu = 4x$;
18. $x^2u' + xu + 1 = 0$;
19. $xu' - 4u = x + 3x^2$;
20. $u' + (\cotg x)u = 3\text{sen } x \cos x$;
21. $xu' + \frac{1}{\ln x}u = 1$ per $x > 0$;
22. $u' = \frac{1}{\cos x}u, u(\pi/4) = (1/\sqrt{2})$;
23. $u' + |x|u = 0, u(0) = 4$;
24. $u' + u = 2x + 5, u(0) = 4$;
25. $u' + 4u = 6 \text{sen}(2x), u(0) = -3/5$;
26. $u' - 2u + e^{2x} = 0, u(0) = 2$;
27. $u' - 2u = x^2e^{2x}, u(0) = 2$;
28. $u' - 2xu - x = 0, u(0) = 1$;
29. $e^u u' + e^u = 4 \text{sen } x, u(0) = 0$ (Consiglio: Sia $v = e^u$);
30. $(x^2 + 1)u' - 2xu = x^2 + 1, u(1) = \pi$;
31. $u' - (\text{tg } x)u = e^{\text{sen}(x)}, 0 < x < \pi/2, u(1) = 0$;
32. $u' + 2xu = e^{-x^2}, u(0) = 1$;
33. $u' - 2u = x^2, u(0) = 2$;
34. $u' + (2u/x) = (\cos x)/x^2, u(\pi) = 0, x > 0$;
35. $u' + 3u = 6x$;
36. $u' - u = 4e^{2x}$;

$$37. u' - (2u/x) = \text{sen } x;$$

$$38. u' - \frac{x}{x^2 + 1}u = 1;$$

$$39. u' - (\text{tg } x)u = 1;$$

$$40. u' + (u/x) = 2 + (1/x^2).$$

Equazioni separabili

$$41. u' = (x^2/u);$$

$$42. u' = x^2\sqrt{u}/(1 + x^2);$$

$$43. u' = e^{x+u};$$

$$44. u' = \frac{x - xu^2}{u + x^2u};$$

$$45. xu' = u^2 - 3u + 2;$$

$$46. u' = \frac{x + 1}{u^4 + 1};$$

$$47. u' = xe^x/(2u);$$

$$48. u' = 1/(\text{tg } x \cos^2 u);$$

$$49. x^2uu' = u - 1;$$

$$50. u' = \frac{\ln x \cos u}{x \text{sen } 2u};$$

$$51. u' = (\text{sen } x)/u, u(0) = -1;$$

$$52. u' = x^2e^{-u}, u(0) = 2;$$

$$53. u' = 1 - u^2, u(0) = 0;$$

$$54. u' = (u^2 + u)/x, u(1) = -1;$$

$$55. u' = \frac{3x^2 + 2}{2(u - 1)}, u(0) = -1;$$

$$56. u' = \sqrt{1 - u^2}, u(\pi/2) = 0;$$

$$57. x^2 u' - \frac{1}{2} \cos 2u = \frac{1}{2}, u(1) = \pi/4;$$

$$58. u' = xu^4 \sqrt{1 + 3x^2}, u(1) = -1;$$

$$59. u' = -3u^{4/3} \text{sen } x, u(\pi/2) = (1/8);$$

$$60. u' + \frac{4u}{1-x^2} = 0, u(2) = 9.$$

Equazioni Lineari Omogenee del Secondo Ordine

$$61. u'' + 5u' + 6u = 0;$$

$$62. u'' - 10u' + 25u = 0;$$

$$63. u'' + 2u' + 5u = 0;$$

$$64. u'' - 2u' + 5u = 0;$$

$$65. u'' + 3u' - 10u = 0;$$

$$66. u'' + 9u = 0.$$

Equazioni Lineari Non Omogenee del Secondo Ordine

$$67. u'' - u' - 2u = x;$$

$$68. u'' - 4u = 6e^x;$$

$$69. u'' - 2u' + u = 4 \text{ sen } x;$$

$$70.* u'' + 5u' + 4u = 2e^{-x};$$

$$71. u'' = x^2 + x + 1;$$

$$72. u'' + 2u' + 5u = 10x^2 - e^{-2x};$$

$$73.* u'' + 4u = x - 2 \text{ sen } 2x;$$

$$74.* u'' + u = x \cos 2x;$$

75. $u'' + u' - 4u = 2 \operatorname{senh} x$;
76. $u'' + u = 8 \cos^3 x$;
77. $u'' - u' - 2u = x, u(0) = 0, u'(0) = 1$;
78. $u'' - 4u = 6e^x, u(0) = 1, u'(0) = 2$;
79. $u'' - 2u' + u = 4 \operatorname{sen} x, u(0) = 1, u'(0) = 0$;
- 80.* $u'' + 5u' + 4u = 2e^{-x}, u(0) = 1, u'(0) = 1$;
- 81.* $u'' + 4u = x - 2 \operatorname{sen} 2x, u(\pi) = 0, u'(\pi) = 1$;
82. $u'' + u = 8 \cos^3 x, u(\pi/6) = 0, u'(\pi/6) = 0$.
83. $u'' - u = 2e^{3x}$;
84. $u'' + 9u = 3x + 2$;
85. $u'' - 7u' + 12u = 5e^{4x}$;
- 86.* $u'' - 4u = 3e^{-2x} - x^2$;
- 87.* $u'' + 4u = 4 \cos^3 2x$.
88. $u'' - u = 2e^{3x}, u(0) = 1, u'(0) = 2$;
89. $u'' + 9u = 3x + 2, u(0) = 0, u'(0) = 1$.
- 90.* $u'' + \omega^2 u = f(x), u(0) = u'(0) = 0$.
91. $u'' + 4u' + 13u = 6 \cos 2x - 5 \operatorname{sen} 2x$;
92. $u'' + 3u' + 2u = \cos 3x + 2 \operatorname{sen} 3x$;
93. $u'' + 4u' + 5u = 7 \cos x + 8 \operatorname{sen} x$;
94. $4u'' + 12u' + 9u = 3 \cos x$;
95. $3u'' + 13u' + 4u = 4 \cos 2x - \operatorname{sen} 2x$;
96. $u'' + cu' + u = A \cos x, c = 2, 0.2, 0.002$;
97. $3u'' + 12u = 5 \cos 3x$;

98. $4u'' + 36u = 3 \operatorname{sen} 2x$;

99. $u'' + \pi^2 u = 2 \operatorname{sen} (22x/7)$;

100.* $u'' + u = A \cos(1 + \epsilon)x$, $\epsilon = 1, 0.1, 0.001$.

Equazioni Lineari Omogenee di Ordine $n \geq 3$

101. $u''' + 3u'' + 2u' = 0;$

102. $u''' + 10u'' + 29u' = 0;$

103. $u''' - 6u'' + 9u' = 0;$

104. $u''' + 8u = 0;$

105. $u''' + 13u'' = 0;$

106. $u''' - 27u = 0;$

107. $u^{(4)} + 5u''' + 6u'' = 0;$

108. $u^{(4)} - 8u''' + 17u'' = 0;$

109. $u^{(4)} + 3u''' + 3u'' + u' = 0;$

110. $u^{(5)} - 3u^{(4)} + 3u''' - u'' = 0;$

111. $u^{(4)} - 5u'' + 4u = 0;$

112. $u^{(4)} + 5u'' + 4u = 0;$

113. $u^{(4)} + 6u'' + 9u = 0;$

114. $u^{(5)} + 8u''' + 16u' = 0;$

115. $u^{(4)} - 81u = 0;$

116. $u^{(5)} + u' = 0;$

117. $u^{(4)} - 6u'' + 9u = 0;$

118. $u^{(4)} + 6u''' + 13u'' = 0;$

119. $u^{(6)} - 64u = 0;$

120. $u^{(6)} + u = 0.$